OXFORD
INTERNATIONAL
AQA EXAMINATIONS

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MATHEMATICS

MA01

(9660/MA01) Unit P1 Pure Mathematics

Mark scheme

June 2022

Version 1.0 Final

226XMA01/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

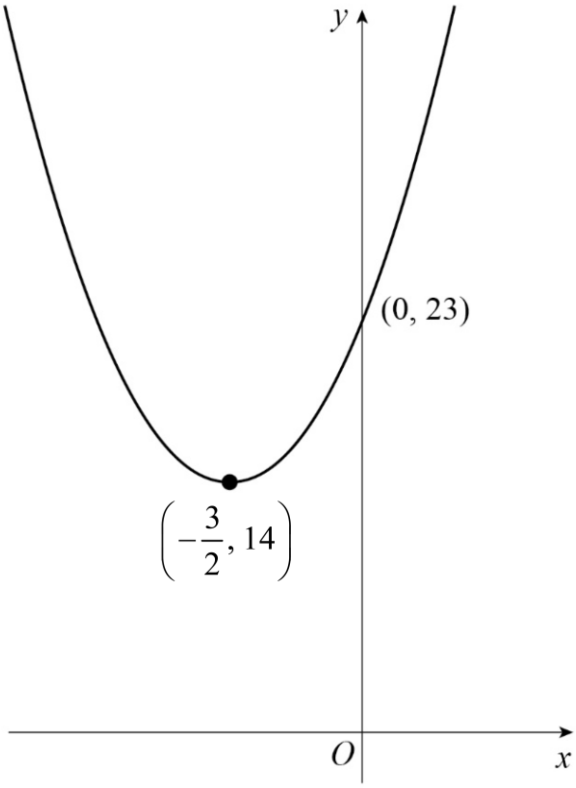
Q	Answer	Marks	Comments
1(a)(i)	$a = 9$	**B1**	
		1	
1(a)(ii)	$a = 15$	**B1**	
		1	
1(b)	$5(x-7)+2(y+2)\ [=6]$	**M1**	**oe** Clear correct use of $(x-7)$ and $(y+2)$, condone slip with constant.
	$5x+2y=37$	**A1**	**oe** but must be in this form.
		2	
	Total	**4**	

Q	Answer	Marks	Comments
2(a)	$\frac{13-1}{4-(-2)}$ $[=2]$	M1	**oe** Use of coordinates with method to find gradient of *J* or a correct pair of simultaneous equations.
	$\frac{y-13}{x-4}=2$ or $\frac{y-1}{x-(-2)}=2$	M1	**oe. ft** their gradient. May see $y=2x+c$ and substitution of coordinates to find c Must be in the form of an equation.
	$y=2x+5$	A1	**CAO** must come from correct working.
		3	
2(b)(i)	$3x^2-6x+3$ $[=0]$	B1	Correct rearrangement.
	$[b^2-4ac=]$ $(-6)^2-4\times3\times3$ or $6^2-4\times3\times3$	M1	Correct substitution into b^2-4ac Could see $a=1$, $b=-2$ and $c=1$ used. **ft** their *a*, *b* and *c* provided rearrangement attempted isolating terms on LHS.
	$(-6)^2-4\times3\times3=0$ One distinct real solution.	A1ft	Correct evaluation of their discriminant & consistent conclusion for number of distinct real solutions.
		3	
2(b)(i) ALT	$3x^2-6x+3$ $[=0]$	B1	Correct rearrangement.
	$3(x-1)^2$ $[=0]$ or $(x-1)^2$ $[=0]$ [$x=1$ is a repeated root]	M1	Correct factorisation or substitution into the quadratic formula. **ft** their quadratic expression. **PI** by $x=1$
	One distinct real solution.	A1ft	Correct evaluation of their correct factorisation or use of quadratic formula & consistent conclusion for number of distinct real solutions.
		3	
2(b)(ii)	*J* is a tangent to the curve or *J* touches the curve [at one point]	E1ft	Correct geometrical description. **ft** their final answer to **part (b)(i)** Allow ‘intersect at one point’
		1	
	Total	**7**	

