OXFORD
INTERNATIONAL
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INTERNATIONAL A-LEVEL
MATHEMATICS
MA03

(9660/MA03) Unit P2 − Pure Mathematics

Mark scheme

January 2020

Version: 1.0 Final

201xma03/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

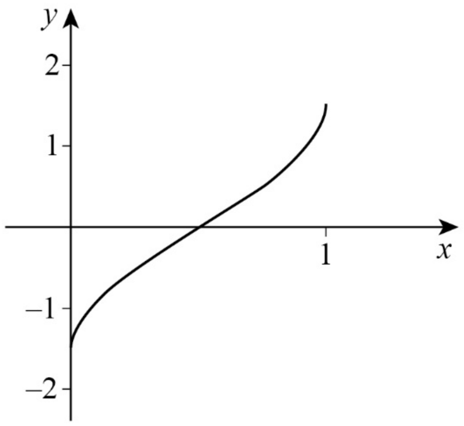
M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)(i)	[$5\sin\theta - 12\cos\theta =$] $R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$ $R = 13$ $\alpha = 67.4$	**M1** **A1** **A1**	PI
1(a)(ii)	$\sin(\theta - 67.4) = -\frac{1}{13}$ $\theta = -108.2$ $\theta = 63.0$	**M1** **A1** **A1**	 **Ft** *their* **(a)** Both correct and no extras in interval (ignore answers outside interval)
1(b)	$2\cot^2 x = 2\text{cosec}^2 x - 2 \quad [= 10 - 5\text{cosec}x]$ $[2\text{cosec}^2 x - 2 = 10 - 5\text{cosec}x]$ $2\text{cosec}^2 x + 5\text{cosec}x - 12 = 0$ $(2\text{cosec}x - 3)(\text{cosec}x + 4)[= 0]$ $\text{cosec}x = \frac{3}{2}, -4$ $\sin x = \frac{2}{3}, -0.25$ $x = 42,\ 138,$ $-14,\ 194$	**M1** **m1** **A1** **B1** **B1**	Correct use of trig identity PI Factorisation or correct use of formula Both correct and no errors seen Either may be seen Sight of **any** of these values correct All four correct and no extras in interval (ignore answers outside interval)
	Total	**11**	

Notes:

(a) May use cos and sin leading to $\sin p = 2/3, -1/4$ for first **M1, m1, A1**

(b) Condone more accurate correct answers, but not –14.5, 194.5

Q	Answer	Marks	Comments
2(a)	$0 \le x \le 1$	B1	
2(b)		M1 A1	Correct shape and position $(0,-\frac{\pi}{2})$ $(1,\frac{\pi}{2})$ stated or marked on diagram
2(c)	Stretch + either I or II Parallel to x-axis I SF $\frac{1}{2}$ II Followed by Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$ $k = \frac{1}{2}$	M1 A1 M1 A1	**Alt:** Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$ **M1** $k = 1$ **A1** Followed by Stretch in x-direction **M1** SF $\frac{1}{2}$ **A1**
2(d)	$\mathrm{f}(x) = \sin^{-1}(2x-1) + x - 1$ $\mathrm{f}(0.6) = -0.198...$ $\mathrm{f}(0.7) = 0.111...$ Change of sign, (the function is continuous), $0.6 < \alpha < 0.7$	M1 A1	Or reverse Both values rounded or truncated to at least 1sf Must have both statements and interval in words or symbols Accept x for α
2(e)	$x_2 = 0.695$ $x_3 = 0.650$	B1 B1	
	Total	**11**	

Q	Answer	Marks	Comments
3(a)	$u = \ln x, \quad dv = x$ $du = \dfrac{1}{x}, \quad v = \dfrac{x^2}{2}$ $\int x \ln x \, dx = \dfrac{x^2}{2}\ln x - \int \dfrac{x^2}{2} \times \dfrac{1}{x} \mathrm{d}x$ $= \dfrac{x^2}{2}\ln x - \dfrac{x^2}{4} \quad [+c]$	M1 A1 A1	Use of parts formula
3(b)	$u = \ln x, \quad dv = 1$ $du = \dfrac{1}{x}, \quad v = x$ $\int \ln x \, dx = x \ln x - \int x \times \dfrac{1}{x} \mathrm{d}x$ $= x \ln x - x \quad [+c]$	M1 A1 A1	Use of parts formula
	Total	**6**	

