OXFORD
INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL AS
MATHEMATICS
MA02

(9660/MA02) Unit PSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

January 2022

Version: 1.0 Final

2 2 1 X M A 0 2 / M S

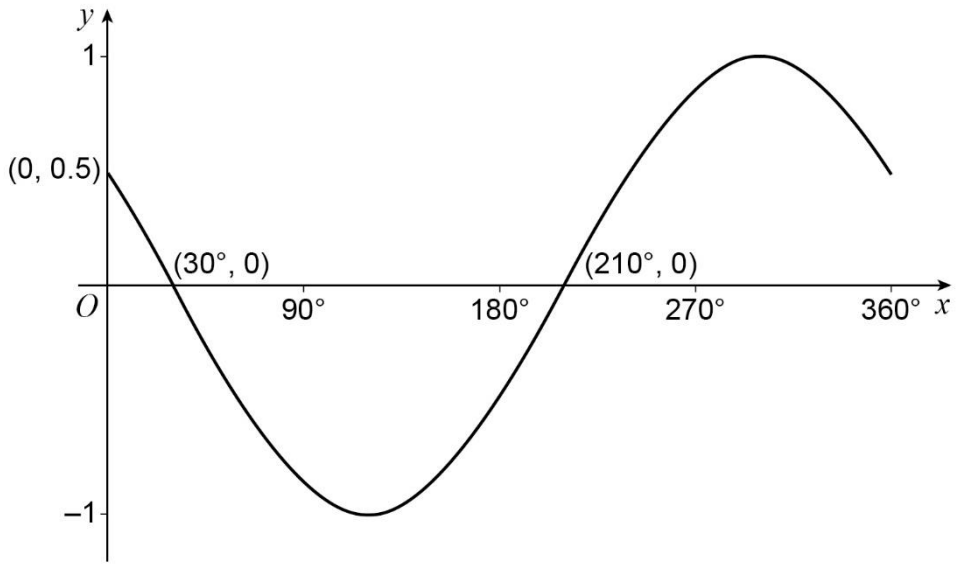
Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from oxfordaqaexams.org.uk

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	**See artwork below**	**B1** **B1** **B1**	Sinusoidal curve of correct form with one minimum below the x-axis and one maximum above, both with the correct amplitude Ignore any section of the curve drawn outside of $0^\circ \leq x \leq 360^\circ$ The curve must intersect the x-axis at exactly two points; once between 0° and 90° and again between 180° and 270° Both correct coordinates for intercepts with the x-axis given and no others Condone values only indicated Correct coordinates of intercept with the y-axis Condone value only indicated
			
		3	

Q	Answer	Marks	Comments
1(b)	$\frac{360°}{4}$ $\left[=90°\right]$ or $360° = \frac{4\times180°}{k}$ **oe**	M1	**PI** Dividing the period of f by 4 Could be implied by indicating that the graph of $y=\tan x$ is mapped onto the graph of $y=\tan(kx)$ by a stretch sf $\frac{1}{2}$ in the x-direction. Condone 2π in place of $360°$ (i.e. working in radians)
	$k = 2$	A1	**CAO** Accept $k=-2$ or $k=\pm2$
		2	

	Question 1 Total	**5**	

Q	Answer	Marks	Comments
2(a)	$\left[BD^2 =\right]\ 9.1^2 + 10.3^2 - 2\times 9.1\times 10.3\times \cos 0.9$	**M1**	Correct substitution into the Cosine Rule May be partially evaluated
	$\left[BD^2 =\right]\ 72.3\left[7299...\right]$ or $\left[BD =\right]\ 8.50\left[723...\right]$ and $8.5\left[\text{cm}\right]$	**A1**	**AG CAO** Correct value of BD^2 rounded or truncated to at least one decimal place, or correct value of BD rounded or truncated to at least two decimal places before **AG**
		2	

Q	Answer	Marks	Comments
2(b)	$\left[\angle BDC =\right]\ \frac{5.1}{8.50723...}\ \left[=0.59948...\right]$ **oe** and $\left[\angle BDC =\right]\ 0.6\ \left[\text{radians}\right]$	**B1**	Condone $BD = 8.5$
		1	

