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(9660/MA01) Unit P1 Pure Mathematics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)(i)	$16a^{\frac{11}{6}}$	**B1**	
		1	

Q	Answer	Marks	Comments
1(a)(ii)	$2a^{\frac{5}{12}}$	**B1**	
		1	

Q	Answer	Marks	Comments
1(b)(i)	$\left[500\times 5^{p}\times\right]\ x^{2p+6}$	**M1**	Correctly applies index rules to obtain a correct single power of x Could be seen embedded in a product. Ignore terms in their product that do not include x **PI** by $2p+6$ or $2p=-6$ or correct answer seen.
	$[p=]\ -3$	**A1**	**CAO**
		2	

Q	Answer	Marks	Comments
1(b)(ii)	$\left[\frac{500}{5^3}=\frac{500}{125}\right]=4$	**B1**	**CAO**
		1	

	Question 1 Total	**5**	

Q	Answer	Marks	Comments
2(a)	$\left[\,\lvert QR\rvert =\right]\ \sqrt{(4-14)^2+(9-(-3))^2}$	M1	**oe** **PI** by $15.6[2049...]$ or correct final answer
	$\sqrt{244}$ or $2\sqrt{61}$	A1	**ISW** Ignore decimal value if given as well.
		2	

Q	Answer	Marks	Comments
2(b)	$[\text{Mid-Point of } QR =]\ (9, 3)$	B1	**PI** in later working.
	$[\text{Gradient of } QR =]\ \dfrac{9-(-3)}{4-14}$	M1	**oe** Correct method for finding the gradient of QR **PI** by $-\dfrac{6}{5}$ **oe** seen.
	$[\text{Gradient of } l =]\ \dfrac{5}{6}$	A1ft	**oe** Possibly seen in later working. **ft** their gradient of QR
	$\dfrac{y-3}{x-9}=\dfrac{5}{6}$ **oe** and $y=\dfrac{5}{6}x-\dfrac{9}{2}$	A1	Forms a correct equation for l before the given answer. May see $y=\dfrac{5}{6}x+p$ and substitution of coordinates of the mid-point of QR to find p but must be a complete method. **AG** Must be convincingly shown
		4	

Q	Answer	Marks	Comments
2(c)	$[k =]\ \frac{5}{6}\times 30-\frac{9}{2}$	M1	Substitutes $x = 30$ into the equation of l **PI** by correct value of k
	$[k =]\ \frac{41}{2}$ or 20.5	A1	**CAO**
	$\left[20.5=\frac{1}{4}\times 30+d\right]$		
	$[d =]\ 13$	B1ft	**ft** follow through their $k-7.5$ Substitutes their k into the equation of the line and evaluates d Condone equivalent fraction
2(c) ALT	$\left[\frac{5}{6}x-\frac{9}{2}=\frac{1}{4}x+d \text{ and } x=30\Rightarrow\right]$ $\frac{5}{6}(30)-\frac{9}{2}=\frac{1}{4}(30)+d$ or $\frac{7}{12}(30)=d+\frac{9}{2}$	M1	**oe** Equates equations of both lines and $x = 30$ substituted into a correct equation. **PI** by correct value of d
	$[d =]\ 13$	A1	**CAO**
	$[k =]\ \frac{41}{2}$ or 20.5	B1ft	**ft** follow through their $7.5+d$ Substitutes their d into the equation of the line and evaluates k Condone equivalent fraction
		3	

	Question 2 Total	9	

Q	Answer	Marks	Comments
3(a)	$\left[S_{30}=\right]\ \frac{1}{2}\times30\times\left(2a+\left(30-1\right)d\right)$ $\left[=30a+435d\right]$ or $\left[S_{10}=\right]\ \frac{1}{2}\times10\times\left(2a+\left(10-1\right)d\right)$ $\left[=10a+45d\right]$	M1	**oe** Could be embedded. Correct expression for S_{30} or S_{10} with values substituted simplified or unsimplified.
	$\frac{1}{2}\times30\times\left(2a+\left(30-1\right)d\right)$ $-\frac{1}{2}\times10\times\left(2a+\left(10-1\right)d\right)$ $\left(30a+435d\right)-\left(10a+45d\right)\ \left[=522\right]$	M1	**oe** Correct expression for $S_{30}-S_{10}$
	$20a+390d=522$ and $10a+195d=261$	A1	Integer multiple of final answer, before given answer **AG** Must be convincingly shown
		3	

