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MA01

(9660/MA01) Unit P1 Pure Mathematics

Mark scheme

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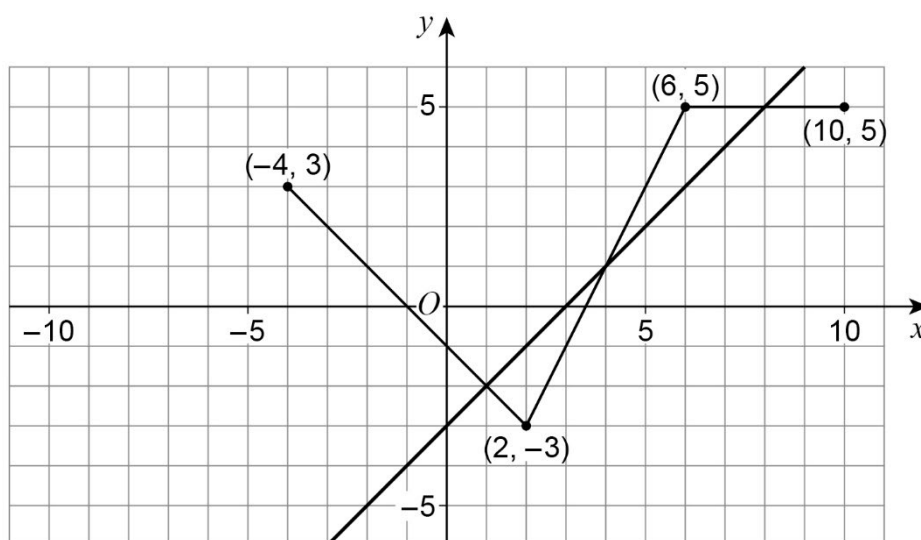
Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
-x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)(i)	$y = f(2x)$	B1	
		1	

Q	Answer	Marks	Comments
1(a)(ii)	$y = f(x+4) - 1$	B1	
		1	

Q	Answer	Marks	Comments
1(b)	Line $y = x - 3$ drawn 1, 4, 8	M1 A1	Evidence of correct line drawn, intersecting $y = f(x)$ at two or more points. CAO



		2	
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	Question 1 Total	4	
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Q	Answer	Marks	Comments
2(a)	$y - 3 = \frac{3}{5}(x - 15)$ oe and $3x - 5y - 30 = 0$	B1	Uses gradient and coordinates of A to form equation leading to the required result. May see $y = \frac{3}{5}x + c$ and substitution of coordinates of A to find c but must be complete method leading to $y = \frac{3}{5}x - 6$ oe seen.
		1	

Q	Answer	Marks	Comments
2(b)	[x-intercept] $x = 10$ [y-intercept] $y = -6$ [Midpoint of BC] $(5, -3)$ [Gradient of l_2] $-\frac{5}{3}$ $y - (-3) = -\frac{5}{3}(x - 5)$ $5x + 3y - 16 = 0$	B1 B1ft B1 M1 A1	Correct values of x & y intercepts B and C May be given as coordinates. PI in later working. ft their values for the x & y intercepts. Condone not given as coordinates but must be clearly identified. PI in later working. PI in later working. oe Uses their gradient of l_2 and their coordinates of the midpoint of BC to form an equation for l_2 May see $y = -\frac{5}{3}x + c$ and substitution of coordinates of their midpoint to find c but must be complete method. Correct equation set equal to zero. Allow any non-zero integer multiple.
		5	

Q	Answer	Marks	Comments
2(c)	$(-1, 7)$	B1	Correct coordinates of D Condone not given as coordinates but must be clearly identified.
		1	

Q	Answer	Marks	Comments
2(d)	$[AD =] \sqrt{(15 - (-1))^2 + (3 - 7)^2}$	M1	oe ft their coordinates of D
	$[AD =] 4\sqrt{17}$	A1	CAO Must be in correct form.
		2	