Q	Answer	Marks	Comments
2(c)	$\left[\frac{1}{2}r^2\theta =\right]\ \frac{1}{2}\times(8.50723...)^2\times 0.59948...$	**M1**	Correct expression for area of sector *BCD* Condone $BD = 8.5$ and $\angle BDC = 0.6$
	$[\text{Area of Sector} =]\ 21.69344...\ [\text{cm}^2]$	**A1**	**AWRT** 21.7 **PI** by correct final answer or 1.28
	$[\text{Area of Triangle} =]$ $\frac{1}{2}\times(8.50723...)^2\times\sin 0.59948...$	**M1**	Correct expression for area of triangle *BCD* Condone $BD = 8.5$ and $\angle BDC = 0.6$
	$[\text{Area of Triangle} =]\ 20.41719...\ [\text{cm}^2]$	**A1**	**AWRT** 20.4 **PI** by correct final answer or 1.28
	$[\text{Shaded Area} =]\ 1.3\ [\text{cm}^2]$	**A1**	**CAO**
		5	

	Question 2 Total	**8**	

Q	Answer	Marks	Comments
3(a)	$\frac{17-5}{9-k}$	**B1**	**PI** Correct expression in terms of k for the gradient of *QR*
	$\frac{17-5}{9-k}\times\frac{1}{3}=-1$	**M1**	**oe** Use of $m\times m'=-1$ and their gradient of *QR* to form an equation in k
	$3k-27=12$ **oe**		**AG CAO**
	and $k=13$	**A1**	Must be a further line of working: linear equation in k and required result stated
3(a) Alt	- 3	**B1**	Correct gradient of *QR* **PI** in later working.
	$y=-3x+44$	**M1**	**oe** Correct equation of the line *QR*
	$5=-3k+44$ **oe**		**CSO**
	and $k=13$	**A1**	Substitutes the coordinates of *R* into the equation of the line *QR* and **AG**
		3	

Q	Answer	Marks	Comments
3(b)(i)	$\sqrt{(13-8)^2+(5-10)^2}$	**M1**	**oe** Correct expression for the radius unsimplified
	$r=5\sqrt{2}$ or $\sqrt{50}$	**A1**	Exact value only
		2	

Q	Answer	Marks	Comments
3(b)(ii)	$(x-8)^2+(y-10)^2=50$	**B1ft**	**ft** their r^2 provided it is an exact value.
		1	

Q	Answer	Marks	Comments
3(c)(i)	$(x-4)^2+\ldots+(y+3)^2+\ldots \quad [=0]$	**M1**	Attempts to complete the square twice by, for example $(x\pm4)^2$ and $(y\pm3)^2$
	$(4,-3)$	**A1**	Condone not given as coordinates but must be clearly identified
		2	

Q	Answer	Marks	Comments
3(c)(ii)	Translation	**E1**	Independent. Correctly identifies the type of transformation
	$\begin{bmatrix}4\\13\end{bmatrix}$	**E1ft**	Correct vector Must be given in vector form **ft** their centre from **part (c)(i)**
		2	

	Question 3 Total	**10**	

Q	Answer	Marks	Comments
4(a)	$[\alpha =]\ 22^\circ$	**B1**	**CAO**
	$[\alpha =]\ 68^\circ$	**B1**	**CAO** **SC1** Both correct values of a given but neither rounded to the nearest degree. $\alpha = 22.213502...^\circ$ $\alpha = 67.786498...^\circ$
		2	

Q	Answer	Marks	Comments
4(b)(i)	$\frac{10}{\sin x} - 3\frac{\sin x}{\cos x} = \frac{11\cos x}{\sin x}$	**M1**	**PI** in later working. Use of $\tan x = \frac{\sin x}{\cos x}$ obtaining a correct equation in $\sin x$ and $\cos x$
	$10 - 3\frac{\sin^2 x}{\cos x} = 11\cos x$ or $10\cos x - 3\sin^2 x = 11\cos^2 x$	**M1**	Manipulation to include $\sin^2 x$ Must be seen
	$10 - 3\frac{\left(1-\cos^2 x\right)}{\cos x} = 11\cos x$ or $10\cos x - 3\left(1-\cos^2 x\right) = 11\cos^2 x$ or $8\cos^2 x + 3\left(\cos^2 x + \sin^2 x\right) - 10\cos x = 0$ and $8\cos^2 x - 10\cos x + 3 = 0$	**A1**	Clear use of $\cos^2 x + \sin^2 x = 1$ and required result stated Be convinced. **AG**
		3	

