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(9660/MA02) Unit PSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	8.4 [cm]	B1	
		1	

Q	Answer	Marks	Comments
1(b)	$\frac{1}{2}\times 12^2\times 0.7\ [=50.4]$ or $\frac{1}{2}\times x^2\times 0.7\left[=0.35x^2\right]$	M1	**oe PI** Correct expression for area of one sector.
	$\frac{1}{2}\times 12^2\times 0.7-\frac{1}{2}\times x^2\times 0.7[=35]$ or $50.4-0.35x^2\ [=35]$	M1	**oe** Correct expression for area of region *ABCD* **PI** by $x^2=44$ or $x=\sqrt{44}$ or $0.35x^2=15.4$
	$[x=]\ 2\sqrt{11}$	A1	**CAO**
		3	

	Question 1 Total	**4**	

Q	Answer	Marks	Comments
2(a)(i)	$\frac{a-7}{a-5}$ or $\frac{a-1}{a-13}$	**M1**	**oe** Correct expression for gradient of radius or tangent.
	$\frac{a-7}{a-5}\times\frac{a-1}{a-13}=-1$ or $\frac{a-7}{a-5}=\frac{13-a}{a-1}$	**M1**	**oe** equation relating gradients of tangent and radius.
	$a^2-8a+7=-a^2+18a-65$	**M1**	**oe** equation with fractions and brackets cleared.
	$2a^2-26a+72=0$ and $a^2-13a+36=0$	**A1**	**AG** Must be convincingly shown **M1M1M1** not scored scores **A0** **oe** Must have further simplification before required result stated.

Q	Answer	Marks	Comments
2(a)(i) ALT	$\left[QR^2 =\right]\ (16-8)^2+(1-7)^2\ [=100]$ or $[QR=]\ \sqrt{(16-8)^2+(1-7)^2}\ [=10]$	**M1**	**oe** Correct use of distance formula to calculate QR
	$(a-5)^2+(a-7)^2+(13-a)^2+(a-1)^2\ [=100]$ or $\sqrt{(a-5)^2+(a-7)^2+(13-a)^2+(a-1)^2}\ [=10]$	**M1**	**oe** Use of Pythagoras' Theorem with correct expression for PR^2+PQ^2
	$a^2-10a+25+a^2-14a+49+169$ $-26a+a^2+a^2-2a+1\ [=100]$ or $\sqrt{a^2-10a+25+a^2-14a+49+169}$ $\sqrt{-26a+a^2+a^2-2a+1}\ [=10]$	**M1**	**oe** Their brackets correctly expanded. Condone one error.
	$4a^2-52a+244=100$ or $4a^2-52a+144=0$ and $a^2-13a+36=0$	**A1**	**AG** Must be convincingly shown **M1M1M1** not scored scores **A0** Must have further simplification before required result stated.
		4	

Q	Answer	Marks	Comments
2(a)(ii)	$a = 9$	**B1**	Condone $a = 4$ given as well **PI** by correct coordinates of *P*
	[Gradient $\frac{9-1}{12-16}[=-2]$ or $\frac{9-1}{9-13}[=-2]$]		
	$(y-9) = -2(x-12)$ or $(y-1) = -2(x-16)$ or $y = -2x + 33$	**M1**	Uses their gradient to find equation of line **ft** their a
	$2x + y = 33$	**A1**	Correct equation in correct form.
		3	

Q	Answer	Marks	Comments
2(b)	$\left[r^2 =\right] (12-8)^2 + (9-7)^2$	**M1**	**oe PI** Follow through their a
	$r^2 = 20$	**A1**	Condone $r = \sqrt{20}$ or $r = 2\sqrt{5}$
	$(x-11)^2 + (y-(-3))^2 = 20$ or $(x-11)^2 + (y+3)^2 = 20$	**B2ft**	**B1** for each correct bracketed term in an equation set equal to 20 **ft** their r^2 Equation must be in the correct form.
		4	

	Question 2 Total	**11**	

Q	Answer	Marks	Comments
3(a)	$\left[A=\right]5$	B1	
		1	

Q	Answer	Marks	Comments
3(b)	$\left[k=\right]2$	B1	
		1	

Q	Answer	Marks	Comments
3(c)	$\left[4=5\times1.5^{3b}+2\right]$ $3b=\log_{1.5}(0.4)$ or $3b=\frac{\log_a(0.4)}{\log_a(1.5)}$ or $b=\frac{\log_a(0.4)}{\log_a 3.375}$ or $b=\frac{\log_a\left(\sqrt[3]{0.4}\right)}{\log_a 1.5}=\frac{\log_a 0.7368...}{\log_a 1.5}$ or $b=\log_{1.5}\left(\sqrt[3]{0.4}\right)=\log_{1.5}0.7368...$	M1	**oe** Expression for $3b$ or b in terms of logarithms with $A=5$ substituted **ft** their value of A from **part (a)** **PI** by $3b=-2.259(851...)$ rounded or truncated to 3 dp Condone $3b=-2.26$
	$b=-0.753$	A1	**CAO**
		2	