Q	Answer	Marks	Comments
3(b)	$a+(36-1)d$ $[=a+35d]$ or $5(a+(9-1)d)+27$ $[=5a+40d+27]$	M1	**PI oe** Correct expression for u_{36} or $5u_9+27$ simplified or unsimplified. Could be embedded.
	$a+(36-1)d=5(a+(9-1)d)+27$ or $a+35d=5a+40d+27$ or $4a+5d=-27$	M1	**oe** Correct equation for $u_{36}=5u_9+27$ in terms of a and d **PI** by a correct value for a or d
	$10a+195d=261$ $4a+5d=-27$	M1	Solves simultaneously with at least one of a or d correct.
	$a=-9$ and $d=\frac{9}{5}$	A1	Both a and d correct.
	$[u_n=]\ \frac{9}{5}n-\frac{54}{5}$	A1ft	**CAO** **ft** their values for a and d Correct expression in the correct form. Accept equivalent fractions or decimals.
		5	

Q	Answer	Marks	Comments
3(c)	$\frac{9}{5}n-\frac{54}{5}<140$	M1	**oe** Correct inequality. Accept given as equality. Condone $\le$ for $<$ **ft** their $\frac{9}{5}n-\frac{54}{5}$ from **part 3(b)**. **PI** by $\frac{754}{9}$ or $83.7(777...)$ or correct final answer
	$[n=]$ 83	A1	**CAO**
		2	

	Question 3 Total	**10**	

Q	Answer	Marks	Comments
4(a)	$\left[(1+6x)^7 =\right]$ $\left[(1)^7\right]+7(1)^6(6x)+21(1)^5(6x)^2\left[+35(1)^4(6x)^3\right]$	M1	For either [1], 7, 21, [35] **oe** unsimplified. or $\binom{7}{1}(1)^6(6x)$ or $\binom{7}{2}(1)^5(6x)^2$ **oe** x not needed. **PI**
	$[a =]$ 42	A1	Condone $42x$ Possibly embedded in expansion.
	$[b =]$ 756	A1	Condone $756x^2$ Possibly embedded in expansion.
		3	

Q	Answer	Marks	Comments
4(b)	$\frac{1}{2}\times 7560\left[x^3\right]$ or and $(-k)\times 756\left[x^3\right]$ $3780\left[x^3\right]$	M1	**ft** their b from **part 4(a)**. Multiplying together two relevant pairs of terms. Condone if seen embedded in a full or partial expansion.
	$(3780-756k)\left[x^3\right]=1512\left[x^3\right]$	M1	**oe** Correct equation.
	$[k =]$ 3	A1	**CAO**
		3	

	Question 4 Total	**6**	

Q	Answer	Marks	Comments
5(a)	$h = 0.4$	B1	**PI**
	$\left[\text{With } \mathrm{f}(x) = 8^{\sqrt{x}}\right]$ $\left[\mathrm{I} \approx \frac{h}{2}\{...\}\right]$ $\left[\{...\} =\right]\ \mathrm{f}(1) + \mathrm{f}(3)$ $+ 2\big(\mathrm{f}(1.4) + \mathrm{f}(1.8) + \mathrm{f}(2.2) + \mathrm{f}(2.6)\big)$	M1	**oe** Summing the areas of the trapezia.
	$\left[\{...\} =\right]\ 8 + 36.6604...$ $+ 2\times(11.7098... + 16.2787...$ $+ 21.8523... + 28.5883...)$	A1	**oe** Accept rounded or truncated to two decimal places. **PI** by **AWRT** 40.3
	$\left[\mathrm{I} \approx 0.2 \times 201.5191... =\right]\ 40.3$	A1	**CAO** Must be 40.3
		4	