	Question 2 Total	9	
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Q	Answer	Marks	Comments
3(a)	$2((x+3)+(2x-10))$ [> 31] oe or $4+(x-6)+(x-1)+(2x-10)$ $+ (x+3)+(x-4)$ [> 31] oe and $6x-14 > 31$ oe and $x > 7.5$	B1	CSO AG Unsimplified expression for the perimeter followed by a line of simplification before required result stated. Must appear as an inequality before the final line.
		1	

Q	Answer	Marks	Comments
3(b)	$(x-1)(x-6)+(x-4)(x+3)$ [< 58] or $4(x-4)+(x-1)(2x-10)$ [< 58] or $(2x-10)(x+3)-4(x-6)$ [< 58] or $4(x-4)+(x-1)(x-6)+(x-1)(x-4)$ [< 58] $x^2-7x+6+x^2-x-12$ [< 58] or $4x-16+2x^2-12x+10$ [< 58] or $2x^2-4x-30-4x+24$ [< 58] or $4x-16+x^2-7x+6+x^2-5x+4$ [< 58] $2x^2-8x-6 < 58$ oe and $x^2-4x-32 < 0$	M1 M1 A1	oe Correct expression for the area. oe All brackets correctly expanded in unsimplified form. For 3 rd example, condone $2x^2-4x-30-(4x-24)$ [< 58] AG Must be convincingly shown. Working to show all terms in x and x^2 collected before required result stated. Must appear as an inequality before the final line.
		3	

Q	Answer	Marks	Comments
3(c)	$(x-8)(x+4) \leq 0$ $-4 < x < 8$	M1 A1	Correct attempt to find the critical values. May see quadratic formula or completing the square used. ISW NMS scores M0 A0 Condone ' $x > -4$ and $x < 8$ ' but not ' $x > -4$ or $x < 8$ '
		2	

Q	Answer	Marks	Comments
3(d)	$7.5 < x < 8$ $1.5 < y < 2$	M1 A1ft	PI ft their 8 provided their 8 is greater than 7.5 Condone ' $x > 7.5$ and $x < 8$ ' but not ' $x > 7.5$ or $x < 8$ ' Do not ISW ft their 8 minus 6 provided their 8 is greater than 7.5 Condone ' $y > 1.5$ and $y < 2$ ' but not ' $y > 1.5$ or $y < 2$ '
		2	

	Question 3 Total	8	
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Q	Answer	Marks	Comments
4(a)	$[p(4) =] 4^2(2 \times 4 - 5) - 48$ or $[p(4) =] 2(4)^3 - 5(4)^2 - 48$ $[p(4) =] 16(8 - 5) - 48 = 0$ or $[p(4) =] 48 - 48 = 0$ or $[p(4) =] 2 \times 64 - 5 \times 16 - 48 = 0$ or $[p(4) =] 128 - 80 - 48 = 0$	M1 A1	oe $p(4)$ attempted. Must use Factor Theorem. CSO oe Correctly shows $p(4) = 0$ Must have powers evaluated and be set equal to zero.
		2	

Q	Answer	Marks	Comments
4(b)	$2x^2 + bx + c$ with $b = 3$ or $c = 12$ $(x - 4)(2x^2 + 3x + 12)$	M1 A1	Must be seen as a quadratic expression. CAO Must be in the form required.
		2	

Q	Answer	Marks	Comments
4(c)	$3^2 - 4 \times 2 \times 12$ $3^2 - 4 \times 2 \times 12 [= -87] < 0$ oe and There is exactly one [real] root. $x = 4$	B1ft E1 B1	oe Attempts to find discriminant for their quadratic factor in part (b) PI by correct value of discriminant. Must be working with correct discriminant. Not necessary for discriminant to be evaluated. Gives indication that discriminant has negative value and gives correct concluding statement. Condone statement implying no real roots. E0 scored if previous B1 not scored. Independent. Correct root clearly stated.
		3	

	Question 4 Total	7	
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Q	Answer	Marks	Comments
5(a)(i)	$[u_1 =]$ 29	B1	
		1	