	Question 3 Total	**4**	

Q	Answer	Marks	Comments
4	$\frac{18}{\sin 30^\circ} = \frac{EG}{7/12}$	M1	**oe** Correct use of Sine Rule.
	$[EG =] 21\ [\text{cm}]$	A1	**PI CAO**
	$\frac{1}{2} \times 21 \times 32 \times \sin\beta [= 210]$	M1	Correct use of $A = \frac{1}{2}ab\sin c$ using their length of *EG*
	$\sin\beta = \frac{5}{8} [= 0.625]$ or $[\beta =] 38.6(8218)...^\circ$	A1	Correct rearrangement to find value for $\sin\beta$ Fraction simplified or unsimplified. **PI** by $38.6(8218)...^\circ$
	$[\beta =] 180^\circ - 38.6(8218)...^\circ$ or $[\beta =] 180^\circ - \sin^{-1}\left(\frac{5}{8}\right)$	m1	Dependent on 2nd **M1** **ft** their acute angle or their $\frac{5}{8}$ **PI** by correct final answer.
	$[\beta =] 141^\circ$	A1	**CAO** Condone omission of units. **AWRT** 141°
		6	

	Question 4 Total	**6**	

Q	Answer	Marks	Comments
5(a)	$2 = \log_7 49$ or $1 = \log_7 7$	**B1**	**PI** i Seen or used at any stage.
	$\log_7 (2x-5)^2$	**M1**	A law of logs used. Must be applied to at least one term involving x **PI** by $\frac{(2x-5)^2}{2x+3}[=49]$
	$\log_7 \frac{(2x-5)^2}{2x+3} = 2$ or $\log_7 \frac{(2x-5)^2}{2x+3} = \log_7 49$ or $\log_7 (2x-5)^2 = \log_7 49(2x+3)$	**M1**	**oe** A second law of logs used leading to a correct equation involving logs. Must be applied to at least one term involving x Condone $\frac{(2x-5)^2}{2x+3} = 49$ for **B1M1M1**
	$4x^2 - 20x + 25 = 49(2x+3)$ or $4x^2 - 20x + 25 = 98x + 147$ or $4x^2 - 118x - 122 = 0$ and $2x^2 - 59x - 61 = 0$	**A1**	**CSO AG** Logs cleared and further manipulation seen before required quadratic equation stated.
		4	

Q	Answer	Marks	Comments
5(b)	$(2x-61)(x+1)\ (=0)$ or $x=\dfrac{-(-59)\pm\sqrt{(-59)^2-4\times2\times(-61)}}{2\times2}$	**M1**	Or correct use of completing the square. Correct attempt to solve the quadratic equation **PI** by $x=\dfrac{61}{2}$ and $x=-1$ seen
	$x=\dfrac{61}{2}$ or 30.5	**A1**	**CSO** **M1** not scored award **M0A0**
		2	

	Question 5 Total	**6**	

Q	Answer	Marks	Comments
6(a)	$\dfrac{33+16\left(1-\sin^2 x\right)}{7+4\sin x}$ or $\dfrac{49-16\sin^2 x}{7+4\sin x}$	M1	Use of $\sin^2 x+\cos^2 x=1$
	$\dfrac{(7-4\sin x)(7+4\sin x)}{7+4\sin x}$ or $\dfrac{\left(49-16\sin^2 x\right)(7-4\sin x)}{49-16\sin^2 x}$	M1	Numerator factorised or numerator and denominator multiplied by $7-4\sin x$ and denominator expanded.
	$7-4\sin x$	A1	**CSO**
		3	

