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(9660/MA01) Unit P1 Pure Mathematics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)(i)	7	**B1**	
		1	

Q	Answer	Marks	Comments
1(a)(ii)	-35	**B1**	
		1	

Q	Answer	Marks	Comments
1(b)	$[y =]\ ((-x)-7)^2-35$ or $[y =]\ (-x)^2-14(-x)+49-35$ or $[y =]\ (x+7)^2-35$ or $[y =]\ x^2+14x+49-35$ $y=x^2+14x+14$	**M1** **A1**	Substitutes $-x$ for x in the equation of *C* or forms the completed square form of the equation using the vertex $(-7,-35)$ of *D* $(-x)^2$ may be **PI** by x^2 **CAO**
		2	

	Question 1 Total	**4**	

Q	Answer	Marks	Comments
2(a)	$a = \frac{1}{2}$ or 0.5	**B1**	
	either $3^{13a-8b} = 3^4$ or $13a - 8b = 4$ **oe** **or** $3^{\frac{13}{2}-8b} = 3^4$ or $\frac{13}{2} - 8b = 4$ **oe**	**M1**	**PI** Either Uses power rules to form an equation in a and b **or** Substitutes their value of a and uses power rules to form an equation in b
	$b = \frac{5}{16}$ or 0.3125	**A1ft**	**ft** their a with $b = \frac{13a-4}{8}$
		3	

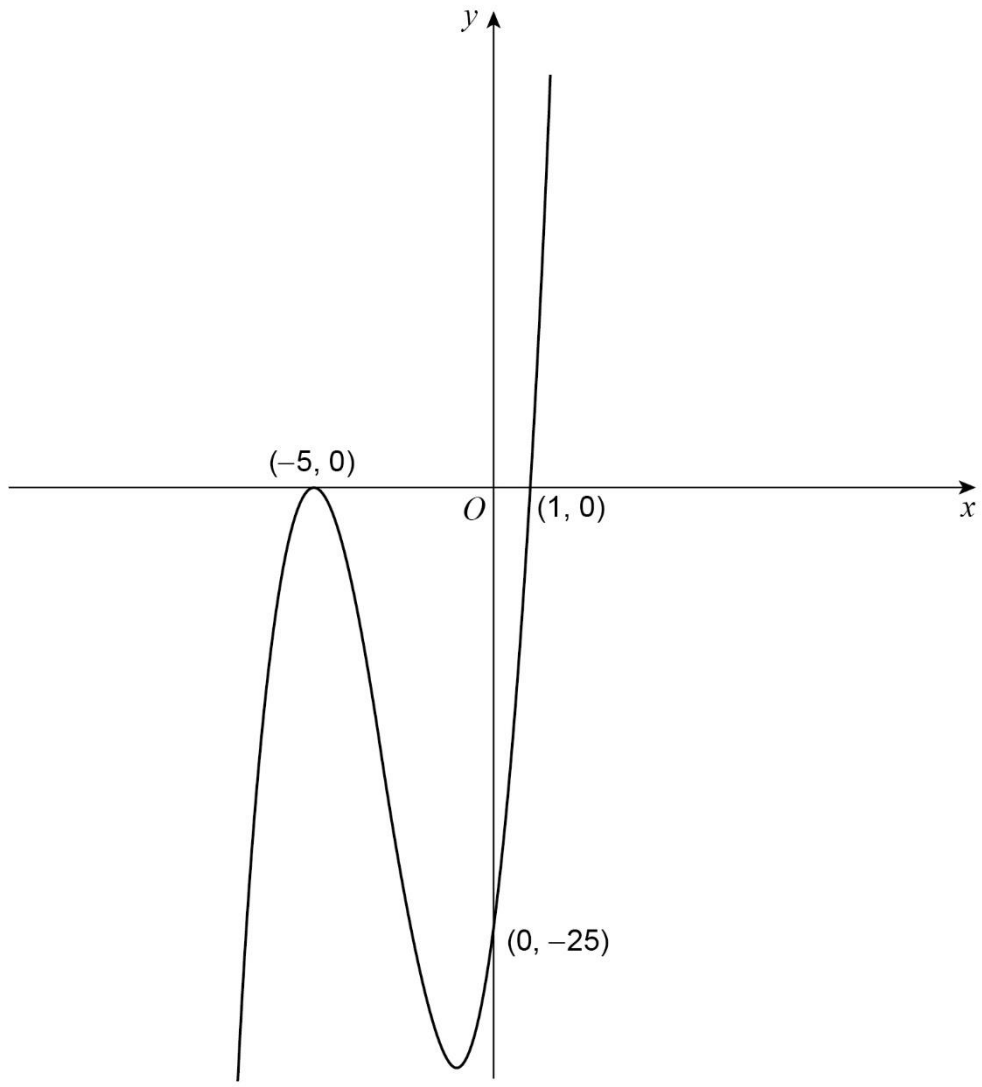
Q	Answer	Marks	Comments
2(b)	$\left[\sqrt[4]{16x^{12}y^8} =\right] 2x^3y^2$	**B1**	**oe** Possibly $2x^{\frac{12}{4}}y^{\frac{8}{4}}$ Correctly writes as a product of powers **PI** by correct final answer
	kx^8y^{-7} or $12x^my^{-7}$ or $12x^8y^n$	**M1**	Use of power rules to write as the product of an integer and powers of x and y Two terms correct in their product
	$12x^8y^{-7}$	**A1**	**CAO** Condone $\pm 12x^8y^{-7}$
		3	

