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MA01

Pure Mathematics Unit P1

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Mark	Comments
1(a)(i)	$u_1 = 12$	B1	
1(a)(ii)	$n = 10$	B1	
1(b)	$p = 7$ $q = 5$	B1 B1ft	ft. Their u_1 minus 7 for q.
	Total	4	

1(b) NB: If using simultaneous equations with $p + q =$ their u_1 from part **(a)** and either $5p + q = 40$ or $pn + q = 75$ for their n from part **(a)** then award B1 for each of their p and q correctly calculated. Must see that simultaneous equation approach is being used.

Q	Answer	Mark	Comments
2(a)	$(2+6)(2^2 + 2b + c)$	M1	$\mathrm{p}(2)$ attempted.
	$8(4 + 2b + c) = -8$ or $32 + 16b + 8c = -8$ or $4 + 2b + c = -1$ and $2b + c = -5$	A1	AG after completely correct working. Must have attempt to simplify with -8 or -1 appearing before last line. Must show intermediate step in working. Remainder Theorem not used scores M0A0
2(b)	$(1+6)(1^2 + b + c)$	M1	$\mathrm{p}(1)$ attempted.
	$7(1 + b + c) = 0$ or $1 + b + c = 0$ and $b + c = -1$	A1	AG after completely correct working. Must have attempt to simplify with $= 0$ or $= -7$ appearing before last line. Must show intermediate step in working. Factor Theorem not used scores M0A0
2(c)	$b = -4$	B1	
	$c = 3$	B1	
2(d)	-21	B1ft	ft their $6b + c$. Condone $-21x$
	Total	7	

Q	Answer	Mark	Comments
3(a)	$a = 8$	B1	
	$c = 2$	B1ft	ft their a provided $ac = 16$
3(b)	$(k-6)^2 - 64 \geq 0$ or $b^2 - 64 \geq 0$	B1	Condition for real roots. PI by later working. Do not condone use of strict inequalities.
	$k^2 - 12k + 36 - 64 \geq 0$ or $(k-6)^2 \geq 64$ or $b^2 \geq 64$	M1	Simplified or unsimplified. Condone if given as strict inequality or equality. Condone use of their a and c from part (a).
	$k = -2$ and $k = 14$	A1	Correct critical values.
	$k \leq -2$ or $k \geq 14$	A1	Must be 'or' not 'and'. Condone omission of 'or'. Do not condone use of strict inequalities.
	Total	6	

Q	Answer	Mark	Comments
4	$\frac{16+7\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$	M1	Multiplies numerator and denominator by the conjugate of the denominator.
	(Numerator =) $48 - 16 \times 2\sqrt{5} + 3 \times 7\sqrt{5} - 7\sqrt{5} \times 2\sqrt{5}$ or $48 - 32\sqrt{5} + 21\sqrt{5} - 70$	M1	Must be a correct four or three term expression. Allow one error.
	(Denominator =) $3^2 - \left(2\sqrt{5}\right)^2$ or $9 - 20$ or -11	B1	Must be seen as a denominator.
	$2 + \sqrt{5}$	A1	Correct width of rectangle.
	$2\left(3+2\sqrt{5}\right) + 2\left(2+\sqrt{5}\right) \quad \left(= 10 + 6\sqrt{5}\right)$ or $x + \left(4-\sqrt{5}\right) + \left(4+3\sqrt{5}\right)$ $\left(= x + \left(8+2\sqrt{5}\right)\right)$	B1ft	oe. For correct perimeter of rectangle or triangle simplified or unsimplified. ft their width of rectangle.
	$x + \left(8+2\sqrt{5}\right) = 10 + 6\sqrt{5}$	M1	oe. For forming equation equating perimeters simplified or unsimplified, or unsimplified expression for x.
	$2 + 4\sqrt{5}$	A1	CAO
	Total	7	

