
INTERNATIONAL A-LEVEL MATHEMATICS

MA04

(9660/MA04) Unit S2 Statistics

Mark scheme

January 2021

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
-x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)(i)	$[E[3X_1 + 2X_2 - X_3] = 3E[X_1] + 2E[X_2] - 3E[X_3]]$ $= 3f + 2h - g$	B1	
		1	

Q	Answer	Marks	Comments
1(a)(ii)	$[\text{Var}[4X_1 - 3X_2] = 4^2 \text{Var}[X_1] + 3^2 \text{Var}[X_2]]$ $= 25g$	B1	
		1	

Q	Answer	Marks	Comments
1(b)	$3h = f$ and $f + g + h = 7.1$ $2g + 2 = 9 \Rightarrow g = 3.5$ $3h = f$, $f + h = 3.6$ $f = 2.7$ and $h = 0.9$	B1 B1 M1 A1	PI oe PI by correct answer. For a clear attempt to find f and h oe
		4	

	Question 1 Total	6	
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Q	Answer	Marks	Comments
2(a)	$X \sim B(25, 0.9)$ $H_0: p = 0.9$ $H_1: p > 0.9$ $[1 - P(X \leq 23) =] \quad 1 - 0.729 = 0.271$ $P(X \geq 24) = 0.271$ $0.271 > 0.1$ Do not reject H_0 No evidence to suggest that Steve has a higher 'at least satisfactory' rating	B1 B1 M1 A1 m1 A1ft E1	PI [or $Y \sim B(25, 0.1)$] $H_0: p = 0.1$ $H_1: p < 0.1$ 0.271 $P(Y \leq 1) = 0.271$ Comparison of probability with 0.1 Allow CR for Y as $\{0\}$ for M1 A1 as is a CR for X is $\{25\}$ Allow 'accept H_0 ' Correct statement must be in context and must follow from fully correct solution.
		7	

Q	Answer	Marks	Comments
2(b)(i)	0.0718	B1	
		1	

Q	Answer	Marks	Comments
2(b)(ii)	Accepting that the proportion of Steve's deliveries that are 'at least satisfactory' is greater than 90% when it is 90%	B1	Must be in context.
		1	

	Question 2 Total	9	
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Q	Answer	Marks	Comments
3(a)	$F(8) = 1$ so $64k - \frac{1}{3} = 1$ oe or $F(4) = 0$ so $16k - \frac{1}{3} = 0$ oe $k = \frac{1}{48}$	M1 A1	Correct substitution of upper or lower limit oe $k = 0.0208\bar{3}$
		2	

Q	Answer	Marks	Comments
3(b)		B1 B1 B1	Use of a straight line with positive gradient between $t = 4$ and $t = 8$ Correct end points for the correct straight line between $t = 4$ and $t = 8$ $\left[\left(4, \frac{1}{6} \right) \text{ and } \left(8, \frac{1}{3} \right) \right]$ Correct graph between for $0 \leq t \leq 4$ and $8 \leq t \leq 10$
		3	

	Question 3 Total	5	
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Q	Answer	Marks	Comments
4	$H_0: \mu = 70 \text{ [days]}$ $H_1: \mu > 70$ $\bar{x} = 70.6$ $s^2 = \frac{1}{99} \times \left(499000 - \frac{7060^2}{100} \right)$ $= 5.6\dot{9}$ $\bar{X} \sim N\left(70, \frac{5.6\dot{9}}{100}\right)$ $z = \frac{70.6 - 70}{\sqrt{\frac{5.6\dot{9}}{100}}}$ $= 2.5[13791745]$ $z_{\text{crit}} = 2.3[26347931]$ Reject H_0 as $2.5[137\dots] > 2.3[263\dots]$ or $z > z_{\text{crit}}$ Evidence to suggest that weed treatment has an increase in the 'mean number of days' claimed [at the 1% level]	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>A1ft</p> <p>E1</p>	<p>Seen or used</p> <p>Attempt at variance formula Allow one slip</p> <p>Implied by correct answer (oe 188/33)</p> <p>ARWT 5.70 accept $s = 2.38[6832566]$</p> <p>$\bar{X} \sim N\left(\mu, \frac{s^2}{100}\right)$</p> <p>Calculates z with their s^2, their \bar{x} and their μ or for $se = 0.238\dots$ or $P(\bar{X} \geq 70.6) = 0.006$</p> <p>or sight of 70.56 for \bar{x} or 0.00597</p> <p>AWRT 2.3 or for CR is $\bar{x} > 70.56$</p> <p>Allow 'accept H_1' Follow through their z and z_{crit} provided signs are consistent or comparison of $70.6 > 70.56$ or comparison of their '0.00597' to 0.01</p> <p>Must be consistent with their conclusion on whether to accept H_1 or not based on their z and z_{crit} if not explicitly stated</p>
		10	