Q	Answer	Marks	Comments
5(a)(ii)	$[u_5 =]$ 1021	B1	
		1	

Q	Answer	Marks	Comments
5(b)(i)	12, 36, 108, 324, 972	B1	CAO
		1	

Q	Answer	Marks	Comments
5(b)(ii)	$(29 - 12), (61 - 36), (141 - 108),$ $(365 - 324), (1021 - 972)$	M1	Intention of subtracting each term in their answer to part (b)(i) from the corresponding term in sequence A Minimum of two subtractions.
	17, 25, 33, 41, 49	A1	ft their 29 and their 1021 from part (a) PI by two correct terms of sequence C CAO
		2	

Q	Answer	Marks	Comments
5(c)(i)	8	B1	
		1	

Q	Answer	Marks	Comments
5(c)(ii)	$8n + 9$	B1	ACF such as $17 + 8(n - 1)$
		1	
	Question 5 Total	7	

Q	Answer	Marks	Comments
6(a)(i)	$\left[\frac{dy}{dx} =\right] 9x^2 + 28x + 17$	B2,1	B1 Two correct terms. B2 Fully correct.
		2	

Q	Answer	Marks	Comments
6(a)(ii)	$9 \times (-2)^2 + 28 \times (-2) + 17 \quad [= 36 - 56 + 17]$ $\left[\frac{dy}{dx} =\right] -3 \quad \text{and} \quad \left[m' =\right] \frac{1}{3}$ $y - 9 = \frac{1}{3}(x + 2) \quad \text{oe}$ and $y = \frac{1}{3}x + \frac{29}{3}$	M1 A1	Attempt to find value of $\frac{dy}{dx}$ at P ft their answer to part (a)(i) Correct value of derivative and gradient of normal at P $m' = \frac{1}{3}$ may be embedded in unsimplified equation of l CSO AG Forming a simplified or an unsimplified equation leading to required equation stated. May see $y = \frac{1}{3}x + c$ and substitution of coordinates of P to find c but must be complete method. Must have previous A1 to be able to be awarded this A1
		3	

Q	Answer	Marks	Comments
6(b)	$3x^3 + 14x^2 + 17x + 11 = \frac{1}{3}x + \frac{29}{3}$ or $3x^3 + 14x^2 + 17x + 11 - \left(\frac{1}{3}x + \frac{29}{3}\right) \quad [= 0]$ $9x^3 + 42x^2 + 51x + 33 = x + 29$ or $3x^3 + 14x^2 + \frac{50}{3}x + \frac{4}{3} = 0$ and $9x^3 + 42x^2 + 50x + 4 = 0$	M1 A1	AG Must be convincingly shown Additional line of simplification before required result stated.
		2	

Q	Answer	Marks	Comments
6(c)(i)	$\left(x + \frac{4}{3}\right)^2 \dots$ or $(3x+4)^2 \dots$ $9\left(x + \frac{4}{3}\right)^2 - 16 + 2$ $9\left(x + \frac{4}{3}\right)^2 - 14$	<p>M1</p> <p>A1</p> <p>A1</p>	<p>Condone $\frac{8}{6}$ or $\frac{12}{9}$ for $\frac{4}{3}$</p> <p>Allow</p> $9\left(\left(x + \frac{4}{3}\right)^2 - \frac{16}{9}\right) + 2$ $9\left(\left(x + \frac{4}{3}\right)^2 - \frac{16}{9} + \frac{2}{9}\right)$ $9\left(\left(x + \frac{4}{3}\right)^2 - \frac{14}{9}\right)$ or $(3x+4)^2 - 14$ <p>Condone $\frac{8}{6}$ or $\frac{12}{9}$ for $\frac{4}{3}$</p> <p>CAO Condone $\frac{8}{6}$ or $\frac{12}{9}$ for $\frac{4}{3}$</p>
		3	