	Question 2 Total	**6**	

Q	Answer	Marks	Comments
3(a)	$\left[\text{f}(6) =\right]\ 6^3 + 9\times 6^2 + 15\times 6 + k$ $216 + 324 + 90 + k = 605$ **oe** and $k = -25$	**M1** **A1**	**oe** $\text{f}(6)$ attempted Terms may be partially evaluated Must include k **PI** by later working **AG CSO** $\text{f}(6)$ with powers and products evaluated set equal to 605 (or better) leading to the required result Remainder Theorem not used scores **M0A0**
		2	

Q	Answer	Marks	Comments
3(b)(i)	$c = 25$	**B1**	
		1	

Q	Answer	Marks	Comments
3(b)(ii)	$\left[b^2 - 4ac = 0 \Rightarrow\right]\ b^2 - 4\times 1\times 25 = 0$ $b = 10$	**M1** **A1**	Discriminant clearly used and set equal to zero. **ft** their c Condone $b^2 - 4\times 25 = 0$ or $b^2 - 100 = 0$ or $b^2 = 100$ Discriminant not used scores **M0A0** Final answer of $b = \pm 10$ scores **M1A0**
		2	

Q	Answer	Marks	Comments
3(c)	**See artwork below**	**B1**	Correct positive cubic graph with two vertices and maximum tangential to the x-axis and minimum in the 3rd quadrant
		B1	Correct coordinates of both x-intercepts. Condone given as values rather than coordinates
		B1	Correct coordinates of y-intercept. Condone given as value rather than coordinates
			
		3	

	Question 3 Total	**8**	

Q	Answer	Marks	Comments
4(a)	$a+2d=2(a+18d)$ or $a+2d=2a+36d$	**M1**	**oe** Correct equation relating a and d in Month 3 and Month 19
	$a+13d=252$	**M1**	**oe** Correct equation in a and d for the number of cars produced in Month 14
	$a=408$	**A1**	**CAO**
	$d=-12$	**A1**	**CAO**
		4	

Q	Answer	Marks	Comments
4(b)	$\frac{1}{2}\times 34(2\times 408+(34-1)\times(-12))$ or $408+(34-1)\times(-12)\quad[=12]$ and $\frac{1}{2}\times 34\times(408+12)$	**M1**	**oe ft** their a and d from **part (a)** seen substituted
	7140	**A1**	**CAO**
		2	

	Question 4 Total	**6**	

Q	Answer	Marks	Comments
5(a)	[Gradient of l_1 =] $\frac{3}{5}$ $\frac{k-(-2)}{18-3} = \frac{3}{5}$ or $\left[y-(-2)=\frac{3}{5}(x-3)\Rightarrow\right]$ $k+2=\frac{3}{5}(18-3)$ $5k+10=45$ **oe** and $k=7$	**B1** **M1** **A1**	**PI** in later working. **oe** Uses the coordinates of *A* and *C* to form an expression equal to their gradient or **oe** Substitutes the coordinates of *C* into the equation of the line through *A* and *C* **CSO AG** Must be clear line of working before required result stated
		3	