Q	Answer	Marks	Comments
4(b)(ii)	$[\cos x =]\ \frac{1}{2}$ and $[\cos x =]\ \frac{3}{4}$	**B1**	**PI** Both correct roots
	$\cos(\theta - 15^\circ) = \frac{1}{2}$ or $[\theta - 15^\circ =] \cos^{-1}\left(\frac{1}{2}\right)$ or $\cos(\theta - 15^\circ) = \frac{3}{4}$ or $[\theta - 15^\circ =] \cos^{-1}\left(\frac{3}{4}\right)$	**M1**	**PI** by $41.4[09622...]^\circ$ or 60° or a correct angle q **ft** their roots.
	$[\theta =]\ 56.4^\circ$	**A1**	**CAO**
	$[\theta =]\ 75^\circ$	**A1**	**CAO** Condone 75.0° If both correct answers given award **A1A0** if their final answer includes incorrect angle(s) in the given interval.
		4	

	Question 4 Total	**9**	

Q	Answer	Marks	Comments
5(a)	$\log_7 7^{p-5} = \log_7 9$ or $p - 5 = \log_7 9$	**M1**	**PI** in later working. Coordinates substituted into the equation and $\log_7$ of both sides taken
	$p-5$ or $2\log_7 3$	**M1**	**PI** Correct application of a logarithm rule to one term
	$[p =]\ 5 + 2\log_7 3$	**A1**	Condone log instead of $\log_7$
		3	

Q	Answer	Marks	Comments
5(b)	$[8 =] \log_n n^8$	**B1**	**PI** Writing 8 as $\log_n n^8$
	$8 + \log_n k + \log_n (2y)^4 = \log_n \left(n^2 y\right)^6$	**M1**	**oe** One correct use of logarithm rule in an equation. Powers may be partially manipulated
	$\log_n \left(n^8 k (2y)^4\right) = \log_n \left(n^2 y\right)^6$	**M1**	**oe** One correct use of second logarithm rule in an equation. Powers may be partially manipulated. Must have single logarithm on both sides
	$\left[\log_n \left(n^8 k (2y)^4\right) = \log_n \left(n^2 y\right)^6\right]$ $\Rightarrow n^8 k (2y)^4 = \left(n^2 y\right)^6$ $\Rightarrow 16 n^8 k y^4 = n^{12} y^6$ $\Rightarrow y^2 = \frac{16 n^8 k}{n^{12}}$	**m1**	**oe** $\log_n$ correctly eliminated and rearranged to isolate term in y Dependant on **M1 M1** being awarded
	$y = \frac{4\sqrt{k}}{n^2}$	**A1**	**CAO** Accept $y = 4k^{\frac{1}{2}} n^{-2}$
		5	

Q	Answer	Marks	Comments
5(b) ALT 1	$[8 =] \log_n n^8$	**B1**	**PI** Writing 8 as its equivalent in $\log_n$
	$\left[\begin{array}{l} 8 + \log_n k + \log_n (2y)^4 = 6\left(\log_n \left(n^2\right) + \log_n y\right) \\ \Rightarrow 8 + \log_n k + \log_n (2y)^4 = 6\log_n \left(n^2\right) + 6\log_n y \end{array}\right]$ $\Rightarrow 8 + \log_n k + \log_n (2y)^4 = \log_n \left(n^2\right)^6 + \log_n y^6$	**M1**	**oe** One correct use of logarithm rules in an equation. Powers may be partially manipulated. Must have correctly manipulated the coefficient 6
	$\left[\log_n n^8 + \log_n k + \log_n (2y)^4 - \log_n y^6 = \log_n \left(n^2\right)^6\right]$ $\Rightarrow \log_n \left(\dfrac{n^8 k (2y)^4}{y^6}\right) = \log_n \left(n^2\right)^6$	**M1**	**oe** One correct use of different logarithm rules in an equation Powers may be partially manipulated. Must have single logarithm on both sides
	$\dfrac{n^8 k (2y)^4}{y^6} = \left(n^2\right)^6$ $\Rightarrow n^8 k (2y)^4 = \left(n^2\right)^6 y^6$ $\Rightarrow 16n^8 k y^4 = n^{12} y^6$ $\Rightarrow y^2 = \dfrac{16n^8 k}{n^{12}}$	**m1**	**oe** $\log_n$ correctly eliminated and rearranged to isolate term in y Dependant on **M1 M1** being awarded
	$y = \dfrac{4\sqrt{k}}{n^2}$	**A1**	**CAO** Accept $y = 4k^{\frac{1}{2}} n^{-2}$
		5	