Q	Answer	Marks	Comments
6(c)(ii)	$\left[9\left(x + \frac{4}{3}\right)^2 - 14 = 0 \Rightarrow \left(x + \frac{4}{3}\right)^2 = \frac{14}{9}\right]$ $\left[x + \frac{4}{3} = \pm \frac{\sqrt{14}}{3}\right]$ $[x =] \frac{-4 \pm \sqrt{14}}{3}$ or $[x =] \frac{-4 + \sqrt{14}}{3}$ and $[x =] \frac{-4 - \sqrt{14}}{3}$	<p>M1</p> <p>A2,1</p>	<p>ft their answer to part (c)(i) provided it is in the correct form.</p> <p>Condone $\frac{8}{6}$ or $\frac{12}{9}$ for $\frac{4}{3}$</p> <p>Sets completed square form equal to zero and rearranges.</p> <p>A1 One correct answer (simplified or unsimplified)</p> <p>A2 Both correct answers (fully simplified)</p> <p>Condone given as sum or difference of two fractions with denominator 3, eg</p> $[x =] -\frac{4}{3} \pm \frac{\sqrt{14}}{3}$
		3	

	Question 6 Total	13	
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Q	Answer	Marks	Comments
7(a)	$\left[\frac{d^2y}{dx^2} = \right] \frac{3}{2} \times 2x^{\frac{1}{2}} - \frac{3}{4} \times 9x^{-\frac{1}{4}}$	M1	At least one term correct
	$\left[\frac{d^2y}{dx^2} = \right] 3x^{\frac{1}{2}} - \frac{27}{4}x^{-\frac{1}{4}}$	A1	ACF with simplified coefficients
		2	

Q	Answer	Marks	Comments
7(b)	$\left[\frac{dy}{dx} = \right] 2t^2 - 9t - 56$	B1	PI Correct substitution.
	$\left[\frac{dy}{dx} = \right] (2t+7)(t-8)$	B1	Correct factorisation.
		2	

Q	Answer	Marks	Comments
7(c)(i)	$[x =] t^{\frac{4}{3}}$	M1	PI Use of x as a power of t
	$[x =] 8^{\frac{4}{3}}$	M1	Finds positive root of $(2t+7)(t-8) = 0$ and raises it to their $\frac{4}{3}$
	$[x =] 16$	A1	ft their c from part (b) . Award SC1 for correct final answer but neither $t^{\frac{4}{3}}$ nor $8^{\frac{4}{3}}$ seen.
		3	

Q	Answer	Marks	Comments
7(c)(ii)	$\left[\frac{d^2y}{dx^2} = 3 \times 16^{\frac{1}{2}} - \frac{27}{4} \times 16^{-\frac{1}{4}} = \right] \frac{69}{8}$ or 8.625 and Since $\frac{d^2y}{dx^2} > 0$ then it is a minimum	E1ft	Evaluates their second derivative with $x = 16$ ft their $x = 16$ and their second derivative provided value of second derivative is positive and gives statement linking positive value of second derivative to it being a minimum.
		1	

Q	Answer	Marks	Comments
7(d)	$0 < x < 16$	B1ft	ft their 16 from part (c)(i) but only if value is positive.
		1	

	Question 7 Total	9	
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Q	Answer	Marks	Comments
8(a)	$4 + 2\sqrt{3} + 2\sqrt{a} + 2\sqrt{3} + 3$ $+ \sqrt{3a} - 2\sqrt{a} - \sqrt{3a} - a$	M1	Complete expansion of brackets, condoning one error
	$7 + 4\sqrt{3} - a$	A1	Correct final form, must be convincingly shown.
		2	

Q	Answer	Marks	Comments
8(a) ALT	$(2 + \sqrt{3})^2 - a$	M1	
	$4 + 2\sqrt{3} + 2\sqrt{3} + 3 - a$ or $4 + 4\sqrt{3} + 3 - a$ and $7 + 4\sqrt{3} - a$	A1	Correct final form, must be convincingly shown. Additional line of simplification before required result stated.
		2	

