Pearson Edexcel

Mark Scheme (Results)

October 2020

Pearson Edexcel IAL Mathematics (WMA13)
Pure Mathematics P3

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Pearson Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ or ft will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper or ag- answer given
   - $\lfloor\,$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:

   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice

Question Number	Scheme	Marks
1	$2\left(2\cos^2 x - 1\right) = 7\cos x$	M1
	$4\cos^2 x - 7\cos x - 2 = 0 \Rightarrow \cos x = -\frac{1}{4}$	M1 A1
	$\Rightarrow x = \arccos\left(-\frac{1}{4}\right) = 104.5°, 255.5°$	dM1 A1
		(5)
		(5 marks)

M1 Attempts to use $\cos 2x = \pm 2\cos^2 x \pm 1$ to form a quadratic equation in $\cos x$

If the other two forms are attempted there must be some attempt to use $\sin^2 x + \cos^2 x = 1$ to form a quadratic equation in $\cos x$

$2 \times 2\cos^2 x - 1 = 7\cos x$ is M0 unless the correct identity has been previously stated or recovery occurs.

M1 Attempts to solve a 3TQ in $\cos x$ using an allowable method (the quadratic need not be correct and may have come from incorrect work)

A1 Reaches $\cos x = -\frac{1}{4}$ or $-\frac{2}{8}$ or −0.25. (May be implied by a correct value for x) Ignore any reference to $\cos x = 2$ Those who use $y = \cos x$ and stop at a $y = -\frac{1}{4}$ score A0.

dM1 Depend on the second method mark. Takes arccos of at least one solution (α) of their quadratic where $|\alpha| < 1$ to find at least one solution in range. If substitution not seen then you will need to check.

NB a radian answer of awrt 1.8 or correct 1d.p. answer for their α can imply the method.

A1 awrt $104.5°, 255.5°$ with no other values in the range. Ignore values outside the range.

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Question Number	Scheme	Marks
2.(a)	Sight of $10^{1.478}$ or $10^{0.0646}$ or $10^{0.0646t+1.478}$	M1
	$(a=)\text{awrt } 30$ or $(b=)\text{awrt } 1.16$	A1
	$\log_{10} N = 0.0646t + 1.478 \rightarrow N = 10^{0.0646t+1.478} = 10^{0.0646t}10^{1.478}$ $= "30"\times"1.16"^t$	dM1
	$N = 30\times 1.16^t$	A1
		(4)
(b)	Attempts $N = 30\times 1.16^{30} = \text{awrt } 2600$	M1 A1
		(2)
		(6 marks)

(a) **NB This shows as MMAA on ePEN but is being marked as MAMA.**

M1 Sight of $10^{1.478}$ or $10^{0.0646}$ or $10^{0.0646t+1.478}$ (allowing slips copying the values) anywhere in their solution. This mark is implied by seeing awrt 30 or awrt 1.16

A1 Sight of either $\text{awrt } 30$ or $\text{awrt } 1.16$

dM1 Applies correct index laws and proceeds to find values for *a* and *b*. $N = \left(10^{0.0646}\right)^t \times 10^{1.478} = "1.16"^t \times "30"$

For this mark there must be evidence of correct index work so expect to see at least $10^{0.0464t+1.478}$ before a final answer and no incorrect index work.

A1 $a = awrt\ 30$ and $b = awrt\ 1.16$ as long as there is no contrary work, or states that $N = 30\times 1.16^t$ (awrt values)

(b)

M1 Attempts $N = 30\times 1.16^{30}$ with their values of *a* and *b*.

Alternatively $\log_{10} N = 0.0646\times 30 + 1.478 \Rightarrow N = 10^{3.416}$

A1 awrt 2600, isw after a correct answer.

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Alt (a) using $N = ab^t$ as a starting point.

M1 As main scheme.

A1 $a = \text{awrt } 30$ or $b = \text{awrt } 1.16$ (must be correctly assigned)

dM1 Takes $\log_{10}$ of both sides and proceeds to at least $\log_{10} N = \log_{10} a + t\log_{10} b$ and attempts to find a value for both of the constants with no incorrect log work.