Q	Answer	Marks	Comments
5(b) ALT 2	$[8 =] \log_n n^8$	**B1**	**PI** Writing 8 as its equivalent in $\log_n$
	$8 = \log_n \left(n^2 y\right)^6 - \log_n k - \log_n (2y)^4$	**M1**	**oe** One correct use of logarithm rule in an equation. Powers may be partially manipulated
	$8 = \log_n \left[\dfrac{\left(n^2 y\right)^6}{k(2y)^4}\right]$	**M1**	**oe** One correct use of second logarithm rule in an equation. Powers may be partially manipulated. Must have single logarithm on the right-hand side
	$n^8 k(2y)^4 = \left(n^2 y\right)^6$ $\Rightarrow 16n^8 k y^4 = n^{12} y^6$ $\Rightarrow y^2 = \dfrac{16n^8 k}{n^{12}}$	**m1**	**oe** $\log_n$ correctly eliminated and rearranged to isolate term in y Dependant on **M1 M1** being awarded
	$y = \dfrac{4\sqrt{k}}{n^2}$	**A1**	**CAO** Accept $y = 4k^{\frac{1}{2}} n^{-2}$
		5	

	Question 5 Total	**8**	

Q	Answer	Marks	Comments
6(a)	$k + 4k + 9k + 16k$	M1	**oe** Award for $30k$ seen.
	$k + 4k + 9k + 16k = 1$ **oe** and $k = \frac{1}{30}$	A1	**AG** Sets the sum of the four probabilities in terms of k equal to 1, leading to correct final answer
		2	

Q	Answer	Marks	Comments
6(b)	$\mathrm{P}(X \geq 3) = \frac{5}{6}$	B1	**oe AWRT** 0.833
		1	

Q	Answer	Marks	Comments
6(c)	$\mathrm{E}\left(\frac{1}{X^3}\right) = \frac{1}{1^3} \times \frac{1}{30} + \frac{1}{2^3} \times \frac{4}{30} + \frac{1}{3^3} \times \frac{9}{30} + \frac{1}{4^3} \times \frac{16}{30}$	M1	Applies formula **PI** by final answer of **AWRT** 0.069
	$= \frac{5}{72}$	A1	**oe** Must be exact value.
		2	

Q	Answer	Marks	Comments
6(d)	$\mathrm{E}(X - Y)\ [= \mathrm{E}(X) - \mathrm{E}(Y)] = \frac{10}{3} - 10$	M1	**PI** by correct final answer
	$= -\frac{20}{3}$	A1	**oe, AWRT** –6.67
		2	

	Question 6 Total	**7**	

Q	Answer	Marks	Comments
7(a)	$[\mathrm{P}(A \cap B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cup B)]$ $\mathrm{P}(A \cap B) = 0.2 + 0.35 - 0.48$ $\mathrm{P}(A \cap B) = 0.07$	M1 A1	oe
		2	

Q	Answer	Marks	Comments
7(b)	A and B are not mutually exclusive as $\mathrm{P}(A \cap B) \neq 0$	E1	Condone 'no' as statement and a correct reason
		1	

Q	Answer	Marks	Comments
7(c)	$\mathrm{P}(B \mid A) = \frac{0.07}{0.2}$ $\mathrm{P}(B \mid A) = 0.35$	M1 A1	PI
		2	

Q	Answer	Marks	Comments
7(d)	A and B are independent as $\mathrm{P}(B \mid A) = \mathrm{P}(B)$	E1	Condone 'yes' as statement and $\mathrm{P}(B \mid A) = \mathrm{P}(B)$ or $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$
		1	

	Question 7 Total	**6**	

Q	Answer	Marks	Comments
8(a)(i)	$\mathrm{E}(X) = p$	**B1**	
		1	

Q	Answer	Marks	Comments
8(a)(ii)	$\mathrm{Var}(X) = p - p^2$ or $p(1-p)$	**B1**	
		1	

Q	Answer	Marks	Comments
8(b)	$\mathrm{E}\left(\sum_{i=1}^{n} X_i\right) = np$	**B1ft**	**PI** in later working. **ft** their answer from **part (a)(i)**
	$\mathrm{Var}\left(\sum_{i=1}^{n} X_i\right) = np(1-p)$	**B1ft**	**oe** **PI** in later working. **ft** their answer from **part (a)(ii)**
	$np = 6.25$ and $np(1-p) = 4.6875$ $6.25(1-p) = 4.6875$	**M1**	Forms two equations in terms of n and p using their expectation and variance, and correctly eliminates p or n **PI** by correct value of p or n Allow q in place of $1 - p$
	$p = 0.25$	**A1**	**oe**
	$n = 25$	**A1**	
		5	

