

## INTERNATIONAL A-LEVEL MATHEMATICS

## **MA04**

(9660/MA04) Unit S2 Statistics

Mark scheme

January 2020

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\*201xma04/MS\*

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## Key to mark scheme abbreviations

	Μ	Mark is for method
	m	Mark is dependent on one or more M marks and is for method
	Α	Mark is dependent on M or m marks and is for accuracy
	В	Mark is independent of M or m marks and is for method and accuracy
	E	Mark is for explanation
$\checkmark$	or ft	Follow through from previous incorrect result
	CAO	Correct answer only
	CSO	Correct solution only
	AWFW	Anything which falls within
	AWRT	Anything which rounds to
	ACF	Any correct form
	AG	Answer given
	SC	Special case
	oe	Or equivalent
	A2, 1	2 or 1 (or 0) accuracy marks
	<i>–x</i> EE	Deduct <i>x</i> marks for each error
	NMS	No method shown
	PI	Possibly implied
	SCA	Substantially correct approach
	sf	Significant figure(s)
	dp	Decimal place(s)

Q	Answer	Marks	Comments
	I		I
1(a)	$E\left(\sum_{i=1}^{3} X_{i}\right) = 8$	B1	
1(b)	$Var\left(\sum_{i=1}^{3} X_{i}\right) = 4.5$	B1	
1(c)	$E(3X_1 - 4X_3) = 3 \times 2 - 4 \times 1$	M1	$E(3X_1 - 4X_3) = 3a - 4b$ where a, b = 2, 5 or 1
	= 2	A1	
1(d)	$Var(5X_1 - 2X_2) = 5^2 \times 3 + 2^2 \times 1$	M1	Var $(5X_1 - 2X_2) = 5^2c + 2^2d$ c, d = 3, 1 or 0.5
	= 79	A1	
	Total	6	

Q	Answer	Marks	Comments
		[	
2(a)	$\frac{1}{\lambda^2} = 400$	M1	
	$\lambda = 0.05$	A1	Do not award if –0.05 seen but not rejected
2(b)	Mean = $\frac{1}{\lambda}$ = 20	B1ft	Follow through their $\lambda$
2(c)	P(X > 25) = 1 - F(25)	M1	ое
	$= 1 - (1 - e^{-0.05 \times 25})$	A1	Attempts to find correct probability using cdf of exponential or integration of pdf
	= 0.287	A1	AWRT, NMS 3/3
2(d)	$1 - e^{-0.05x} = 0.6$	M1	Forms equation using cdf of exponential or integration of pdf
	<i>x</i> = 18.3	A1	AWRT
	Total	8	

Q	Answer	Marks	Comments
	$Y \sim Po(1)$	B1	PI
3(a)	$P(X > 3) \approx P(Y > 3) = 1 - P(Y \le 3)$	M1	Attempt at correct probability
	= 0.019	A1	AWRT
2(h)	<i>n</i> is large	B1	ое
3(D)	p is small	B1	oe
	Total	5	

Q	Answer	Marks	Comments
4(a)	$\overline{x} = 2.5$	B1	
-(a)	$s^2 = \frac{1}{20 - 1} \left( 173 - \frac{50^2}{20} \right)$	М1	Attempt at variance formula Allow one slip Implied by correct answer
	= 2.53	A1	AWRT
	Because Mean $\approx$ Variance	E1	Values and conclusion must be seen Allow "values of sample mean and variance are close"
4(b)	$P(X = 4) = \frac{e^{-2.5} \times 2.5^4}{4!}$	М1	Applies Poisson formula Implied by correct answer Condone using different value of $\lambda$
	= 0.134	A1	AWRT
	Total	6	

Q	Answer	Marks	Comments
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5(a)	$H_0: p = 0.2$ or $H_0: p = 0.8$ $H_1: p < 0.2$ $H_1: p > 0.8$	B1	Both hypotheses
	X~B(15, 0.2) or X~B(15, 0.8)	M1	PI
	$P(X \le 1) = 0.1671 \text{ or}$ $P(X \ge 14) = 0.1671$	A1	AWRT 0.17
	0.1671 > 0.1	M1	Compares their probability with 0.1
	Accept H <sub>0</sub>	A1ft	Follow through their probability provided both M marks awarded Implied by correct conclusion in context
	Evidence to suggest/support that the proportion of customers giving an "Excellent" rating has not increased	E1	Must be consistent with their conclusion on whether to accept $H_0$ or not or their probability if not explicitly stated
5(b)	Concluding that the proportion of customers giving an "Excellent" rating has not increased when it has	E1	oe
	Total	7	

Q	Answer	Marks	Comments
			•
6(a)	$\frac{83-\mu}{\sigma} = 1.6449$	M1	Setting up formula with a <i>z</i> -value or $\Phi^{-1}(0.95)$
	$83 - \mu = 1.6449\sigma$ (AG)	A1	No errors seen 1.6449 must be seen before the final line
C(b)	1.4051	B1	Sight of anywhere in part (b)
0(D)	$\frac{10-\mu}{\sigma} = \pm 1.4051$	M1	Oe, condone sign error for -1.4051
	$10 - \mu = -1.4051\sigma$	A1	oe
	$\mu$ = 43.6 or $\sigma$ = 23.9	M1	AWRT
	$\mu$ = 43.6 and $\sigma$ = 23.9	A1	
6(c)	85 is within two standard deviations of the mean	E1	oe
	or Negative scores are within two standard deviations of the mean		Allow non-zero probability (0.042) using normal distribution model of gaining a score higher than 85
	Total	8	