A1 $a = awrt\ 30$ and $b = awrt\ 1.16$ as long as there is no contrary work, or states that $N = 30\times 1.16^t$ (awrt values)

Question Number	Scheme	Marks
3. (a)	$\dfrac{dy}{dx}=\dfrac{(4x-1)^{\frac{1}{2}}\times 2-(2x+3)\times 2(4x-1)^{-\frac{1}{2}}}{(4x-1)}$	M1 A1
	$\dfrac{(4x-1)^{\frac{1}{2}}\times 2-(2x+3)\times 2(4x-1)^{-\frac{1}{2}}}{(4x-1)}\times\dfrac{(4x-1)^{\frac{1}{2}}}{(4x-1)^{\frac{1}{2}}}=\dfrac{4x-8}{(4x-1)^{\frac{3}{2}}}$	dM1 A1
		(4)
(b)	Turning point where $\dfrac{dy}{dx}=0\Rightarrow x=2$	M1
	Find value of f at $x=2\Rightarrow f(x)=\sqrt{7}$ Hence range is $f\geqslant\sqrt{7}$	dM1 A1
		(3) **(7 marks)**

(a)

M1 Attempts the quotient rule and **achieves** $\dfrac{dy}{dx}=\dfrac{p(4x-1)^{\frac{1}{2}}-q(2x+3)(4x-1)^{-\frac{1}{2}}}{(4x-1)}$ condoning slips in coefficients

Allow the denominator to appear as $\left((4x-1)^{\frac{1}{2}}\right)^2$ or even $(4x-1)^{\frac{1}{4}}$

Alternatively attempts the product rule with $(2x+3)$ and $(4x-1)^{-\frac{1}{2}}$ and **achieves**

$\dfrac{dy}{dx}=A(4x-1)^{-\frac{1}{2}}\pm B(2x+3)(4x-1)^{-\frac{3}{2}}$

A1 For a correct un-simplified $\dfrac{dy}{dx}$. Look for $\dfrac{dy}{dx}=\dfrac{(4x-1)^{\frac{1}{2}}\times 2-(2x+3)\times 2(4x-1)^{-\frac{1}{2}}}{(4x-1)}$ oe via the quotient rule

Look for $\dfrac{dy}{dx}=2(4x-1)^{-\frac{1}{2}}-2(2x+3)(4x-1)^{-\frac{3}{2}}$ oe via the product rule

dM1 Multiplies both numerator and denominator by $(4x-1)^{\frac{1}{2}}$ and simplifies to a linear numerator, or correctly takes a common factor of $(4x-1)^{-\frac{1}{2}}$ out of the numerator to leave a linear factor.

If the product rule is used look for a correct attempt to factor out $(4x-1)^{-\frac{3}{2}}$

A1 $\dfrac{4x-8}{(4x-1)^{\frac{3}{2}}}$ or exact simplified equivalent Must have a single term in $(4x-1)$

NB Allow marks for (a) if simplification of f'(x) is seen in (b) (but not having set f'(x) = 0 and rearranging).

(b)

M1 Attempts $\dfrac{dy}{dx}=0\Rightarrow x=\ldots$

dM1 Finds the value of $f(\text{their } 2)$, where their 2 > $\dfrac{1}{4}$

A1 $f\geqslant\sqrt{7}$ Allow e.g. $y\geqslant\sqrt{7}$, $f(x)\geqslant\sqrt{7}$ or range is $\left[\sqrt{7},\infty\right)$ Allow with $\dfrac{7}{\sqrt{7}}$ oe or awrt 2.65

Do not allow $x\geqslant\sqrt{7}$ or just $\geqslant\sqrt{7}$

Special Case: If candidates square the function first in (a) they will get $\left(f(x)^2\right)'=1-\dfrac{49}{(4x-1)^2}=\dfrac{16x^2-8x-48}{(4x-1)^2}$.