	Question 8 Total	**7**	

Q	Answer	Marks	Comments
9(a)	$10\left[\text{N s}\right]$	**B1**	
		1	

Q	Answer	Marks	Comments
9(b)	$(5\times 2)-(-1.5\times 2)$ $13\left[\text{N s}\right]$	**M1** **A1**	**oe** Condone one sign error for **M1**
		2	

	Question 9 Total	**3**	

Q	Answer	Marks	Comments
10	$T-0.2\times 9.8=0.2\times 1.25$	**M1**	**PI** in later working. Correct equation of motion for *P*
	$m\times 9.8-T=m\times 1.25$	**M1**	**PI** in later working. Correct equation of motion for *Q*
	$9.8m-1.96=0.25+1.25m$ or $9.8m-2.21=1.25m$	**M1**	Forms correct equation in m only, possibly by adding both equations of motion or by finding $T=2.21$ and using it to form the equation of motion for *Q*
	$m=0.26$	**A1**	**AWRT**
		4	

	Question 10 Total	**4**	

Q	Answer	Marks	Comments
11(a)	$\left[a = \frac{\mathrm{d}v}{\mathrm{d}t} = \right] \ \frac{3t^2}{2} - 6t + 5$ $\frac{3 \times 2^2}{2} - 6 \times 2 + 5$ $-1 \left[\mathrm{m\,s^{-2}}\right]$	**M1** **m1** **A1**	**oe** At least two correct terms. Substitutes $t = 2$ into their differentiated expression **CAO**
		3	

Q	Answer	Marks	Comments
11(b)	$\frac{t^3}{2} - 3t^2 + 5t = \frac{t}{2}\left(t^2 - 6t + 10\right)$ $\left[\frac{t}{2}(\ \ )(t-3)^2 + 1[\ \ ]\right]$ or $[\Delta =](-6)^2 - 4 \times 1 \times 10 = -4$ $\left[v \geq 0 \text{ or } (t-3)^2 + 1 > 0 \text{ or } \frac{t}{2}(t-3)^2 + \frac{t}{2} \geq 0\right]$ Velocity is never negative, so the particle does not change direction or As the quadratic equation has no real roots, the velocity is never zero [for $t > 0$] Hence, the particle does not change direction.	**M1** **A1** **E1**	**oe** Factorising by t or attempt to complete the square of the quadratic. **PI** by $t^2 - 6t + 10$ **oe** Correct completed square form or shows the correct discriminant is negative
		3	

Q	Answer	Marks	Comments
11(c)	$\left[\int\left(\frac{t^3}{2} - 3t^2 + 5t\right)\mathrm{d}t = \right] \frac{t^4}{8} - t^3 + \frac{5t^2}{2} \left[+c\right]$	**M1**	At least two terms correct.
	$\frac{4^4}{8} - 4^3 + \frac{5\times 4^2}{2} \quad \left[-\left(\frac{0^4}{8} - 0^3 + \frac{5\times 0^2}{2}\right)\right]$	**M1**	Sight of a correct attempt to evaluate their integral between $t = 0$ and $t = 4$ **PI** by correct answer
	8 [m]	**A1**	**CAO**
		3	

	Question 11 Total	**9**	

Q	Answer	Marks	Comments
12(a)	$\left[s = ut + \frac{1}{2}at^2 \right]$ $150 = 0 \times 3.4 + \frac{1}{2} \times a \times 3.4^2$ or $150 = 5.78a$ $26\ \text{ms}^{-2}$	**M1** **A1** **B1**	**oe** Use of one or more constant acceleration formulae to obtain a correct equation in a **AWRT** 26 Correct value for the acceleration. Independent. Correct units.
		3	

Q	Answer	Marks	Comments
12(b)	Any correct reason, such as: • Initial speed unlikely to be 0 [due to momentum of the firework or due to the explosion] • The part may be moving in a direction different to the vertical	**E1**	Any valid reason that implies any of $u \neq 0$, $s \neq 150$ or non-linear motion Do not award for 'answer is rounded', 'errors in the calculation' or '9.81 should be used for acceleration due to gravity'
		1	

	Question 12 Total	**4**	