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MA01

(9660/MA01) Unit P1 Pure Mathematics

Mark scheme

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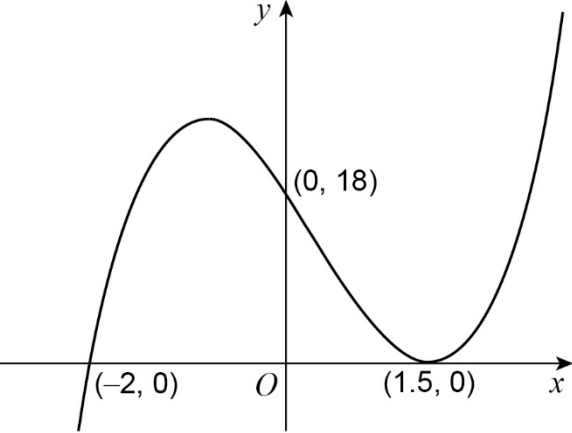
Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
-x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)(i)	18	B1	
		1	

Q	Answer	Marks	Comments
1(a)(ii)	-15	B1	
		1	

Q	Answer	Marks	Comments
1(a)(iii)	-4	B1	
		1	

Q	Answer	Marks	Comments
1(b)	See image below	<p>B1</p> <p>B1</p> <p>B1ft</p>	<p>Correct positive cubic graph with maximum in the second quadrant and minimum tangential to the positive x-axis.</p> <p>Correct coordinates of both x-intercepts and no others. Condone given as values rather than coordinates.</p> <p>ft their 18 from part (a)(i)</p> <p>Correct coordinates of y-intercept. Condone given as value rather than coordinates.</p>
			
		3	
Question 1 Total		6	

Q	Answer	Marks	Comments
2(a)	$\frac{1}{2} \times 90 \times (90 + 1)$	M1	Award M1 for $\frac{1}{2} \times 90 \times (0 + 89)$
	4095	A1	CAO NMS Scores M1A1
		2	

Q	Answer	Marks	Comments
2(b)(i)	$a + 11d = 25$	M1	oe M1 implied by $(d =) \frac{57 - 25}{28 - 12}$ oe
	$a + 27d = 57$	M1	oe
	$a = 3 \quad d = 2$	A1	
		3	

Q	Answer	Marks	Comments
2(b)(ii)	$\frac{1}{2} \times 65 \times (2 \times 3 + (65 - 1) \times 2)$	M1	Substitutes their values for a and d and $n = 65$ into a correct formula for the sum of the first n terms of an arithmetic series.
	or $3 + (65 - 1) \times 2 (= 131)$ and $\frac{1}{2} \times 65 \times (3 + 131)$ oe		
	4355 dollars	A1	CAO condone units omitted
		2	

	Question 2 Total	7	
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Q	Answer	Marks	Comments
3(c)	$(CD =) \sqrt{(17-3)^2 + (4-8)^2}$ $(CD =) 2\sqrt{53}$ or $\sqrt{212}$ $\frac{1}{2} \times 2\sqrt{53} \times 3\sqrt{53} \Rightarrow \text{Area} = 159$	M1 A1 A1	oe ft their coordinates of D CAO Must be seen to use $AB = 3\sqrt{53}$ PI by correct $ CD $ and area seen with no working showing that they have not used the length of AB CAO
		3	
	Question 3 Total	10	

Q	Answer	Marks	Comments
4(a)	$\left(x \pm \frac{9}{2}\right)^2 \dots$ $\left(x - \frac{9}{2}\right)^2 - \frac{61}{4}$ Translation $\begin{bmatrix} \frac{9}{2} \\ \frac{69}{4} \end{bmatrix}$	M1	PI by correct x -component in vector.
		A1	PI by correct vector.
		E1	Stating translation and no other transformations.
		E1ft	Strict ft their $-\frac{9}{2}$ and their $-\frac{61}{4}$ minus 2 Must be given in vector form.
		4	

Q	Answer	Marks	Comments
4(b)(i)	$(f(3k) =) 4(3k)^3 + 5(3k)^2 + 32k^3 - 20k^2$ $(f(3k) =) 140k^3 + 25k^2$	M1	$f(3k)$ attempted If polynomial division used it must be correct.
		A1	CAO
		2	

Q	Answer	Marks	Comments
4(b)(ii)	$(f(-2k) =) 4(-2k)^3 + 5(-2k)^2 + 32k^3 - 20k^2$ $(f(-2k) =) -32k^3 + 20k^2 + 32k^3 - 20k^2 = 0$	M1	$f(-2k)$ attempted. Must use Factor Theorem.
		A1	CSO Correctly shows $f(-2k) = 0$ All coefficients must be seen simplified.
		2	