Q	Answer	Mark	Comments
5(a)	$\frac{-1-7}{16-2}=-\frac{4}{7}$	B1	Correct gradient found using coordinates of *A* and *B*. Accept equivalent fractions.
	$y-7=-\frac{4}{7}(x-2)$ oe and $4x+7y-57=0$	B1	Uses gradient and coordinates of *A* or *B* to form equation leading to correct equation in the correct form
5(b)	$y=-\frac{3}{2}x+10$ or Gradient of $L_2=-\frac{3}{2}$	B1	For writing equation of L_2 in the form $y=mx+c$ or stating the gradient of L_2. Condone incorrect constant term.
	$y=\frac{2}{3}x-3$ or Gradient of $L_3=\frac{2}{3}$	B1	For writing equation of L_3 in the form $y=mx+c$ or stating the gradient of L_3. Condone incorrect constant term.
	$-\frac{3}{2}\times\frac{2}{3}=-1$ therefore perpendicular	E1	Concluding statement that the lines are perpendicular and justification relating gradients of L_2 and L_3 needed.
5(c)	$4x+7y-57=0$ and $2x-3y-9=0$ $\Rightarrow$ equation in x or y only	M1	Any correct method Use of midpoint to find *D* scores M0
	$x=9$	A1	
	$y=3$	A1	
	$\sqrt{(6-9)^2+(1-3)^2}$	M1ft	ft their coordinates of *D* provided previous M1 scored
	$\sqrt{13}$	A1	Correct length of *CD*
	Area = 13	B1ft	ft their length of CD provided second M1 scored
	Total	11	

Q	Answer	Mark	Comments
6(a)	3 roots	B1	
	The line $y = 2.5$ crosses the curve at exactly three points	E1	Correct explanation.
6(b)	Translation	E1	
	$\begin{pmatrix} 2 \\ 5 \end{pmatrix}$	E1	Correct vector.
6(c)	Three line segments with at least one correct	M1	
	Three correct line segments	A1	
	Total	6	

Q	Answer	Mark	Comments
7(a)	$y = \frac{3}{2}x + 6$	B1	Rearranges equation of *L* to make y the subject. PI by later working.
	$\frac{3}{2}x + 6 = 4 + \frac{7}{2}x - \frac{1}{2}x^2$	M1	Eliminates y in the equation of *C*. Uses their $\frac{3}{2}x + 6$.
	$\frac{1}{2}x^2 - 2\text{x} + 2 = 0$ or $x^2 - 4x + 4 = 0$	A1ft	ft. Correctly rearranges their $\frac{3}{2}x + 6 = 4 + \frac{7}{2}x - \frac{1}{2}x^2$ into equation set equal to zero.
	$P = (2{,}9)$	B2	Correct x-coordinate only scores B1B0. Condone values not given as coordinates.
7(a) ALT	$\frac{3}{2}$	B1	Correct gradient of *L*. PI in later working or by rearranging equation of *L* as $y = \frac{3}{2}x + 6$
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x} =\right) \frac{7}{2} - x$	B1	Correct derivative of *C*.
	$\frac{7}{2} - x = \frac{3}{2}$	M1	Sets their gradient of *L* equal to their derivative of *C*.
	$P = (2{,}9)$	A2	Correct x-coordinate only scores A1A0. Condone values not given as coordinates.
7(b)	$\int \left(4 + \frac{7}{2}x - \frac{1}{2}x^2\right) \mathrm{d}x =$ $4x + \frac{7}{4}x^2 - \frac{1}{6}x^3 + c$	B2	Simplified or unsimplified. For B2 condone omission of $+c$. Two correct terms only scores B1B0
7(c)	$\int_{-1}^{2} \left(4 + \frac{7}{2}x - \frac{1}{2}x^2\right) \mathrm{d}x =$ $=\left(4(2) + \frac{7}{4}(4) - \frac{1}{6}(8)\right) - \left(4(-1) + \frac{7}{4}(1) - \frac{1}{6}(-1)\right)$	M1	Identifies correct definite integral and attempts $\mathrm{F}(2) - \mathrm{F}(-1)$. ft their indefinite integral from part (b) and their x-coordinate of *P* from part (a).
	15.75	A1	PI oe. Correct area under *C* between $x = -1$ and $x = 2$.
	Area of shaded region = $\frac{1}{2} \times 9 \times 6 - 15.75$	M1	Uses their y-coordinate of *P* for 9 and 4 + their x-coordinate of *P* for 6 from part (a). Uses their 15.75 provided attempt at evaluation of $\int_a^b \left(4 + \frac{7}{2}x - \frac{1}{2}x^2\right) \mathrm{d}x$ for their a and b.
	11.25	A1	oe
	Total	11	