Q	Answer	Marks	Comments
5(b)(i)	$[CE=]\quad \sqrt{(18-13)^2+(7-4)^2}\quad \left[=\sqrt{34}\right]$ $\left(2\sqrt{17}\right)^2=\left(\sqrt{34}\right)^2+(BE)^2$ $(BE)^2 = 68 - 34 = 34$ $BE = CE\left[=\sqrt{34}\right]$	**M1** **M1** **A1**	**oe PI** Forms an expression for the length or the square of the length of *CE* **oe** Applies Pythagoras' Theorem to triangle *BCE* **CSO** Extra line of working and concludes $BE=CE$ Extra line of working could be $BE=\sqrt{34}$ if it comes from correct working.
		3	

Q	Answer	Marks	Comments
5(b)(ii)	Use of: (Translation *E* to *B*) $\begin{bmatrix} -3 \\ 5 \end{bmatrix}$ or (Translation *E* to *D*) $\begin{bmatrix} 3 \\ -5 \end{bmatrix}$	**M1**	Use of translation or equivalent to find: correct coordinates for *B* or *D* or both correct x-coordinates for *B* and *D* or both correct y-coordinates for *B* and *D*
	$B(10, 9)$	**A1**	
	$D(16, -1)$	**A1**	**SC2** for both correct coordinates but incorrectly identified
		3	

Q	Answer	Marks	Comments
5(b)(iii)	$m' = -\frac{5}{3}$	**B1ft**	Correct gradient of l_2 **ft** their gradient of l_1 from **part (a)** Possibly embedded in later working
	$\frac{y-4}{x-13} = -\frac{5}{3}$	**M1**	**oe** **ft** their gradient of l_1 from **part (a)** May use coordinates of *B*, *D* or *E* May see $y = -\frac{5}{3}x + p$ and substitution of coordinates of *B*, *D* or *E* to find p but must be a complete method
	$5x + 3y = 77$ **or** $y = -\frac{5}{3}x + \frac{77}{3}$	**A1**	**oe** Correct equation
	$\left(\frac{17}{2}, \frac{23}{2}\right)$	**m1, A1**	**m1** solving equations simultaneously to obtain one correct value for x or y **A1** for correct coordinates Accept decimal equivalents Accept if not given as coordinates but must be clearly identified
		5	

	Question 5 Total	**14**	

Q	Answer	Marks	Comments
6(a)(i)	*P* is a minimum point since $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0$	**E1**	States that it is a minimum point and indicates that the second derivative is positive
		1	

Q	Answer	Marks	Comments
6(a)(ii)	$\left[4x - 5 = 11 \Rightarrow\right]\ \ x = 4$ $2 \times 4^2 - 5 \times 4 + d = 0$ **oe** and $d = -12$	**M1** **A1**	**PI** May be seen embedded in expression for first derivative. Correct x-coordinate of *P* **AG CSO** Substitutes $x = 4$ into the expression for the first derivative and sets equal to zero before required result stated
		2	

Q	Answer	Marks	Comments
6(b)	$m = 30$	**B1**	Correct gradient of tangent at Q seen or used.
	$\left[2a^2 - 5a - 12 = 30\right]$ $2a^2 - 5a - 42 = 0$	**M1**	Forms quadratic equation set equal to zero, using $m = \pm 30$ **PI** by $a = 6$ or correct x-coordinate of Q
	$\left[(2a + 7)(a - 6) = 0\right]$ $a = 6$	**A1**	**CAO** Correct x-coordinate of Q Ignore if $a = -\frac{7}{2}$ given as well
	$\left[\int (2x^2 - 5x - 12)\mathrm{d}x = \right]$ $\frac{2}{3}x^3 - \frac{5}{2}x^2 - 12x + c$	**B2,1**	**B2** for fully correct integration **B1** for two correct terms Condone $+c$ omitted. Simplified or unsimplified.
	$\frac{2}{3}(6)^3 - \frac{5}{2}(6)^2 - 12 \times 6 + c = 14$	**M1**	**oe** Substitutes $x = 6$ into their integral and sets equal to 14 in an attempt to evaluate c. Must have $+c$ term **ft** their $a = 6$
	$y = \frac{2}{3}x^3 - \frac{5}{2}x^2 - 12x + 32$	**A1**	**CAO**
		7	