Q	Answer	Marks	Comments
4(c)(i)	$-6k$	B1	CAO
		1	

Q	Answer	Marks	Comments
4(c)(ii)	$\left(f\left(\frac{1}{3}x\right) = 4\left(\frac{1}{3}x\right)^3 + 5\left(\frac{1}{3}x\right)^2 + 32k^3 - 20k^2 \right)$ $\frac{4}{27}x^3 + \frac{5}{9}x^2 + 32k^3 - 20k^2$	M1 A1	Substitutes $\frac{1}{3}x$ for x in $f(x)$ PI by one correct coefficient of x^3 or x^2 in their final answer.
		2	

	Question 4 Total	11	
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Q	Answer	Marks	Comments
5(a)	$\left(\frac{dy}{dx}\right) 2ax - b$ $2a \times 2 - b = 16$ and $4a - b = 16$	B1	Correct derivative
		B1	AG Intention to use $x = 2$ and required result stated.
		2	

Q	Answer	Marks	Comments
5(b)	$\left[\frac{a}{3}x^3 - \frac{b}{2}x^2 + 5x\right]_2^3 = 23$ $\left(\frac{a}{3}(3)^3 - \frac{b}{2}(3)^2 + 5(3)\right) - \left(\frac{a}{3}(2)^3 - \frac{b}{2}(2)^2 + 5(2)\right)$ $\frac{19}{3}a - \frac{5}{2}b + 5 = 23$ and then $38a - 15b = 108$	M1	At least two terms correct.
		A1	Fully correct integration. Ignore limits and do not need to see = 23
		m1	Substitution into $F(3) - F(2)$ ft their expression. Condone one sign error.
		A1	Simplifying $F(3) - F(2)$ and set equal to 23
		4	CSO AG

Q	Answer	Marks	Comments
5(c)	$a = 6 \quad b = 8$	B1	Both correct values.
		1	

Q	Answer	Marks	Comments
5(d)	$(6x^2 - 8x + 5 = x + d \Rightarrow) 6x^2 - 9x + 5 - d (= 0)$ $(-9)^2 - 4 \times 6 \times (5 - d)$ $d = \frac{13}{8} (= 1.625)$ $d < \frac{13}{8} \quad \text{or} \quad d < 1.625$	B1ft M1 A1 A1	ft their a and b Correct discriminant for their quadratic containing d . May be partially simplified. oe Correct critical value. oe Must be seen as an inequality.
5(d) Alt	$12x - 8 = 1$ $\left(\frac{3}{4}, \frac{19}{8}\right)$ $\left(\frac{19}{8} = \frac{3}{4} + d \Rightarrow\right) d = \frac{13}{8}$ $d < \frac{13}{8} \quad \text{or} \quad d < 1.625$	M1 A1 A1 A1	Correct derivative of equation for C set equal to 1 ft their a and b oe Correct coordinates for the point on C where the gradient of the tangent is 1 May not be given as coordinates. PI in later working. oe Correct critical value. oe Must be seen as an inequality.
		4	

	Question 5 Total	11	
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Q	Answer	Marks	Comments
6(b)(i)	$\left[(2)^8 \right] + 8(2)^7 \left(-\frac{1}{4}x \right) + 28(2)^6 \left(-\frac{1}{4}x \right)^2$ $+ 56(2)^5 \left(-\frac{1}{4}x \right)^3 + \left[70(2)^4 \left(-\frac{1}{4}x \right)^4 + \dots \right]$ $\left(256 + 256x + 112x^2 + 28x^3 \left[+ \frac{35}{8}x^4 + \dots \right] \right)$ $- \left(256 - 256x + 112x^2 - 28x^3 \left[+ \frac{35}{8}x^4 + \dots \right] \right)$ $512x + 56x^3$	<p>M1</p> <p>M1</p> <p>A1</p>	<p>Substitution of $-x$ for x in any term in their expansion in part (a). Ignore terms of order 4 and higher.</p> <p>$\left(2 + \frac{1}{4}x \right)^8 - \left(2 - \frac{1}{4}x \right)^8$ with expansions simplified.</p> <p>AG Be convinced. Must see evidence of second M1</p>
		3	

Q	Answer	Marks	Comments
6(b)(ii)	$x = 0.4$ 208.38	<p>B1</p> <p>B1</p>	<p>Must be seen or PI by final answer.</p> <p>CAO</p>
		2	

	Question 6 Total	9	
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Q	Answer	Marks	Comments
7(a)	$4x^2 + 2xh + 8xh$ $4x^2 + 10xh = 75$ $xh = \frac{1}{10}(75 - 4x^2)$ or $h = \frac{1}{10x}(75 - 4x^2)$ $(V = 4x^2h =) 4x \times \frac{1}{10}(75 - 4x^2)$ and $V = 30x - \frac{8}{5}x^3$	<p>M1</p> <p>A1</p> <p>B1ft</p> <p>B1</p>	<p>oe Attempt at surface area introducing variable for height of tank. Simplified or unsimplified. Condone one slip.</p> <p>oe Correct equation linking dimensions of tank to surface area.</p> <p>oe ft their surface area equation.</p> <p>oe Correct unsimplified expression for volume of tank with height variable eliminated and AG Be convinced.</p>
		4	