Q	Answer	Marks	Comments
5(b)(i)	$\left[8^{\left(\frac{1}{3}+\sqrt{x}\right)} =\right]\ 8^{\frac{1}{3}}\left[\times 8^{\sqrt{x}}\right]$ or $2\left[\times 8^{\sqrt{x}}\right]$	B1	**PI** by correct scale factor of stretch.
	Stretch in the y-direction.	E1	Both ‘stretch’ and ‘direction’ needed.
	[Scale] factor 2	E1	Accept ‘sf’. Allow $8^{\frac{1}{3}}$ for 2
		3	

Q	Answer	Marks	Comments
5(b)(ii)	$\left[\int_1^3 8^{\left(\frac{1}{3}+\sqrt{x}\right)}\mathrm{d}x = 2\int_1^3 8^{\sqrt{x}}\,\mathrm{d}x \approx\right]\ 2\times 40.3$	M1	Their trapezium rule value multiplied by a scale factor **PI** by 80.6 or 2 × part **(a)** trapezium rule value but not 80.2 (from calculator use)
	80.6	A1	**CAO** Second use of trapezium rule is **M0 A0**
		2	

	Question 5 Total	**9**	

Q	Answer	Marks	Comments
6(a)	$[\mathrm{f}(4)=]\ 4^3+a\times 4^2-6b\times 4+7$ $64+16a-24b+7=23$ $71+16a-24b=23$ **oe** and $2a-3b=-6$	M1 A1	Correctly substitutes $x=4$ into $\mathrm{f}(x)$ Must use the Remainder Theorem. **AG** Must be convincingly shown Expression for $\mathrm{f}(4)$ set equal to 23 with products and powers evaluated and **AG** Must be at least one extra line of working given before **AG**
		2	

Q	Answer	Marks	Comments
6(b)	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]\ 3x^2+2ax-6b$ $3(-5)^2+2a(-5)-6b=21$ $[75-10a-6b=21]$ $5a+3b=27$	M1 m1 A1	Condone one error in a term or one term omitted. Substitutes $x=-5$ into their derivative and sets equal to 21 **CAO** **oe** must be in the correct form
		3	

Q	Answer	Marks	Comments
6(c)	$[a=]\ 3$ and $[b=]\ 4$	B1	**CAO**
		1	

Q	Answer	Marks	Comments
6(d)	$[\mathrm{g}'(x) =]\ 48 + 2x - x^2$	M1	Allow one error in a term or one term omitted.
	$48 + 2x - x^2 > 0$	M1	**PI** correct inequality or correct critical values. Condone given as equality. **ft** their $\mathrm{g}'(x)$
	$[x =]\ -6$ and $[x =]\ 8$	A1	Both correct critical values.
	$-6 < x < 8$	A1ft	Correct solution to $\mathrm{g}'(x) > 0$ **PI** by both correct intervals in final answer. **ft** their two critical values.
	$-6 < x < -4$ or $2 < x < 8$	M1 A1	**M1**: One correct interval. Ignore other incorrect intervals given. **A1**: Both correct intervals and no others. Do not condone 'and' for 'or'.
		6	

	Question 6 Total	12	

Q	Answer	Marks	Comments
7(a)	$[y=]\ x^2-6x^{\frac{4}{3}}+16$	B1	Correct expansion. **PI** by correct derivative.
	$\left[\dfrac{dy}{dx}=\right]\ 2x-8x^{\frac{1}{3}}$	B1ft	**oe** Simplified or unsimplified. **ft** their expansion provided it contains a fractional power of x
		2	

Q	Answer	Marks	Comments
7(b)	$2x-8x^{\frac{1}{3}}=0$	M1	**oe ft** their first derivative equal to zero.
	$2x\left(1-4x^{-\frac{2}{3}}\right)=0 \Rightarrow x=0$ When $x=0,\ y=16$ and $(0, 16)$	A1	Statement that $x=0$ from correct first derivative Correct coordinates of *P* Condone not given as coordinates but must be clearly identified.
	$x^{\frac{2}{3}}-4=0$ or $1-4x^{-\frac{2}{3}}=0$ or $x^2=64$	M1	**oe PI**
	$[x_Q=]\ 8$	A1	Correct x-coordinate of *Q*
	$(8, -16)$	A1	Correct coordinates of *Q*
		5	