Q	Answer	Marks	Comments
9(a)	$\left[(3)^3 \right] + 3(3)^2(-2\sqrt{x}) + 3(3)(-2\sqrt{x})^2$ $\left[+(-2\sqrt{x})^3 \right]$ <p>$p = 54$</p> <p>$q = 36$</p>	<p>M1</p> <p>A1</p> <p>A1</p>	<p>For either (1), 3, 3, (1) oe unsimplified.</p> <p>or $\binom{3}{1}(3)^2(-2\sqrt{x})$ or $\binom{3}{1}(3)^2(2\sqrt{x})$</p> <p>or $\binom{3}{2}(3)(-2\sqrt{x})^2$ or $\binom{3}{2}(3)(2\sqrt{x})^2$ oe</p> <p>x not needed, PI</p> <p>If correct working for p seen condone $-54\sqrt{x}$ seen in expansion and p not explicitly stated.</p> <p>CAO</p> <p>Condone $36x$ seen in expansion and q not explicitly stated.</p> <p>If working for p not shown and correct q seen with NMS, award M1 A0 A1</p>
		3	

Q	Answer	Marks	Comments
9(b)(i)	$\frac{27}{\sqrt{x}} - 42 + 36\sqrt{x} - 8x$ <p>or</p> $27x^{-\frac{1}{2}} - 42 + 36x^{\frac{1}{2}} - 8x$ $27x^{-\frac{1}{2}} - 42 + 36x^{\frac{1}{2}} - 8x$ $\left[2 \times 27x^{\frac{1}{2}} - 42x + \frac{2}{3} \times 36x^{\frac{3}{2}} - \frac{1}{2} \times 8x^2 + c \right]$ $54x^{\frac{1}{2}} - 42x + 24x^{\frac{3}{2}} - 4x^2 + c$ <p>or</p> $54\sqrt{x} - 42x + 24x\sqrt{x} - 4x^2 + c$	<p>M1</p> <p>A1</p> <p>B2,1ft</p>	<p>ft their 54 and 36</p> <p>Writing their quotient as a simplified sum of terms in \sqrt{x} and x. Possibly given in index form.</p> <p>Condone one error.</p> <p>Condone $-42 = -54 + 12$</p> <p>CAO</p> <p>Correct expression given in index form.</p> <p>Condone $-42 = -54 + 12$</p> <p>ft their integral provided previous M1 scored.</p> <p>Correct answer given in index or surd form.</p> <p>B2 Fully correct.</p> <p>B1 Three terms correct.</p> <p>Condone omission of $+c$</p>
		4	

Q	Answer	Marks	Comments
9(b)(ii)	$\left(54(9)^{\frac{1}{2}} - 42(9) + 24(9)^{\frac{3}{2}} - 4(9)^2 \right)$ $- \left(54(4)^{\frac{1}{2}} - 42(4) + 24(4)^{\frac{3}{2}} - 4(4)^2 \right)$ [= 108 – 68] 40	M1 A1	Attempts $F(9) - F(4)$ ft their indefinite integral from part (b)(i) .
		2	

Q	Answer	Marks	Comments
9(c)	Area of Trapezium = $\frac{1}{2}(5.75 + 1.5) \times 5$ $\left[= 18.125 \text{ or } \frac{145}{8} \text{ or } 18\frac{1}{8} \right]$ $\frac{1}{2} \times 40 - \frac{1}{2}(5.75 + 1.5) \times 5$ 1.875	B1 B1ft	oe such as fractional equivalents Correct expression for the required area. Must see working using their part (b)(ii) oe ft their $\frac{1}{2} \times 40$ (PI by use of 20) provided it is greater than 18.125 Re-integrating is awarded B0 B0 Accept $\frac{15}{8}$ or $1\frac{7}{8}$
		2	