They will score no marks in (a) but can achieve full marks in (b) as the stationary point occurs at the same x coordinate.

Special Case: A correct range given following no attempt, or an incorrect answer to (a), can score SC M1M0A0.

Question Number	Scheme	Marks
4 (a)	$\mathrm{ff}(6)=\mathrm{f}(13)=-1$	M1 A1
		(2)
(b)	Attempts $21+2(2-x)=5x \Rightarrow x=...$ or $21-2(x-2)=5x \Rightarrow x=...$	M1
	$x=\frac{25}{7}$ only	A1
		(2)
(c)	Either $k<21$ or $k \geqslant 17$	M1
	$17 \leqslant k<21$	A1
		(2)
(d)	$a=\frac{1}{7}$ $b=4$	B1 B1
		(2)
		(8 marks)

(a)

M1 For attempting f "twice". Award for sight of $\mathrm{f}(13)$ or $21-2\left|2-\left(21-2\left|2-x\right|\right)\right|$ followed by a value.

A1 For -1

(b)

M1 Attempts $21+2(2-x)=5x \Rightarrow x=...$

A1 For $x=\frac{25}{7}$ only. Do not isw if students go on to find additional solutions unless they identify this as the only answer by rejecting others.

(c)

M1 For either $k<21$ or $k \geqslant 17$ Condone for this mark $k \leqslant 21$ or $k>17$

Alt: Allow for **both** k = 17 and k = 21 identified as critical values even if no inequalities are given.

A1 $17 \leqslant k<21$ May be written as separate inequalities. Accept alternative notations for the range such as $[17,21)$

Do not accept $17 \leqslant \mathrm{f}(x)<21$

(d)

B1 Either $a=\frac{1}{7}$ **or** $b=4$

B1 Both $a=\frac{1}{7}$ **and** $b=4$ Allow both marks if $y=\frac{1}{7}\mathrm{f}(x-4)$ is given.

Question Number	Scheme	Marks
5 (a)	$\sin 3x \equiv \sin(2x+x) \equiv \sin 2x\cos x + \cos 2x\sin x$	M1
	$\equiv 2\sin x\cos x\cos x + \left(1-2\sin^2 x\right)\sin x$	M1
	$\equiv 2\sin x\left(1-\sin^2 x\right) + \left(1-2\sin^2 x\right)\sin x$	ddM1
	$\equiv 3\sin x - 4\sin^3 x$ *	A1*
		(4)
(b)	$\int_0^{\frac{\pi}{3}} \sin^3 x \, dx = \int_0^{\frac{\pi}{3}} \frac{3}{4}\sin x - \frac{1}{4}\sin 3x \, dx$	M1
	$= \left[-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x\right]_0^{\frac{\pi}{3}}$	dM1 A1
	$= \frac{5}{24}$	A1
		(4) **(8 marks)**

(a)

M1 Uses $\sin 3x = \sin(2x+x) = \pm\sin 2x\cos x \pm \cos 2x\sin x$

M1 Uses the correct identity for $\sin 2x = 2\sin x\cos x$ and any correct identity for $\cos 2x$

ddM1 Dependent upon both previous M's. It is for using $\cos^2 x = 1-\sin^2 x$ to get an equation in only $\sin x$

A1* Fully correct solution with correct notation within their proof. Examples of incorrect notation include use of $\sin x^2$ instead of $\sin^2 x$ or use of sin instead of sin x and so on. Penalise in the A mark only for such.

Note: The ddM mark and final A mark may be score by substituting $\sin^2 x = 1-\cos^2 x$ into the right hand side of the equation to reach an identical expression to an expanded left hand side ("working from both sides"), with the A mark then awarded for correct work leading to identical expressions **and** an minimal conclusion given (e.g. //)

Note You may see use of De Moivre's Theorem from an FP3 candidate. This can score full credit if carried out correctly. If there are errors or you are unsure then send to review.