	Question 6 Total	**10**	

Q	Answer	Marks	Comments
7(a)	$h = 1$	**B1**	**PI** in later working
	$\left[I \approx \frac{h}{2}\{...\} \right]$ $\{...\} = \frac{k}{2} + \frac{k}{6} + 2\left(\frac{k}{3} + \frac{k}{4} + \frac{k}{5}\right)$	**M1**	**oe**
	$\frac{k}{2}\left(\frac{1}{2} + \frac{1}{6} + 2\left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right)\right) = 14.07$ or $\frac{67k}{60} = 14.07$	**M1**	**oe** Sets equal to 14.07 and correctly factorises out k or forms correct equation with one term in k
	$k = 12.6$ or $\frac{63}{5}$	**A1**	**CAO**
		4	

Q	Answer	Marks	Comments
7(b)(i)	Over-estimate	**E1**	Over-estimate stated
	The tops of the trapezia/strips are above the curve	**E1**	Any valid explanation, such as 'convex' or 'concave upwards' **E0E1** not possible.
		2	

Q	Answer	Marks	Comments
7(b)(ii)	Increase the number of strips/ordinates/trapezia	**E1**	Valid explanation Condone 'make h smaller' **oe**
		1	

	Question 7 Total	**7**	

Q	Answer	Marks	Comments
8(a)(i)	$\left[\mathrm{f}'(x)=\right]\ 3x^2-12x+57$	**B1**	Correct derivative
		1	

Q	Answer	Marks	Comments
8(a)(ii)	$(x-2)^2$...	**M1**	**PI**
	$3(x-2)^2-12+57$	**A1**	Allow $3\left((x-2)^2-4\right)+57$ $3\left((x-2)^2-4+19\right)$ $3\left((x-2)^2+15\right)$
	$3(x-2)^2+45$	**A1**	**CAO**
	$(x-2)^2\geq 0$ [for all real values of x]	**E1ft**	**PI** by, for example, $\mathrm{f}'(x)\geq 45$ or a statement that implies that the curve of $\mathrm{f}'(x)$ is always on or above the line $y = 45$, such as (2, 45) is a minimum **ft** their $(x+b)^2$ Condone $3(x-2)^2 \geqslant 0$
	$\left[\mathrm{f}'(x)=\right]\ 3(x-2)^2+45>0$ [for all real values of x] and hence f is increasing [for all x]	**E1ft**	**ft** their $a(x+b)^2+c$ provided a and c are both positive Statement saying $\mathrm{f}'(x)$ is positive for all x and concluding statement $3(x-2)^2+45\geq 0$ scores **E0** (Possible to award **E0E1**)
		5	

Q	Answer	Marks	Comments
8(b)	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]\ \frac{3}{8}x^{\frac{1}{2}}-8x^{-\frac{3}{2}}$	**B1**	Correct derivative simplified or unsimplified
	$\frac{3}{8}(16)^{\frac{1}{2}}-8(16)^{-\frac{3}{2}}$	**M1**	Substitutes $x=16$ into their derivative. May be partially evaluated
	$m=\frac{11}{8}$ or 1.375	**A1**	Correct gradient of tangent
	$y-13=\frac{11}{8}(x-16)$	**m1**	**oe** Forms correct equation for the tangent at *P* **ft** their gradient
	$\left(\frac{72}{11},0\right)$	**A1**	Must be exact value of x-coordinate. Condone not given as coordinates if clearly identified
		5	

	Question 8 Total	**11**	

Q	Answer	Marks	Comments
9	$na = -\frac{14}{5}$	**B1**	Condone $-\frac{14}{5} = -2.8$ **PI** by correct subsequent substitution seen
	$\frac{n(n-1)}{2}a^2 = \frac{84}{25}$	**B1**	**oe**, eg $\frac{(na)^2 - (na)a}{2} = \frac{84}{25}$ and $\frac{196}{25} + \frac{14}{5}a = \frac{168}{25}$ Condone $\frac{84}{25} = 3.36$ Condone $a = -\frac{14}{5n}$ substituted
	$\left[a = -\frac{14}{5n} \Rightarrow\right] \quad \frac{n(n-1)}{2} \times \left(-\frac{14}{5n}\right)^2 = \frac{84}{25}$	**M1**	**oe** Correct substitution for a into their $\frac{n(n-1)}{2}a^2 = \frac{84}{25}$
	$\frac{98n(n-1)}{n^2} = 84$ **oe** and $98(n-1) = 84n$ **oe** and $n = 7$	**A1**	**CSO AG** Be convinced.
	$a = -\frac{2}{5}$	**B1**	**CAO** Condone $a = -0.4$
	$\frac{7(7-1)(7-2)}{6} \times \left(-\frac{2}{5}\right)^3 = -b$ **oe** or $\binom{7}{3} \times \left(-\frac{2}{5}\right)^3 = -b$ **oe**	**M1**	Correct substitution of $n = 7$ and their a
	$b = \frac{56}{25}$ or 2.24	**A1**	**CAO** If $n = 7$ assumed then **SC1** for correct a and **SC2** for correct b
		7	