	Question 9 Total	11	
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Q	Answer	Marks	Comments
10	$\frac{1}{2}(10)(2 \times 12 + (10 - 1)d) \quad [= 480]$ $d = 8$ $\frac{1}{2}k(2 \times 12 + (k - 1) \times 8)$ $= \frac{1}{2}(k - 10)(2 \times 12 + (k - 11) \times 8) + 3360$ or $\frac{1}{2}k(2 \times 12 + (k - 1) \times 8)$ $- \frac{1}{2}(k - 10)(2 \times 12 + (k - 11) \times 8) \quad [= 3360]$ $4k^2 + 8k = 4k^2 - 72k + 320 + 3360$ or $4k^2 + 8k - 4k^2 + 72k - 320 \quad [= 3360]$ $k = 46$ $\left[\sum_{n=1}^{46} u_n = \right] \frac{1}{2}(46)(2 \times 12 + (46 - 1) \times 8)$ $\left[\sum_{n=1}^{46} u_n = \right] 8832$	M1 A1 M1 M1 A1 m1 A1	oe PI Correct expression for $\sum_{n=1}^{10} u_n$ in terms of d with values substituted. CAO oe Values correctly substituted. Correct equation in terms of k setting sum of the sequence equal to the sum of the first $k - 10$ terms and the last 10 terms. or Correct expression in terms of k for the difference between the sum of the sequence and the sum of the first $k - 10$ terms. Condone one error. oe Brackets correctly expanded in an unsimplified equation or expression. CAO oe Substitutes correctly into the formula for the sum of an arithmetic sequence. ft their 46 This is dependent on second M1 only CAO
		7	

Q	Answer	Marks	Comments
<p>10 ALT 1</p>	$\frac{1}{2}(10)(2 \times 12 + (10 - 1)d) [= 480]$ $d = 8$ $12 + (k - 1) \times 8 = 8k + 4$ $\frac{1}{2}(10)(2(8k + 4) + 9 \times (-8)) = 3360$ or $16k - 64 = 672$ $k = 46$ $\left[\sum_{n=1}^{46} u_n = \right] \frac{1}{2}(46)(2 \times 12 + (46 - 1) \times 8)$ or $\left[\sum_{n=1}^{46} u_n = \right] \frac{1}{2}(46)(2(8 \times 46 + 4) + (46 - 1) \times (-8))$ $= \frac{1}{2}(46)(2 \times 372 + 45 \times (-8))$ $\left[\sum_{n=1}^{46} u_n = \right] 8832$	<p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p>	<p>oe PI Correct expression for $\sum_{n=1}^{10} u_n$ in terms of d with values substituted.</p> <p>CAO</p> <p>oe PI Simplified or unsimplified. Expression for the kth term of the sequence.</p> <p>oe Correct equation in terms of k setting the sum of the first 10 terms of the reversed sequence equal to 3360 Simplified or unsimplified. ft their $-d$ and $8k + 4$</p> <p>oe Substitutes correctly into the formula for the sum of an arithmetic sequence. May apply to sequence in reversed or original order. ft their 46 This is dependent on second M1 only</p> <p>CAO</p>
		7	

Q	Answer	Marks	Comments
<p>10 ALT 2</p>	$\frac{1}{2}(10)(2 \times 12 + (10 - 1)d) [= 480]$ <p>$d = 8$</p> $12 + (k - 10) \times 8$ <p>or</p> $8k - 68$ $\frac{1}{2}(10)(2(8k - 68) + 9 \times (8)) = 3360$ <p>or</p> $16k - 64 = 672$ $k = 46$ $\left[\sum_{n=1}^{46} u_n = \right] \frac{1}{2}(46)(2 \times 12 + (46 - 1) \times 8)$ $\left[\sum_{n=1}^{46} u_n = \right] 8832$	<p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p> <p>7</p>	<p>oe PI Correct expression for $\sum_{n=1}^{10} u_n$ in terms of d with values substituted.</p> <p>CAO</p> <p>oe PI Simplified or unsimplified. Expression for the $(k - 9)$ th term of the sequence.</p> <p>oe Correct equation in terms of k setting the sum of the last 10 terms of the sequence equal to 3360. Simplified or unsimplified. ft their d and $8k - 68$</p> <p>CAO</p> <p>oe Substitutes correctly into the formula for the sum of an arithmetic sequence. ft their 46 This is dependent on second M1 only</p> <p>CAO</p>
	Question 10 Total	7	