If attempted in reverse:

M1 $3\sin x - 4\sin^3 x = 3\sin x - 2\sin^2 x\sin x - 2\sin x(1-\cos^2 x) = \sin x - 2\sin x\frac{1}{2}\left(1-\cos 2x\right) + \sin 2x\cos x$

Uses $\sin^3 x = \sin x \times \sin^2 x$ with either $\sin^2 x = \frac{1}{2}(1-\cos 2x)$ OR $\sin^2 x = 1-\cos^2 x$ and $2\sin x\cos x = \sin 2x$

M1 Uses both steps above to get to an equation with sin$2x$ and cos$2x$

ddM1 Gathers terms to reach $=\cos 2x\sin x + \sin 2x\cos x$

A1 Completes the proof uses $\cos 2x\sin x + \sin 2x\cos x = \sin 3x$ with no errors seen.

(b)

M1 Attempts to use part (a) to simplify. Accept $\int \sin^3 x \, dx = \int A\sin x + B\sin 3x \, dx$

dM1 $\int A\sin x + B\sin 3x \, dx \to a\cos x + b\cos 3x$

A1 $-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x$ oe (not a multiple of this (unless recovered))

A1 CSO $=\frac{5}{24}$

Note an answer of A1 $\frac{5}{24}$ with no supporting working scores no marks as algebraic integration is specified. But alternative methods of integration are permissible. Two alternatives are:

Question Number	Scheme	Marks
5 (b) Alt 1	$\int_0^{\frac{\pi}{3}} \sin^3 x\,\mathrm{d}x = \int_0^{\frac{\pi}{3}} \sin x\left(1-\cos^2 x\right)\mathrm{d}x = \int_0^{\frac{\pi}{3}} \sin x - \sin x\cos^2 x\,\mathrm{d}x$	M1
	$= \left[-\cos x - \left(-\frac{\cos^3 x}{3}\right)\right]_0^{\frac{\pi}{3}}$	dM1A1
	$= \frac{5}{24}$	A1
		(4)
(b) Alt 2	$u = \cos x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = -\sin x \Rightarrow \int_0^{\frac{\pi}{3}} \sin^3 x\,\mathrm{d}x = \int_1^{\frac{1}{2}} u^2 - 1\,\mathrm{d}u$	M1
	$= \left[\frac{u^3}{3} - u\right]_1^{\frac{1}{2}}$	dM1 A1
	$= \frac{5}{24}$	A1
		(4) **(8 marks)**

Notes: First three marks as follows (final A is as main scheme).

Alt 1

M1: Splits as $\sin^3 x = \sin x \sin^2 x$ and applies $\sin^2 x = 1 - \cos^2 x$ to get the integrand into and integrable form.

dM1 for $\sin x \to \pm\cos x$ and $\sin x\cos^2 x \to K\cos^3 x$

A1 $-\cos x - \left(-\frac{\cos^3 x}{3}\right)$ oe (not a multiple of this (unless recovered))

Alt 2

M1 Sets $u = \cos x$, finds $\frac{\mathrm{d}u}{\mathrm{d}x} = \pm\sin x$ and makes a full substitution using both of these to get an integral in terms of u only. (Limits not needed for this mark).

dM1 for $au^2 - b \to Au^3 - bu$

A1 For reaching $\left[\frac{u^3}{3} - u\right]_1^{\frac{1}{2}}$ including correct limits or for undoing the substitution and reaching $\frac{\cos^3 x}{3} - \cos x$

Question Number	Scheme	Marks
6 (a)	$5e^{x-1}+3=18 \Rightarrow e^{x-1}=3$	M1
	$\Rightarrow x=\ln 3+1$ or $e^{x}=3e$	A1
	$\Rightarrow x=\ln 3e$	A1
		(3)
(b)	Sets $5e^{x-1}+3=10-x^{2}$ and proceeds to find and use a suitable function. Eg $\left(f(x)=\right)7-x^{2}-5e^{x-1}$	B1
	Attempts $f(1.1335)=0.001$ and $f(1.1345)=-0.007$	M1
	Correct values with reason(change of sign and continuous) and conclusion, hence α is 1.134 to 3dp	A1
		(3)
(c)	$x_2=-\sqrt{7-5e^{-3-1}}=-2.628388$	M1 A1
	$\beta=-2.620330$	A1
		(3)
		(9 marks)