Q	Answer	Marks	Comments
7(c)(i)	$\left[\dfrac{d^2y}{dx^2}=\right]\ 2-\dfrac{8}{3}x^{-\frac{2}{3}}$	B1ft	**oe ft** their $\dfrac{dy}{dx}$ provided it contains a fractional power of x
		1	

Q	Answer	Marks	Comments
7(c)(ii)	$\left[\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2 - \dfrac{8}{3} \times 8^{-\frac{2}{3}} = \right]$ $\dfrac{4}{3}$ and Since $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0$ then it is a minimum.	**E1ft**	**oe** Evaluates second derivative with $x = 8$ and gives statement linking positive value of second derivative to it being a minimum. Accept 1.33 or better for $\dfrac{4}{3}$ **ft** their second derivative and their x-coordinate of Q provided the value of the second derivative is positive.
		1	

Q	Answer	Marks	Comments
7(d)(i)	[Substituting $x = 0$ into the second derivative would give $2 - \dfrac{8}{0}$ and] division by zero is not possible.	**E1**	Be convinced.
		1	

Q	Answer	Marks	Comments
7(d)(ii)	$\left[x = -0.1 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \right]$ $3.5[1327...]$ and $\left[x = 0.1 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \right]$ $-3.5[1327...]$	**B1**	Both correct values rounded to 1 dp or better.
	Since the gradient is positive [close to and] to the left of P but negative [close to and] to the right of P then P is a maximum.	**E1**	Correct explanation comparing signs of the gradient or behaviour of the function, and deduction that P is a maximum must be seen.
		2	

	Question 7 Total	**12**	

Q	Answer	Marks	Comments
8	$\left[\frac{6x+5x^2}{x^2\sqrt{x}} = \frac{6}{x\sqrt{x}} + \frac{5}{\sqrt{x}} = \right]\ 6x^{-\frac{3}{2}} + 5x^{-\frac{1}{2}}$	B1	Correctly written as a sum of powers of x **PI** by correct integration
	$\left[\int \frac{6x+5x^2}{x^2\sqrt{x}}\,dx = \right]$ $-12x^{-\frac{1}{2}} + 10x^{\frac{1}{2}}\,[+c]$	B2,1ft	**oe ft** their $6x^{-\frac{3}{2}} + 5x^{-\frac{1}{2}}$ provided each term they integrate has a fractional powers of x **B2** both terms correct or **B1** for one term correct. Simplified or unsimplified.
	$\left[\int_{a^2}^{25a^2} \frac{6x+5x^2}{x^2\sqrt{x}}\,dx = \right]$ $\left(-12(25a^2)^{-\frac{1}{2}} + 10(25a^2)^{\frac{1}{2}}\right) - \left(-12(a^2)^{-\frac{1}{2}} + 10(a^2)^{\frac{1}{2}}\right)\ [=44]$	M1	Forms $\mathrm{F}(25a^2) - \mathrm{F}(a^2)$ for their integration.
	$-\frac{12}{5a} + 50a + \frac{12}{a} - 10a\ [=44]$ or $\frac{48}{5a} + 40a\ [=44]$	M1	**oe** Simplifies the powers of a and removes the brackets.
	$200a^2 - 220a + 48 = 0$ or $50a^2 - 55a + 12 = 0$	M1	**oe** Correctly rearranges to form a quadratic equation in a. Must '$=0$' **PI** by correct final answer.
	$[(10a-3)(5a-4) = 0 \Rightarrow]$ $a = \frac{3}{10}$ or $a = \frac{4}{5}$	A1	**CAO oe** Both correct values.
		7	