	Question 4 Total	10	
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Q	Answer	Marks	Comments
5(a)	$P(X=7) = \frac{e^{-4} \times 4^7}{7!}$ or $P(X=5) = \frac{e^{-4} \times 4^5}{5!}$ $P(X=7) = kP(X=5)$ $\frac{e^{-4} \times 4^7}{7!} = k \frac{e^{-4} \times 4^5}{5!}$ $k = \frac{4^2 \times 5!}{7!}$ $k = \frac{8}{21} \text{ or } k = 0.38095\dot{2}$	<p style="text-align: center;">B1</p> <p style="text-align: center;">M1</p> <p style="text-align: center;">A1</p>	or correct use of $P(X=n)$ and $P(X=n+2)$ PI by sight of $k = \frac{16}{42}$ eg $P(X=7) = \frac{\lambda^2}{7.6} P(X=5)$ If B0 M0 scored then SC2 for use of statistical tables which lead to a value for k as 0.381 or better
		3	

Q	Answer	Marks	Comments
5(b)(i)	$P(X < 3) = P(X \leq 2)$ $= 0.6767$	<p style="text-align: center;">M1</p> <p style="text-align: center;">A1</p>	oe CAO
		2	

Q	Answer	Marks	Comments
5(b)(ii)	$\lambda = 14$ $[P(8 < X < 17) =] \quad P(X \leq 16) - P(X \leq 8)$ $= 0.6938 \text{ or } 0.6939$	<p style="text-align: center;">B1</p> <p style="text-align: center;">M1</p> <p style="text-align: center;">A1</p>	Seen or used PI by correct final answer Condone one slip oe 0.7559 – 0.0621 CAO
		3	

Q	Answer	Marks	Comments
5(c)(i)	$\sum fx = 446$ or $\sum fx^2 = 3554$ $\bar{x} = 6.97$ (3 sf) $s^2 = 7.08$ (3 sf)	M1 A1 A1	Summary statistics found PI by at least one correct answer AWRT 6.97 AWRT 7.08 Note: $s^2 = 6.97$ is A0 [using n rather than $n - 1$ as denominator]
		3	

Q	Answer	Marks	Comments
5(c)(ii)	$\bar{x} \approx s^2$ [so this supports Dave's claim] comparison of their values \bar{x} or s^2 to $(5 + 2) = 7$ [so this supports Dave's claim]	B1ft B1ft	Comment directly comparing their mean and variance B0 for conflicting statements
		2	

	Question 5 Total	13	
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Q	Answer	Marks	Comments
6(a)	$\left[\text{Variance} = \frac{1}{0.2^2} = \right]$ 25 [mins]	B1	
		1	

Q	Answer	Marks	Comments
6(b)	$P(T < 15) = 1 - e^{-0.2 \times 15}$ [= 0.9502129316] = 0.950	M1 A1	Attempts to find correct probability using cdf of exponential or integration of pdf
		2	

Q	Answer	Marks	Comments
6(c)	$P(T > 30 T > 20) = P(T > 10)$ $P(T > 10) = e^{-10 \times 0.2}$ [= 0.1353352832] = 0.135	M1 M1 A1	A clear attempt at the no memory rule or $\left(\frac{e^{-30 \times 0.2}}{e^{-20 \times 0.2}} \right)$ or use of $P(T > 10)$ or $P(T < 10)$ Calculates their probability
		3	

Q	Answer	Marks	Comments
6(d)	$e^{-0.2t} = 0.6$ $-0.2t = \ln(0.6)$ $t = -5 \ln(0.6)$ $t = 2.55$ [mins]	M1 A1	
		2	