Q	Answer	Marks	Comments
9 ALT	$na = -\frac{14}{5}$	**B1**	Condone $-\frac{14}{5} = -2.8$ **PI** by correct subsequent substitution seen
	$\frac{n(n-1)}{2}a^2 = \frac{84}{25}$	**B1**	**oe** Condone $\frac{84}{25} = 3.36$ Condone $n = -\frac{14}{5a}$ substituted
	$\left[n = -\frac{14}{5a} \Rightarrow\right] \quad \frac{\left(-\frac{14}{5a}\right)\left(-\frac{14}{5a}-1\right)}{2} \times a^2 = \frac{84}{25}$ or $\frac{14(14+5a)}{25a^2} \times a^2 = \frac{168}{25}$ or $14(14+5a) = 168$	**M1**	**oe** Correct substitution for n into their $\frac{n(n-1)}{2}a^2 = \frac{84}{25}$ **PI** by elimination of n with $a = \frac{na^2}{na}$ where $na^2 = \frac{28}{25}$
	$a = -\frac{2}{5}$	**A1**	**CAO** Condone $a = -0.4$
	$-\frac{2}{5}n = -\frac{14}{5}$ and $n = 7$ or $n = -\frac{14}{5a}$ seen, $a = -\frac{2}{5}$ and $n = 7$	**B1**	**AG** Be convinced
	$\frac{7(7-1)(7-2)}{6} \times \left(-\frac{2}{5}\right)^3 = -b$ **oe** or $\binom{7}{3} \times \left(-\frac{2}{5}\right)^3 = -b$ **oe**	**M1**	Correct substitution of $n = 7$ and their a
	$b = \frac{56}{25}$	**A1**	**CAO** Condone $\frac{56}{25} = 2.24$
		7	

	Question 9 Total	**7**	
Q	**Answer**	**Marks**	**Comments**
10(a)	$\left[r = \right]\dfrac{8 - x}{12}$	**B1**	Correct expression for the common ratio. **PI** in later working
	$-1 < \dfrac{8-x}{12} < 1$ or $-12 < 8 - x < 12$	**M1**	**oe** Use of condition for convergence of an infinite geometric series to form an inequality. Accept $\lvert 8 - x \rvert < 12$
	$-4 < x < 20$	**A1**	**CAO** Condone $x > -4$ and $x < 20$ but not $x > -4$ or $x < 20$
		3	

Q	**Answer**	**Marks**	**Comments**
10(b)	$S_\infty = \dfrac{12}{1 - \dfrac{8-x}{12}}$ or $S_\infty = \dfrac{144}{4+x}$	**B1ft**	**PI oe** Expression for S_∞ in terms of x **ft** their r from **part (a)** **PI** by use of $\dfrac{12}{1 - r}$ with $r = -1$ or $r = \dfrac{2}{3}$
	$\left[x = 0 \Rightarrow S_\infty = \right] \dfrac{144}{4+0}$ $\left[= 36\right]$	**M1**	**oe** Attempt to evaluate S_∞ at $x = 0$
	$\left[x < 20 \Rightarrow S_\infty = \dfrac{144}{4+x} > \right]$ 6	**M1**	**oe** Use of $x = 20$ with $\dfrac{8-x}{12}$ to find the corresponding critical value **ft** their $x = 20$ from **part (a)** provided it is positive
	$6 < S_\infty < 36$	**A1**	**CAO** Condone $S_\infty > 6$ and $S_\infty < 36$ but not $S_\infty > 6$ or $S_\infty < 36$
		4	

	Question 10 Total	**7**	