Q	Answer	Marks	Comments
3	$9+5\sqrt{7}=a\left(3-\sqrt{7}\right)$	**M1**	Appreciates that y-coordinate of the object point needs to be multiplied by the scale factor.
	$[a=]\ \dfrac{9+5\sqrt{7}}{3-\sqrt{7}}$	**A1**	Correct scale factor.
	$\left[a=\dfrac{9+5\sqrt{7}}{3-\sqrt{7}}\times\dfrac{3+\sqrt{7}}{3+\sqrt{7}}\right]$		
	$27+9\sqrt{7}+15\sqrt{7}+35$ or $27+9\sqrt{7}+15\sqrt{7}+5\sqrt{7}\times\sqrt{7}$ or $27+24\sqrt{7}+35$ or $62+24\sqrt{7}$	**B1ft**	Multiplies their numerator by the conjugate of their denominator and expands. Condone one slip.
	$3^2-\left(\sqrt{7}\right)^2$ or $9-7$ or 2	**B1ft**	Multiplies their denominator by its conjugate and expands.
	$31+12\sqrt{7}$	**B1**	
3 ALT	$\left[\left(3-\sqrt{7}\right)\left(b+c\sqrt{7}\right)=\right]\ 3b+3c\sqrt{7}-b\sqrt{7}-7c$	**M1**	**oe** Correct expansion. Condone one error.
	$\left[\left(3-\sqrt{7}\right)\left(b+c\sqrt{7}\right)=\right]\ (3b-7c)+(3c-b)\sqrt{7}$	**A1**	Correct simplification. **PI** in later working.
	$\left[(3b-7c)+(3c-b)\sqrt{7}=9+5\sqrt{7}\Rightarrow\right]$ $3b-7c=9$ and $3c-b=5$	**B1ft**	Forms a correct pair of simultaneous equations. **ft** their $(3b-7c)+(3c-b)\sqrt{7}$
	$b=31$ or $c=12$	**B1**	At least one value correct.
	$31+12\sqrt{7}$	**B1**	
	Total	**5**	

Q	Answer	Marks	Comments
4(a)	$\left(x+\frac{3}{2}\right)^2 \ldots$	**M1**	Condone exact decimal equivalents to fractions throughout. Allow $4\left[\left(x+\frac{3}{2}\right)^2-\frac{9}{4}\right]+23$
	$4\left(x+\frac{3}{2}\right)^2-9+23$	**A1**	$4\left[\left(x+\frac{3}{2}\right)^2-\frac{9}{4}+\frac{23}{4}\right]$ $4\left[\left(x+\frac{3}{2}\right)^2+\frac{14}{4}\right]$
	$4\left(x+\frac{3}{2}\right)^2+14$	**A1**	**CAO**
		3	

Q	Answer	Marks	Comments
4(b)	Correctly orientated symmetrical quadratic parabola.	**B1**	
	$(0,\ 23)$ labelled on y-axis	**B1**	Condone label given as y-value only.
	Vertex labelled as $\left(-\frac{3}{2},\ 14\right)$	**B1ft**	**ft** their $(-b, c)$ from **part (a)**. Accept correctly positioned vertex with $x = -\frac{3}{2}$ and $y = 14$ indicated on axes.



		3	
4(c)	$k < 14$	**B1ft**	Correct inequality stated for their curve and their 14
		1	
4(d)	28	**B1ft**	**ft** $2 \times$ their 14 provided the curve does not intersect the x-axis. Final answer must be positive.
		1	
	Total	**8**	

Q	Answer	Marks	Comments
5(a)	$u_2 = 25k + 15$	B1	Correct expression for u_2 May be seen embedded.
	$u_3 = k(25k + 15) + 15$	M1	Multiplying their u_2 by k and adding 15, e.g. expression for u_3 unsimplified with expression for u_2 substituted in.
	$u_3 = 25k^2 + 15k + 15$	A1	**AG** Be convinced **NMS** scores 0
		3	
5(b)(i)	$25k^2 + 15k - 18 = 0$ $(5k - 3)(5k + 6)\ [= 0]$	M1	Value for u_3 substituted, rearranged with zero on RHS and correct factorisation of LHS. **PI** by correct factorisation only or correct substitution into the quadratic formula unsimplified.
	$k = \frac{3}{5}$ and $k = -\frac{6}{5}$	A1	**oe CAO**
		2	
5(b)(ii)	$T = \frac{3}{5}T + 15$	M1	Identifies suitable positive value of k from their **part (b)(i)** for $0 < k < 1$ **oe**
	$T = 37.5$	A1	**oe CAO**
		2	
	Total	**7**	