Q	Answer	Marks	Comments
7(a)	$H_0: \mu = 14$ $H_1: \mu \neq 14$	B1	Both hypotheses
	$\overline{X} \square N\left(14, \frac{0.25^2}{25}\right)$	M1	Use of PI
	$z = \frac{13.892 - 14}{\frac{0.25}{\sqrt{25}}}$	M1	Attempts to find <i>z</i> Allow one slip
	<i>z</i> = -2.16	A1	AWRT Allow 2.16
	$z_{\rm crit}$ = 2.3263 or -2.3263	B1	Seen anywhere AWRT 2.33 Accept P( $Z < -2.16$ ) = 0.015
	Accept H <sub>0</sub>	A1ft	Follow through their $z$ and $z_{crit}$ or their probability and 0.01 Implied by correct conclusion in context
	Evidence to suggest/support that the mean diameter of the pipes produced by machine A has not changed following the power cut	E1	Must be consistent with their conclusion on whether to accept $H_0$ or not or their <i>z</i> and <i>z</i> <sub>crit</sub> (or their probability and 0.01) if not explicitly stated

Q	Answer	Marks	Comments
7(b)	$H_0: \mu = 14$ $H_1: \mu > 14$	B1	Both hypotheses
	dof <i>v</i> = 8	M1	PI
	$t = \frac{14.02 - 14}{\frac{0.04}{\sqrt{9}}}$	M1	Attempts to find $t$ Allow one slip Condone $z =$
	<i>t</i> = 1.5	A1	Condone <i>z</i> =
	<i>t</i> <sub>crit</sub> = 1.397	B1	Seen anywhere
	Reject H₀	A1ft	Follow through their $t$ and $t_{crit}$ (but not $z_{crit}$ ) Implied by correct conclusion in context
	Evidence to suggest/support that the mean diameter of the pipes produced by machine B is greater than 14 millimetres	E1	Must be consistent with their conclusion on whether to accept $H_0$ or not or their <i>t</i> and <i>t</i> <sub>crit</sub> if not explicitly stated
	Total	14	

Q	Answer	Marks	Comments
8(a)	$\int_{1}^{6} k(x^2 - 7x + 6)  \mathrm{d}x = 1$	M1	Sets integral over whole range equal to 1
	$k\left[\frac{x^{3}}{3} - \frac{7x^{2}}{2} + 6x\right]_{1}^{6} = 1$	М1	Correct integration
	$k \left[ \left( \frac{6^3}{3} - \frac{7 \times 6^2}{2} + 6 \times 6 \right) - \left( \frac{1^3}{3} - \frac{7 \times 1^2}{2} + 6 \times 1 \right) \right] = 1$ $\Rightarrow k = -\frac{6}{125} \text{ (AG)}$	A1	Requires intermediate line after integration
8(b)	$E(X) = \int_{1}^{6} kx(x^2 - 7x + 6)  \mathrm{d}x$	M1	Identifies correct integral
	$=k\left[\frac{x^{4}}{4}-\frac{7x^{3}}{3}+\frac{6x^{2}}{2}\right]_{1}^{6}$	M1	Correct integration
	$= k \left[ \left( \frac{6^4}{4} - \frac{7 \times 6^3}{3} + \frac{6 \times 6^2}{2} \right) - \left( \frac{1^4}{4} - \frac{7 \times 1^3}{3} + \frac{1 \times 1^2}{2} \right) \right]$ $\Rightarrow E(X) = 3.5 \text{ (AG)}$	A1	Requires intermediate line after integration
8(c)	$E(X^2) = \int_{1}^{6} kx^2 (x^2 - 7x + 6)  \mathrm{d}x$	M1	Identifies correct integral
	$=k\left[\frac{x^{5}}{5}-\frac{7x^{4}}{4}+\frac{6x^{3}}{3}\right]_{1}^{6}$	M1	Correct integration
	$E(X^2) = 13.5$	A1	PI
	$Var(X) = E(X^{2}) - (E(X))^{2}$ = 13.5 - 3.5 <sup>2</sup>	M1	Applies formula
	= 1.25	A1	
	Total	11	

Q	Answer	Marks	Comments
9(a)(i)	$\overline{X} \sim N\left(20, \ \frac{6.5536}{25}\right)$	B1	ое
9(a)(ii)	$P(\bar{X} > 21) = P\left(Z > \frac{21 - 20}{\sqrt{\frac{6.5536}{25}}}\right)$	М1	Standardises
	= $1 - \Phi(1.95)$ = $1 - 0.97441$	M1	PI
	= 0.0254 to 0.0256	A1	AWFW
9(b)(i)	$\sum_{i=1}^{25} X_i \square N(500, 163.84)$	B1	oe
9(b)(ii)	$P\left(\sum_{i=1}^{25} X_i > 525\right) = P\left(Z > \frac{525 - 500}{\sqrt{163.84}}\right)$	M1	Standardises
	= $1 - \Phi(1.95)$ = $1 - 0.97441$	M1	PI
	= 0.0254 to 0.0256	A1	AWFW
0(-)	The probabilities are the same	E1	oe
9(C)	This is because a total height of 525 centimetres is the same as a mean height of 21 cm as $\frac{525}{25} = 21$	E1	oe
	Total	10	

Q	Answer	Marks	Comments
			•
10(a)(i)	All the car owners in the town	E1	Must include 'all' or 'every'
10(a)(ii)	A sample where each member of the population has an equal probability of being chosen		
	or	E1	oe
	Each sample of the required size from the population has an equal probability of being chosen		
10(b)	average number of events in an interval	B1	oe
40(-)	mean	B1	
10(C)	standard deviation	B1	Accept variance
			SC1 for $\mu$ and $\sigma$ or $\mu$ and $\sigma^2$
	Total	5	