Q	Answer	Marks	Comments
6(b)(i)	$7-4\times -1$ **(oe)** and 11	B1	
		1	

Q	Answer	Marks	Comments
6(b)(ii)	$[x=]270^\circ$	B1	
		1	

Q	Answer	Marks	Comments
6(c)	$7-4\sin 2\theta = 6$ $\Rightarrow \sin 2\theta = \frac{1}{4}$	M1	For use of **part (a)** **ft** their $p - q\sin 2\theta$ from **part (a)** with 2θ substituted. Condone x or $2x$ used instead of 2θ
	$[2\theta =]14.4(77512...)^\circ$ or $[2\theta =]165.5(22487...)^\circ$	m1	Anything that rounds or truncates correctly to 1dp If more accurate value(s) seen but not rounded to 1 dp scores **A1**: $\theta = 7.23875...^\circ$ $\theta = 82.76124...^\circ$ Ignore values outside the given interval.
	$[\theta =]$ 7.2° , 82.8°	A2,1	If both correct answers given deduct 1 mark for each extra incorrect value in the given interval to a minimum of **A0** If one correct answer only given then **A1** if there is no more than one incorrect answer in the given interval. Condone 7.3 for 7.2 Both correct answers stated with **M0** scored scores **M0m0A0A0**
		4	

	Question 6 Total	**9**	

Q	Answer	Marks	Comments
7(a)	79.24	**B1**	Accept $\frac{1981}{25}$ Condone 79.2 as final answer if 79.24 seen,
		1	

Q	Answer	Marks	Comments
7(b)	$12^2 \times 0.573^2$	**M1**	**PI** by 47.2 Condone $12^2 \times 0.573 \left[= 82.512 \right]$
	$= 47.3$	**A1**	**AWRT**
		2	

	Question 7 Total	**3**	

Q	Answer	Marks	Comments
8(a)	$\frac{11}{90}$	B1	**oe** or **AWRT** 0.122
		1	

Q	Answer	Marks	Comments
8(b)	$\frac{1}{20}$	B1	**oe**
		1	

Q	Answer	Marks	Comments
8(c)	$\frac{1}{62}$	B1	**oe** or **AWRT** 0.016
		1	

Q	Answer	Marks	Comments
8(d)	$\text{P}(A)=\frac{112}{180}\left[=\frac{28}{45}=0.62(2...)\right]$ or $\text{P}(B)=\frac{96}{180}\left[=\frac{8}{15}=0.53(3...)\right]$	B1	$\text{P}(A)$ or $\text{P}(B)$ correct **oe** **PI** in subsequent working
	$\text{P}(A)\times\text{P}(B)=\frac{224}{675}\left[=0.33(185...)\right]$	B1	**oe** **AWRT** 0.33
	$\text{P}\left(A\cap B\right)=\frac{14}{45}\left[=0.31(111...)\right]$	B1	**oe**
	$\text{P}(A)\times\text{P}(B)=\frac{224}{675}\neq\frac{14}{45}=\text{P}\left(A\cap B\right)$ So not independent.	E1ft	**oe** Compares their $\text{P}(A)\times\text{P}(B)$ and $\text{P}\left(A\cap B\right)$ and makes correct conclusion for their values.
		4	

Q	Answer	Marks	Comments
8(d) **ALT**	$\mathrm{P}(A)=\frac{28}{45}\left[=0.62(2...)\right]$	**B1**	**oe**
	$\mathrm{P}(A\mid B)=\frac{56}{96}\left[=\frac{7}{12}=0.583(3...)\right]$	**B2**	**CAO** **oe**
	$\mathrm{P}(A\mid B)=\frac{7}{12}\neq\mathrm{P}(A)=\frac{28}{45}$ So not independent.	**E1ft**	**oe** Compares their $\mathrm{P}(A)$ and $\mathrm{P}(A\mid B)$ and makes correct conclusion for their values.
		4	

	Question 8 Total	**7**	

Q	Answer	Marks	Comments
9(a)	$\left[\mathrm{E}(X)=\right]\ p$	B1	**PI** Accept $0(1-p)+1\times p$
	$\left[\mathrm{E}(X^2)=\right]\ 0^2(1-p)+1^2\times p$	M1	Correct expression seen for $\mathrm{E}(X^2)$ **oe**
	$\left[\mathrm{Var}(X)=\right]\ p-p^2=p(1-p)$	A1	**AG** must see $p-p^2$ or $np(1-p)$ with $n=1$
		3	

Q	Answer	Marks	Comments
9(b)	Bernoulli distribution with probability p	B1	Accept $\mathrm{B}(1,p)$
		1	

Q	Answer	Marks	Comments
9(c)	$\left[p(1-p)=0.2176\right]$ $p^2-p+0.2176=0$	M1	Forms correct equation and expands brackets **oe** **PI** by 0.68 or 0.32 seen.
	$\left[p=\right]\ 0.68$	A1	Accept 17/25 **oe** If 0.32 **oe** seen it must be rejected.
		2	