Q	Answer	Marks	Comments
6(a)	$\text{p}(m) = m^3 + 8m^2 + 14m - 8$ or $\text{p}(2m) = (2m)^3 + 8(2m)^2 + 14(2m) - 8$	M1	$\text{p}(m)$ or $\text{p}(2m)$ attempted correctly.
	$\text{p}(2m) = 8m^3 + 32m^2 + 28m - 8$	A1	Coefficients correctly evaluated.
	$(2m)^3 + 8(2m)^2 + 14(2m) - 8 = 4(m^3 + 8m^2 + 14m - 8)$ or $8m^3 + 32m^2 + 28m - 8 = 4(m^3 + 8m^2 + 14m - 8)$ or $8m^3 + 32m^2 + 28m - 8 = 4m^3 + 32m^2 + 56m - 32$	M1	Forms correct equality relating their $\text{p}(m)$ and $\text{p}(2m)$ Condone one error in coefficients.
	$2m^3 + 8m^2 + 7m - 2 = m^3 + 8m^2 + 14m - 8$ $\Rightarrow m^3 - 7m + 6 = 0$ or $4m^3 - 28m + 24 = 0$ $\Rightarrow m^3 - 7m + 6 = 0$	A1	**AG NMS** scores 0
		4	
6(b)(i)	$2^3 - 7(2) + 6 \quad [= 8 - 14 + 6]$	M1	Substitution of $m = 2$ attempted. Must use Factor Theorem.
	$8 - 14 + 6 = 0$	A1	**CSO** Power and product evaluated and shown that it equals zero.
		2	
6(b)(ii)	$m^2 + 2m - 3$	M1	Quadratic factor by inspection, long division or comparing coefficients. Condone one sign error.
	$[m^3 - 7m + 6 =] \ (m-2)(m^2 + 2m - 3)$ $= (m-2)(m-1)(m+3)$	A1	Product of correct linear factor and their quadratic factor or Product of three correct linear factors provided the quadratic factor seen in earlier working
	$[m =] \ -3, 1, 2$	A1	**CSO** **NMS** scores 0
		3	
	Total	**9**	

Q	Answer	Marks	Comments
7(a)(i)	$2x+\frac{7}{2}x^2-\frac{1}{3}x^3\ [+c]$	M1 A1	**M1**: two terms correct. **A1**: all terms correct.
		2	
7(a)(ii)	$\left(2(3)+\frac{7}{2}(3^2)-\frac{1}{3}(3^3)\right)-\left(2(1)+\frac{7}{2}(1^2)-\frac{1}{3}(1^3)\right)$ or $\left(6+\frac{63}{2}-9\right)-\left(2+\frac{7}{2}-\frac{1}{3}\right)$	M1	**oe** Correct substitution into their $F(3)-F(1)$ Terms may be fully or partially evaluated.
	$\left(6+\frac{63}{2}-9\right)-\left(2+\frac{7}{2}-\frac{1}{3}\right)=23\frac{1}{3}$	A1	**CSO**, be convinced Terms fully evaluated and **AG** or working shown to $\frac{70}{3}=23\frac{1}{3}$
		2	
7(b)	$h=0.5$	B1	
	[With $f(x)=6+2^x$] $\left[I\approx\frac{h}{2}\{...\}\right]$ $\{...\}=f(1)+f(3)+2\left(f(1.5)+f(2)+f(2.5)\right)$	M1	**oe** Summing the areas of trapezia using five terms.
	$\{...\}=8+14+2\left(8.8284...+10+11.6568...\right)$	A1	**oe** Accept 3 dp rounded or truncated. **PI** by correct final answer.
	$[I\approx]\ 0.25\times 82.9705...=20.743$ [to 3dp]	A1	**CAO** Must be 20.743
		4	
7(c)(i)	$23\frac{1}{3}-20.743$	M1	**ft** their $20.743<23\frac{1}{3}$ Condone $23.333...$
	[Area ≈] 2.59	A1	**CAO**
		2	
7(c)(ii)	The area found using the trapezium rule is an over-estimate.	E1	Explanation implying the value for the area found in **part (b)** is an over-estimate.
	Under-estimate.	E1	Must have been awarded previous **E1** to be able to be awarded this mark.
		2	
	Total	**12**	