Q	Answer	Mark	Comments
8(a)(i)	r^2=25 or $25k = r^2k$ or $\frac{250}{k} = \frac{25k}{250}$ or $k = (\pm)50$	M1	
	$r = 5$	A1	CAO
8(a)(ii)	$a \times 5^3 = 250$	M1ft	ft their r.
	$a = 2$	A1	CAO
8(b)	$\dfrac{6\left(1-\left(\frac{1}{3}\right)^{15}\right)}{1-\frac{1}{3}}$	B1	Correct substitution.
	$3^2(1-3^{-15})$ or $9(1-3^{-15})$	M1	oe. Clears denominator. Condone $\left(\frac{1}{3}\right)^{15}$ for 3^{-15}
	$p = 2$	A1	
	$q = -13$	A1	
	Total	8	

Q	Answer	Mark	Comments
9(a)(i)	$y = 4x^{\frac{4}{3}} - 12x^{\frac{1}{3}} + 11$	B1	For expanding brackets.
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{16}{3}x^{\frac{1}{3}} - 4x^{-\frac{2}{3}}$	B2ft	B1 One term correct, simplified or unsimplified. B2 All terms correct, simplified or unsimplified. ft their y provided both terms in x have fractional powers.
9(a)(ii)	$\frac{16}{3}x^{\frac{1}{3}} - 4x^{-\frac{2}{3}} = 0$	M1ft	Setting their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to equal zero. ft their $\frac{\mathrm{d}y}{\mathrm{d}x}$.
	$\frac{16}{3}x - 4 = 0$	M1	Multiplying by $x^{\frac{2}{3}}$ throughout.
	$x = \frac{3}{4}$	A1	oe.
9(b)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{16}{9}x^{-\frac{2}{3}} + \frac{8}{3}x^{-\frac{5}{3}}$	B2ft	B1 One term correct, simplified or unsimplified. B2 Fully correct second derivative simplified or unsimplified. ft their $\frac{\mathrm{d}y}{\mathrm{d}x}$ provided it is a two-term expression with both terms fractional powers of x.
	$\left(x = \frac{3}{4} \Rightarrow\right)$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{16}{9}\left(\frac{3}{4}\right)^{-\frac{2}{3}} + \frac{8}{3}\left(\frac{3}{4}\right)^{-\frac{5}{3}}$ $(= 6.46087 \ldots)$	M1	For clear attempt to substitute $x = \frac{3}{4}$ into the second derivative. Uses their non-zero $\frac{3}{4}$ from part (b) and their second derivative provided at least B1 scored. Be convinced.
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0$ so *P* is a minimum.	E1ft	Correct conclusion indicating that value of second derivative is positive. ft Dep M1 scored. Correct conclusion based upon their value for the second derivative.
	Total	10	

Q	Answer	Mark	Comments
10(a)	$\frac{n(n-1)(k+1)^2}{1\times 2}$ or $\frac{n(n-1)(n-2)(k+1)^3}{1\times 2\times 3}$	M1	One correct coefficient. Condone inclusion of x^2 or x^3. No ^nC_r
	$\frac{n(n-1)(n-2)(k+1)^3}{1\times 2\times 3} = \frac{3n(n-1)(k+1)^2}{1\times 2}$	A1	Correct equation. Numerator unsimplified. No ^nC_r
	$\frac{(n-2)(k+1)^3}{6} = \frac{3(k+1)^2}{2}$	M1	Cancels at least $n(n-1)$.
	$(n-2)(k+1) = 9$	A1	CSO
10(b)	$n(k+1) = 12$ or $k+1 = \frac{12}{n}$	B1	PI by later working.
	$(n-2)\times\frac{12}{n} = 9$	M1	oe. Forms equation in n.
	$n = 8$	A1	
	$k = \frac{1}{2}$	B1	
	$^8C_5\left(\frac{3}{2}\right)^5$	M1	oe
	425.25	A1	oe
	Total	10	