Q	Answer	Marks	Comments
4(a)	$8[(1.5)^3]+b[(1.5)^2]+c[1.5]+6=-3.75$ $8[(0.5)^3]+b[(0.5)^2]+c[0.5]+6=5.25$	**M1**	One correct substitution OR for **M1** use of long division
	$2.25b+1.5c=-36.75$ $0.25b+0.5c=-1.75$	**A1**	
		m1	Attempt to solve
	$b=-21$ $c=7$	**A1**	Both answers
4(b)	$(4x^2-1)(3x-2)$ $=12x^3-8x^2-3x+2$	**B1** **B1**	
	$(12x^3-8x^2+x+7)-(12x^3-8x^2-3x+2)$	**M1**	
	$=4x+5$	**A1**	
	OR		
	$12x^3-8x^2+x+7=(4x^2-1)(3x+d+\dfrac{ex+f}{4x^2-1})$	**(M1)**	Accept other correct approaches
	$4d=-8,\ \ d=-2$ $-3+e=1,\ \ e=4$	**(B1)** **(B1)**	
	$7=f-d,\ \ f=5$	**(A1)**	
	Total	**8**	

Q	Answer	Marks	Comments
5(a)	$\dfrac{12}{(3-u)(3+u)} = \dfrac{A}{3-u} + \dfrac{B}{3+u}$ oe	M1	
	$A = 2, \quad B = 2$	A1	
5(b)	$[\dfrac{du}{dx} =]\ \cos x$	B1	
	$[\int \dfrac{12\cos x}{8+\cos^2 x} dx =] \int \dfrac{12du}{8+1-u^2}$	M1	All in terms of u, condone omission of du
	$= \int \dfrac{12\,du}{(3-u)(3+u)}$		
	$= \int \dfrac{A}{3-u} + \dfrac{B}{3+u} du$	A1	Must see du here, or earlier
	$[\int \quad =] -2\ln(3-u) + 2\ln(3+u)$	M1	Correct integration
	$= 2\ln \dfrac{3+u}{3-u}$		
	$[x]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = [u]_{0.5}^{1}$	B1	Change of limits, maybe seen earlier (may change back to x and not change limits)
	$[\int \quad =]\ 2\ln 2 - 2\ln 1.4$		
	$= \ln \dfrac{100}{49}$	A1	
	Total	**8**	

Q	Answer	Marks	Comments
6(a)(i)	$(1+2x)^{0.5} =$ $1+0.5\times 2x+[0.5\times -0.5\times (2x)^2]/2$ $=1+x-\dfrac{x^2}{2}$	**M1** **A1**	
6(a)(ii)	$(1-4x)^{-0.5} =$ $1+(-0.5)(-4x)+[(-0.5)(-1.5)(4x)^2]/2$ $=1+2x+6x^2$	**M1** **A1**	
6(b)(i)	$\sqrt{f(x)}=(1+x-0.5x^2)(1+2x+6x^2)$ $1+3x+7.5x^2$	**M1** **A1**	
6(b)(ii)	$\lvert x\rvert < 0.25$	**B1**	Accept $-0.25 \le x < 0.25$
6(c)	$\dfrac{1+2x}{1-4x}=2$ $x=0.1$ $2^{0.5}=1+0.3+0.075$ $=1.375$	**M1** **A1** **A1**	
	Total	**10**	

Q	Answer	Marks	Comments
7(a)	$[\frac{\mathrm{d}y}{\mathrm{d}x}=]3\mathrm{e}^{3x}-24$	M1	
	$[x=0,\ \frac{\mathrm{d}y}{\mathrm{d}x}=]-21$	A1	
	$y=-21x+1$	A1	
7(b)	$3\mathrm{e}^{3x}=24$	M1	
	$3x=\ln 8$ $x=\ln 2$	A1	
	$y=8-24\ln 2$ **ACF**	A1	
7(c)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=9\mathrm{e}^{3x}$ $[=72]$	B1	
	$[x=\ln 2,]$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}>0$	B1	
	Hence, min point	E1	Must have scored **B1B1** to score this mark
	Total	**9**	