Q	Answer	Marks	Comments
7(b)(i)	$\left(\frac{dV}{dx} =\right) 30 - \frac{24}{5}x^2$ $30 - \frac{24}{5}x^2 = 0$ $x = 2.5$ $\left(x = 2.5 \Rightarrow V = 30 \times 2.5 - \frac{8}{5} \times 2.5^3 =\right) 50$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>Correct derivative. PI by later working.</p> <p>PI By $x = 2.5$ Sets their derivative equal to zero.</p> <p>oe CAO Ignore $x = -2.5$ if seen. PI by correct final answer.</p> <p>CAO</p>
		4	

Q	Answer	Marks	Comments
7(b)(ii)	$\left(\frac{d^2V}{dx^2} =\right) -\frac{48}{5}x$ $-\frac{48}{5} \times 2.5 \left[= -24 \right]$ <p>and</p> <p>Since $\frac{d^2V}{dx^2} < 0$ then it is a maximum value of V</p>	<p>B1ft</p> <p>E1ft</p>	<p>ft their first derivative.</p> <p>Substitutes $x = 2.5$ into their second derivative. ft their $x = 2.5$ provided it is positive and the value of their second derivative would be negative.</p> <p>Indicates that the value of the second derivative is negative and states it is a maximum.</p>
		2	

	Question 7 Total	10	
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Q	Answer	Marks	Comments
8	$x^{\frac{1}{2}} + \frac{1}{6}x^{-\frac{1}{2}}$ $\frac{2}{3}x^{\frac{3}{2}} + \frac{1}{3}x^{\frac{1}{2}}$ or $\frac{2}{3}x\sqrt{x} + \frac{1}{3}\sqrt{x}$ $\left(\frac{2}{3}(16k)^{\frac{3}{2}} + \frac{1}{3}(16k)^{\frac{1}{2}}\right) - \left(\frac{2}{3}(64)^{\frac{3}{2}} + \frac{1}{3}(64)^{\frac{1}{2}}\right)$ or $\left(\frac{2}{3}(16k)\sqrt{16k} + \frac{1}{3}\sqrt{16k}\right) - \left(\frac{2}{3}(64)\sqrt{64} + \frac{1}{3}\sqrt{64}\right)$ $(16k)^{\frac{1}{2}} = 4k^{\frac{1}{2}} \quad \text{or} \quad \sqrt{16k} = 4\sqrt{k}$ $c = 344$ $a = \frac{128}{3} \quad \text{or} \quad b = \frac{4}{3}$ $\sqrt{k}\left(\frac{128}{3}k + \frac{4}{3}\right) - 344$	<p>M1</p> <p>M1</p> <p>A1ft</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>A1</p>	<p>Correct expression for integrand in index form.</p> <p>M1 for one correct term in integral simplified or unsimplified. ft their integrand.</p> <p>Attempted substitution into $F(16k) - F(64)$ using their integral. Condone incorrect calculation of $F(64)$</p> <p>PI by subsequent working.</p> <p>Correct value of c Or equivalent fraction.</p> <p>Either correct value. Ignore powers of k</p> <p>Accept values of a and b rounded or truncated to at least 2dp In the correct form with $a = \frac{128}{3}$ and $b = \frac{4}{3}$</p> <p>Accept values of a and b rounded or truncated to at least 2dp Ignore incorrect value of c</p>
		8	
	Question 8 Total	8	

Q	Answer	Marks	Comments
9(b)	$(3r^2 + 10r + 3 = 0 \Rightarrow) (3r + 1)(r + 3)(= 0)$ $a = 162 \text{ and } r = -\frac{1}{3}$ and $r = -3$ rejected $\sum_{n=k}^{\infty} u_n = \frac{162}{1 - \left(-\frac{1}{3}\right)} - \frac{162 \left(1 - \left(-\frac{1}{3}\right)^{k-1}\right)}{1 - \left(-\frac{1}{3}\right)}$ or $\frac{162 \left(-\frac{1}{3}\right)^{k-1}}{1 - \left(-\frac{1}{3}\right)}$ $\frac{243}{2} (-3)^{1-k}$ $\sum_{n=k}^{\infty} u_n = \frac{(-1)^{k-1} 3^{6-k}}{2}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>Attempt to solve quadratic equation. Could be correct substitution into the quadratic formula. PI by $r = -\frac{1}{3}$ and $r = -3$</p> <p>Correct values for a and r Rejecting $r = -3$ PI by later working</p> <p>Correct expression with values substituted.</p> <p>Uses $\left(\frac{1}{3}\right)^{k-1} = 3^{1-k}$ and the denominator simplified.</p> <p>Be convinced. NMS scores M1A1M0M0A0</p>
		5	
	Question 9 Total	8	