	Question 8 Total	**7**	

Q	Answer	Marks	Comments
9(a)	$\left[u_1 = 27^{2p+1} =\right]$ 3^{6p+3} or $3^{3(2p+1)}$	B1	**PI** Writing u_1 as a power of 3
	$\left[r = \dfrac{u_2}{u_1} = \dfrac{3^{18p}}{27^{2p+1}} = \dfrac{3^{18p}}{3^{6p+3}} =\right]$ 3^{12p-3} or $\left[r = \dfrac{u_3}{u_2} = \dfrac{3^{6p+1}}{3^{18p}} =\right]$ 3^{1-12p}	M1	**oe PI** A correct expression for the common ratio as a single power of 3
	$\dfrac{3^{18p}}{27^{2p+1}} = \dfrac{3^{6p+1}}{3^{18p}}$ or $\dfrac{3^{18p}}{3^{6p+3}} = \dfrac{3^{6p+1}}{3^{18p}}$ or $3^{36p} = 3^{6p+3} \times 3^{6p+1}$ or $3^{12p-3} = 3^{1-12p}$	M1	**oe PI** Correct ratios equated.
	$18p - 6p - 3 = 6p + 1 - 18p$ or $36p = 12p + 4$ or $12p - 3 = 1 - 12p$ and $p = \dfrac{1}{6}$	A1	**oe** Correctly equates powers of 3 to form a linear equation in p before **AG CSO**

9(a) ALT	$\left[u_1 = 27^{2p+1} =\right]$ 3^{6p+3} or $3^{3(2p+1)}$	**B1**	**PI** Writing u_1 as a power of 3
	$\left[r = \frac{u_2}{u_1} = \frac{3^{18p}}{27^{2p+1}} = \frac{3^{18p}}{3^{6p+3}} =\right]$ 3^{12p-3} or $\left[r = \frac{u_3}{u_2} = \frac{3^{6p+1}}{3^{18p}} =\right]$ 3^{1-12p}	**M1**	**oe PI** by $r^2 = \frac{1}{9}$ A correct expression for the common ratio (possibly squared) as a single power of 3
	$3^{6p+1} = 3^{6p+3} \times \left(3^{12p-3}\right)^2$ or $3^{6p+1} = 3^{6p+3} \times 3^{24p-6}$ or $3^{6p+1} = 3^{6p+3} \times \left(3^{1-12p}\right)^2$ or $3^{6p+1} = 3^{6p+3} \times 3^{2-24p}$	**M1**	**oe PI** Correct equation in terms of p only for $u_3 = u_1 \times r^2$ Allow u_1 and r^2 unsimplified.
	$6p+1 = 30p-3$ or $6p+1 = 5-18p$ or $24p = 4$ and $p = \frac{1}{6}$	**A1**	**oe** Correctly equates powers of 3 to form a linear equation in p before **AG CSO**
		4	

Q	Answer	Marks	Comments
9(b)	$[u_1 = a =]$ 81	B1	Allow $a = 3^4$
	$r = \frac{1}{3}$	B1	
	$\left[[54\times]\sum_{n=k+1}^{6k} u_n = [54\times]\left(\sum_{n=1}^{6k} u_n - \sum_{n=1}^{k} u_n\right)\right]$ $[54\times]\left(\frac{81\left(1-\left(\frac{1}{3}\right)^{6k}\right)}{1-\frac{1}{3}} - \frac{81\left(1-\left(\frac{1}{3}\right)^{k}\right)}{1-\frac{1}{3}}\right)$	M1	**oe** Correct substitution into $\sum_{n=1}^{6k} u_n - \sum_{n=1}^{k} u_n$ **ft** their a and r Allow $a = 3^4$
	$\left[54\sum_{n=k+1}^{6k} u_n =\right]$ $81\left(81\left(\frac{1}{3}\right)^{k}\left(1-\left(\frac{1}{3}\right)^{5k}\right)\right)$ or $6561\times\left(\frac{1}{3}\right)^{k}\left(1-\left(\frac{1}{3}\right)^{5k}\right)$	M1	**oe** Multiplication of $\sum_{n=1}^{6k} u_n - \sum_{n=1}^{k} u_n$ by 54, fractions cleared and $\left(\frac{1}{3}\right)^{k}$ taken out as a factor.
	$\left[54\sum_{n=k+1}^{6k} u_n =\right]$ $3^{8-k}\left(1-3^{-5k}\right)$	A2,1	In correct form. **A1** b and c or b and d correct. **A2** Fully correct answer.
		6	

	Question 9 Total	**10**	