Q	Answer	Marks	Comments
8(a)	$\left(\frac{dx}{dt}\right)$ rate of change of x (k) is proportional to $(80-x)$ amount of substance remaining	E1	Complete explanation
8(b)	$\int \frac{\mathrm{d}x}{80-x} = \int k\mathrm{d}t$ $-\ln(80-x) = kt + c$ $t=0,\ x=0,\quad c = -\ln 80$ $t=60,\ x=30,\quad -\ln 50 = 60k - \ln 80$ $k = \frac{1}{60}\ln 1.6 \qquad [= 0.00783...]$	M1 m1 A1 M1 A1	Separate variables Attempt to find k
8(c)(i)	$-\ln(80-x) = 2\ln 1.6 - \ln 80$ $80 - x = \frac{80}{1.6^2}$ $x = 48.75$	M1 A1	
8(c)(ii)	$-\ln(80-70) = \frac{t}{60}\ln 1.6 - \ln 80$ $\frac{t}{60} = \frac{\ln 8}{\ln 1.6}$ $t = 265$	M1 m1 A1	 Accept 265 - 266
	Total	**11**	

Notes: 8(c) correct answers with **NMS** scores full marks

Q	Answer	Marks	Comments
9	$2y + xy = 1$ $x = \dfrac{1-2y}{y}$	B1	
	[Vol =] $\pi \int_{0.2}^{0.25} (\dfrac{1-2y}{y})^2 \, \mathrm{d}y$	B1	Correct including π, limits, $\mathrm{d}y$
	$\int = \int y^{-2} + 4 - \dfrac{4}{y}$ [dy]	M1	Attempt to expand
	$= -y^{-1} + 4y - 4\ln y$	A1	Correct simplified integral
	$= [-4 + 1 - 4\ln 0.25] - [-5 + 0.8 - 4\ln 0.2]$ $= 1.2 + 4\ln 0.8$	m1	Correct substitution of correct limits into expression in correct form (PI by final answer of 0.964 – 0.966)
	$\mathrm{Vol} = \pi(1.2 + 4\ln 0.8)$ **ACF**	A1	
	Total	**6**	

<table>
<tr><th>Q</th><th>Answer</th><th>Marks</th><th>Comments</th></tr>
<tr>
<td>10(a)</td>
<td>
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>1.625</td><td>$1.625^{-1.625} = 0.45432$</td></tr>
<tr><td>1.875</td><td>$1.875^{-1.875} = 0.30770$</td></tr>
<tr><td>2.125</td><td>$2.125^{-2.125} = 0.20154$</td></tr>
<tr><td>2.375</td><td>$2.375^{-2.375} = 0.12817$</td></tr>
<tr><td>2.625</td><td>$2.625^{-2.625} = 0.07939$</td></tr>
<tr><td>2.875</td><td>$2.875^{-2.875} = 0.04802$</td></tr>
</table>
$0.25[0.45432 + 0.30770 + 0.20154$
$+0.12817 + 0.079394 + 0.04802]$

$= 0.305$
</td>
<td>B1

M1

m1

A1</td>
<td>All six correct x values (and no extra used) PI by five correct y values

At least five correct y values in exact form or decimals, rounded or truncated to three dp or
better (in table or formula)
(PI by AWRT correct answer)

Correct sub into formula with $h = 0.25$ OE and at least five correct y values either listed, with + signs, or totalled.
(PI by AWRT correct answer)
CAO, must see this value exactly and no error seen</td>
</tr>
<tr>
<td>10(b)</td>
<td>$y = x^{-x}$
$\ln y = -x \ln x$
$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = -x \times \frac{1}{x} - \ln x$
$= -1 - \ln x$
$\frac{\mathrm{d}y}{\mathrm{d}x} = (-1 - \ln x)y$
$= (-1 - \ln x)x^{-x}$</td>
<td>B1
M1