	Question 6 Total	8	
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Q	Answer	Marks	Comments
7	$\left[\int \frac{1}{x} f(x) dx = \right] \quad \frac{1}{60} \int_1^3 \frac{1}{x} \times (10x + x^3) dx$ $= \frac{1}{60} \left[10x + \frac{x^3}{3} \right]_1^3$ $= \frac{1}{60} \left(39 - \left(10 + \frac{1}{3} \right) \right)$ $= \frac{43}{90}$	<p>M1</p> <p>m1</p> <p>A1</p>	<p>PI</p> <p>correct integration with correct limits</p> <p>oe</p>
		3	
	Question 7 Total	3	

Q	Answer	Marks	Comments
8	$H_0: \mu = 200$ $H_1: \mu \neq 200$ $[\text{dof}] \quad \nu = 9$ $t_{\text{crit}} = \pm 2.262$ $t = [\pm] \frac{200.3 - 200}{\left(\frac{0.55}{\sqrt{10}}\right)}$ $= 1.72 [487872\dots]$ Do not reject H_0 as $[-t_{\text{crit}} <] t < t_{\text{crit}}$ No evidence to suggest that the mean mass of the spheres produced by the machine has changed (at the 5% level)	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1ft</p> <p>E1</p>	<p>Both hypotheses</p> <p>PI by correct t_{crit}</p> <p>Seen or used</p> <p>Seen or used must be +ve</p> <p>Follow through their t and t_{crit} Implied by correct conclusion in context</p> <p>Allow 'accept H_0' Must be consistent with their conclusion on whether to accept H_0 or not or their t and t_{crit} if not explicitly stated</p> <p>Must be in context and must not be definite.</p>
		7	
	Question 8 Total	7	

Q	Answer	Marks	Comments
9(a)(i)	The midpoint of 170 and 200 is 185	E1	Allow calculation from 2 standardised equations
	and the normal distribution is symmetric	E1	
		2	

Q	Answer	Marks	Comments
9(a)(ii)	$P(Z < 2) - (1 - P(Z < 2))$	M1	PI by a correct sketch. Allow standardising with their ' μ ' or $2P(Z < 2) - 1$
	= 0.9545	A1	
		2	CAO

Q	Answer	Marks	Comments
9(b)(i)	$[\Phi^{-1}(0.45) = z =] - 0.1257$	B1	Seen or used. Condone +0.1256613431
	$P\left(Z < \frac{190 - 200}{\sigma}\right) = 0.45$	M1	
	$\frac{190 - 200}{\sigma} = -0.1257$	A1	
		3	Attempt to standardise with a z-value

Q	Answer	Marks	Comments
9(b)(ii)	1	B1	
		1	

	Question 9 Total	8	
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Q	Answer	Marks	Comments
10(a)	[from symmetry] $x = 5$	B1	
		1	

Q	Answer	Marks	Comments
10(b)	$a = \frac{0.1 - 0.05}{4 - 0} \quad \text{or} \quad b = \frac{0.05 - 0.1}{10 - 6}$ $a = 0.0125$ $b = -0.0125$	<p>M1</p> <p>A1</p> <p>B1</p>	<p>PI. Attempt to find gradient of a or b Allow one slip</p> <p>For a or b correct, oe</p> <p>ft for ‘–their a’ or ‘–their b’</p>
		3	

Q	Answer	Marks	Comments
10(c)	$\int_4^6 (q - n(x-5)^2) dx = \left[qx - \frac{n(x-5)^3}{3} \right]_4^6$ <p>[Area beneath pdf between 0 and 4 = 0.3]</p> $\left[qx - \frac{n(x-5)^3}{3} \right]_4^6 = 1 - 2 \times 0.3$ $2q - \frac{2n}{3} = 0.4$ <p>[f(4) =] $q - n = 0.1$</p> $q = 0.25$ $n = 0.15$	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>or $\left[qx - \frac{n(x-5)^3}{3} \right]_4^5 = 0.2$</p> <p>sub in $x = 4$ or $x = 6$ into $f(x)$</p>
		5	

Q	Answer	Marks	Comments
10(d)	$\int_0^4 (ax + 0.05) dx$ $+ \int_4^{4.5} (q - n(x-5)^2) dx = 0.3 + \left[qx - \frac{n(x-5)^3}{3} \right]_4^{4.5}$ $= \frac{61}{160}$	<p>M1</p> <p>A1</p>	<p>Attempt to add two correct probabilities can be a trapezium for area or sight of 0.3 and $\frac{13}{160}$</p> <p>oe 0.38125</p>
		2	
	Question 10 Total	11	