	Question 9 Total	**6**	

Q	Answer	Marks	Comments
10	$\left[\text{Var}\left(X_2\right)=\right]$ 0.7569	B1	**PI** Accept 0.87^2
	$\left[\text{Var}\left(X_3\right)=\right]$ 0.48	B1	**PI** Accept $7.0336 - 2.56^2$
	$\left[\text{Var}\left(\sum_{i=1}^{3} X_i\right)=\text{Var}\left(X_1\right)+\text{Var}\left(X_2\right)+\text{Var}\left(X_3\right)\right]$ $0.51+0.7569+0.48$	M1	**PI** Attempts to add their three variances.
	$\left[\text{Var}\left(\sum_{i=1}^{3} X_i\right)=\right]$ 1.7469	A1	
		4	

	Question 10 Total	**4**	

Q	Answer	Marks	Comments
11(a)(i)	Friction $= 0.15g$ or 1.47	**B1**	Can be implied.
	Equation of motion for *P* $0.2g - T = 0.2a$	**M1**	**ft** their friction. Forms both correct equations of motion for *P* and *Q* Condone F or μR for $0.15g$
	Equation of motion for *Q* $T - 0.15g = a$	**A1**	**M1**: One correct equation. **M1A1**: Both correct equations. **PI** by $0.2g - (0.15g + a) = 0.2a$ **oe**
	$[a =]\ 0.41 \quad [\text{m s}^{-2}]$	**A1**	
		4	

Q	Answer	Marks	Comments
11(a)(ii)	$v^2 = 2 \times 0.41 \times 0.4$ or $v = \sqrt{2 \times 0.41 \times 0.4}$	**M1**	Suitable calculation for v^2 or v If two constant acceleration formulae used then must be a complete method. **ft** their acceleration from **part (a)(i)**.
	$[v =]\ 0.57\ [\text{m s}^{-1}]$	**A1**	**AWRT** $v = 0.57\ [\text{m s}^{-1}]$
		2	

Q	Answer	Marks	Comments
11(b)	The tension in the string is the same either side of the peg.	**E1**	Any suitable explanation eg The magnitude of the force the string exerts on *P* and *Q* is the same.
		1	

	Question 11 Total	**7**	

Q	Answer	Marks	Comments
12(a)	$[s =] \ 0.5 \times 8 \times 10^2$ $[s =] \ 400 \ [\text{m}]$	**M1** **A1**	If two constant acceleration formulae used then must be a complete method. **CAO**
		2	

Q	Answer	Marks	Comments
12(b)	$\left[p = \frac{(-1)-(-5)}{(30)-(10)} = \right] \ 0.2$	**B1**	**CAO**
		1	

Q	Answer	Marks	Comments
12(c)(i)	$[v =] 80 \ [\text{at } t = 10]$ $\left[v = \int (0.2t - 7) dt = \right] \ 0.1t^2 - 7t + c$ $\left[\begin{array}{l} 80 = 0.1 \times 10^2 - 7 \times 10 + c \\ c = 140 \end{array} \right]$ $[v =] 0.1t^2 - 7t + 140$	**B1** **M1** **A1**	Speed at the end of first section of motion. **PI** Correct integration. **ft** their 0.2 Condone omission of $'+c'$ **CAO**
		3	

Q	Answer	Marks	Comments
12(c)(ii)	$[v =] \ 0.1(30)^2 - 7(30) + 140$ $= 20 \ \left[\text{m s}^{-1} \right]$	**B1**	$t = 30$ substituted into the correct expression for the velocity from **part (c)(i)** and **AG**
		1	

Q	Answer	Marks	Comments
12(d)	$[v = 20 + (-1)(15)]$ $[v_{60} =]$ 5 $[\text{ms}^{-1}]$	B1	**CAO**
	Package does **not** land safely	E1	Correct conclusion. **B0E1** not possible.
		2	

	Question 12 Total	**9**	

Q	Answer	Marks	Comments
13(a)	$6m = -3vm + 3vkm$	**M1**	Conservation of linear momentum applied. Allow one sign error. m may not appear throughout.
	$2 + v = vk$ $k = \frac{2}{v} + 1$	**A1**	Further line of working before **AG**
		2	

Q	Answer	Marks	Comments
13(b)	$12 = km(v - (-3v))$ or $12 = 4kmv$	**M1**	Impulse = change in momentum Allow one sign error.
	$3 = \left(\frac{2}{v} + 1\right)mv$ $3 = (2 + v)m$ $m = \frac{3}{2 + v}$	**A1**	$k = \frac{2}{v} + 1$ substituted and one further rearrangement before **AG**
		2	

	Question 13 Total	**4**	