A1
A1</td>
<td>

ACF</td>
</tr>
<tr><td></td><td>Total</td><td>8</td><td></td></tr>
<tr><td colspan="4">Notes:
10(a) 0.305 with NMS scores 4/4
(b) Correct answer without using implicit differentiation scores SC2</td></tr>
</table>

Q	Answer	Marks	Comments
11(a)	[Equation $\vec{AB}$] $\mathbf{r} = \begin{pmatrix} 10 \\ 2 \\ -3 \end{pmatrix} + f\begin{pmatrix} -8 \\ -4 \\ 8 \end{pmatrix}$	**M1**	
	$= \begin{pmatrix} 10 \\ 2 \\ -3 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$	**A1**	
	$\lambda = -4f$ $\lambda = -3$	**m1**	
	$(4,\ -1,\ 3)$ is on line QED	**A1**	
11(b)(i)	Coords of *C* $(4+2c, -1+c, 3-2c)$ $\vec{DC} = \begin{pmatrix} 6+2c \\ -2+c \\ -4-2c \end{pmatrix}$ oe	**M1**	
	$\begin{pmatrix} 6+2c \\ -2+c \\ -4-2c \end{pmatrix} . \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = 0$	**m1**	
	$12+4c-2+c+8+4c=0$	**A1**	
	$c=-2$ $C=(0,-3,7)$	**m1** **A1**	
11(b)(ii)	$CD^2 = (0--2)^2 + (-3-1)^2 + (7-7)^2$	**M1**	
	$=4+16$ $CD = \sqrt{20}$	**A1**	
11(c)	$CP^2 = (4+2p)^2 + (2+p)^2 + (-4-2p)^2$	**M1**	
	$=9p^2+36p+36$	**A1**	
	$9p^2+36p+16=0$	**M1**	
	$p = \dfrac{-36 \pm \sqrt{36^2 - 4\times 9\times 16}}{2\times 9}$	**M1**	
	$p=-2+\frac{1}{3}\sqrt{q}, \quad p=-2-\frac{1}{3}\sqrt{q}$	**A1**	oe
	Total	**16**	

Q	Answer	Marks	Comments
12(a)	$12y\frac{\mathrm{d}y}{\mathrm{d}x}+8\mathrm{e}^{4x}=y^3\mathrm{e}^x+\mathrm{e}^x 3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$	M1 A1	Either implicit differential correct
	$(12y-\mathrm{e}^x 3y^2)\frac{\mathrm{d}y}{\mathrm{d}x}=y^3\mathrm{e}^x-8\mathrm{e}^{4x}$	M1	Or using $\frac{dy}{dx}=0$
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y^3\mathrm{e}^x-8\mathrm{e}^{4x}}{(12y-\mathrm{e}^x 3y^2)}$		
	$\frac{dy}{dx}=0,\quad y^3\mathrm{e}^x=8\mathrm{e}^{4x}$	A1	
	$q^3=8\mathrm{e}^{3p}$		
	$q=2\mathrm{e}^p$	A1	**AG**
12(b)	$6\times 4e^{2p}+2e^{4p}=8e^{3p}e^p$	M1	Equation all in terms of one variable
	$24e^{2p}=6e^{4p}$	A1	
	$e^{2p}=4$	m1	
	$p=\ln 2$		
	$q=4$	A1	ACF
	Total	**9**	

Q	Answer	Marks	Comments
13	$\frac{\mathrm{d}x}{\mathrm{d}t} = 2at$ $\frac{\mathrm{d}y}{\mathrm{d}t} = 2a$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2a}{2at} = \frac{1}{t}$	M1	Either differential correct
	$At\ P,\ y - 2ap = \frac{1}{p}(x - ap^2)$	m1	Equation of a tangent with t replaced
	$yp = x + ap^2$ $yq = x + aq^2$	A1	Both correct (unsimplified)
	$At\ R,\ y(p-q) = a(p^2 - q^2)$ $y = a(p+q)$	m1	Attempt to solve
	$x = apq$	A1	Both correct
	$y^2 = a^2(p^2 + q^2 + \frac{2x}{a})$ $= a(ap^2 + aq^2 + 2x)$	m1	
	$p^2 + q^2 = 1,$ $y^2 = a(a + 2x)$	A1	
	Total	**7**	