Q	Answer	Marks	Comments
8(a)	$y = 3x^{\frac{8}{3}} - 3bx^{\frac{4}{3}} + 12$	B1	Condone $12x^0$
	$\left[\frac{\mathrm{d}y}{\mathrm{d}x} =\right] \frac{8}{3} \times 3x^{\frac{5}{3}} - \frac{4}{3} \times 3bx^{\frac{1}{3}} = 8x^{\frac{5}{3}} - 4bx^{\frac{1}{3}}$	M1 A1	**M1**: At least one term correct, simplified or unsimplified. **ft** their expansion provided there are three terms, at least two having fractional powers. **A1**: Fully correct derivative, simplified or unsimplified.
	$8 \times 8^{\frac{5}{3}} - 4b \times 8^{\frac{1}{3}} = 0$	M1	**oe** Substitutes $x = 8$ into their derivative and sets it equal to zero. May have powers evaluated. **PI**
	$[b =]\ 32$	A1	**CAO**
		5	
8(b)	$[c =]\ 3 \times 8^{\frac{8}{3}} - 3 \times 32 \times 8^{\frac{4}{3}} + 12$ or $[c =]\ 3 \times 8^{\frac{2}{3}}\left(8^2 - 32 \times 8^{\frac{2}{3}} + 4 \times 8^{-\frac{2}{3}}\right)$	M1	Substitutes $x = 8$ into the equation of the curve with their b substituted. **ft** their expansion from **part (a)**.
	$[c =]\ -756$	A1	**CAO**
		2	
	Total	**7**	

Q	Answer	Marks	Comments
9(a)(i)	$y = 2(5-2y)^2 - 10(5-2y) + 13$	**M1**	Substitutes for *x*
	$y = 50 - 40y + 8y^2 - 50 + 20y + 13$	**M1**	**oe** Unsimplified. Brackets on RHS expanded.
	$8y^2 - 21y + 13 = 0$	**A1**	**AG NMS** scores 0 Must see evidence of both **M** marks.
		3	
9(a)(ii)	$(8y-13)(y-1)\ [=0]$ or $(x-3)(4x-7)\ [=0]$	**M1**	Correct factorisation of *x* or *y* **PI** by one correct *x*-value or *y*-value, or correct substitution into quadratic formula unsimplified.
	$y=1,\ y=\frac{13}{8}$ or $x=3,\ x=\frac{7}{4}$	**A1**	**oe** Both *x*-values or *y*-values correct
	$(3, 1)$ and $\left(\frac{7}{4}, \frac{13}{8}\right)$	**A1**	**oe** Condone not given as coordinates only if correct *x* and *y* values correctly linked.
		3	
9(b)	$-\frac{1}{2}$	**B1**	**oe** Correct gradient of *L* identified. May be seen embedded in rearranged equation of *L*.
	$\left[\frac{dy}{dx}=\right]\ 4x-10$	**M1**	
	$x=3 \Rightarrow \frac{dy}{dx}=2$ or $x=\frac{7}{4} \Rightarrow \frac{dy}{dx}=-3$	**A1ft**	Finds value for the gradient at either of the points of intersection. **ft** *x* values from **part (a)**.
	$2\times -\frac{1}{2} = -1 \Rightarrow$ *L* is a normal [at (3, 1)]	**E1**	Correct concluding statement with 2 and $-\frac{1}{2}$ clearly seen and no others. **CSO**
		4	
9(c)	$4x-10=-\frac{1}{2}$	**M1**	**ft** their $\frac{dy}{dx}$ and gradient of *L*
	$\frac{19}{8}$ or 2.375	**A1**	**CAO**
		2	
	Total	**12**	

Q	Answer	Marks	Comments
10(a)	$\dfrac{a}{1-\frac{3}{5}}=20$	M1	**oe** Use of $S_{\infty}=\dfrac{a}{1-r}$ with values correctly substituted.
	[Diameter $= a =$] 8	A1	**CAO**
		2	
10(b)(i)	$\dfrac{7x^2+8x+3}{x^2+1}=\dfrac{3}{5}$	M1	**oe** Forms correct equation.
	$8x^2+10x+3=0$	M1	**oe** Forms correct quadratic equation set equal to zero.
	$x=-\dfrac{1}{2}$ and $x=-\dfrac{3}{4}$	A1	**CAO**
		3	
10(b)(ii)	$\dfrac{7x^2+8x+3}{x^2+1}<1$	B1	**oe**, **PI** by correct final answer Correct strict inequality using condition for convergence for positive common ratio. Simplified or unsimplified. Condone $\left\|\dfrac{7x^2+8x+3}{x^2+1}\right\|<1$ **oe**
	$6x^2+8x+2[<0]$	M1	**oe**, correct quadratic expression. **PI** by correct critical values or correct final answer
	-1 and $-\dfrac{1}{3}$	A1	Both critical values correct. Condone -0.33 or better for $-\dfrac{1}{3}$ throughout.
	$-1<x<-\dfrac{1}{3}$	A1	**CAO, ACF**
		4	
	Total	**9**	