(a)

M1 For attempting to proceed from $5e^{x-1}+3=18$ to $e^{x-1}=...$ or $e^{x}=...$

A1 $x=\ln 3+1$ or for $e^{x}=3e$

A1 $x=\ln 3e$ Accept $x=\ln 3e^{1}$

(b)

B1 Sets the equations equal to each other and finds a suitable function which is then used.

Suitable functions are $f(x)=\pm\left(7-x^{2}-5e^{x-1}\right)$, $g(x)=\pm\left(x-\sqrt{7-5e^{x-1}}\right)$ or $h(x)=\pm\left(x-1-\ln\left(\frac{7-x^{2}}{5}\right)\right)$

M1 Substitutes $x=1.1335$ and $x=1.1345$, or suitable values for a tighter interval (either side of 1.133634..), into a suitable function and obtains one correct value to 1sf rounded or truncated.

FYI for the + options above

$f(1.1335)=0.001$ and $f(1.1345)=-0.007$

$g(1.1335)=-0.0005$ and $g(1.1345)=0.003$

$h(1.1335)=-0.0002$ and $g(1.1345)=0.001$

A1 Requires both values to be correct (1sf rounded or truncated), a reason (change of sign and continuous) and a minimal conclusion

(c)

M1 For an attempt at substituting -3 into the iterative equation.
This is implied by the sight of the expression or awrt -2.63

A1 awrt -2.628388

A1 $\beta=$ awrt -2.620330 condone -2.62033

....

It is possible in part (b) to score B0 M1 A1 by comparing y values for $y=5e^{x-1}+3$ and $y=10-x^{2}$ at $x=1.1335$ and $x=1.1345$ For the A1 apply the similar criteria as for the main scheme with values to 3d.p..

At $x=1.1335$, $y|_{1.1335}=5e^{x-1}+3=8.714$ and $y|_{1.1335}==10-x^{2}=8.715$

At $x=1.1345$, $y|_{1.1345}=5e^{x-1}+3=8.720$ and $y|_{1.1345}=10-x^{2}=8.713$

Question Number	Scheme	Marks
7.(a)	$R=\sqrt{17}$	B1
	$\tan\alpha=4\Rightarrow\alpha=\text{awrt }1.326$	M1A1
		(3)
(b)	Minimum height $=\dfrac{24}{3+"R"}=3.37$ (metres)	M1 A1
		(2)
(c)	Uses part (a) $10=\dfrac{24}{3+\sqrt{17}\cos\left(\frac{1}{2}t-1.326\right)}\Rightarrow\cos\left(\frac{1}{2}t-1.326\right)=\dfrac{-0.6}{\sqrt{17}}$	M1 A1
	$t=\text{awrt }6.09$	M1 A1
		(4)
		(9 marks)

(a)

B1 $R=\sqrt{17}$

Condone $R=\pm\sqrt{17}$ (Do not allow decimals for this mark Eg 4.12 but remember to isw after $\sqrt{17}$)

M1 $\tan\alpha=\pm4,\ \tan\alpha=\pm\frac{1}{4}\Rightarrow\alpha=...$

If R is used to find α accept $\sin\alpha=\pm\frac{4}{R}$ or $\cos\alpha=\pm\frac{1}{R}\Rightarrow\alpha=...$

A1 $\alpha=\text{awrt }1.326$ Note that the degree equivalent $\alpha=\text{awrt }75.96°$ is A0

(b)

M1 Attempts minimum height by stating or finding $\dfrac{24}{3+"R"}$

Attempts via differentiation must be complete methods with correct work up to slips in coefficients. They are unlikely to succeed.

FYI $\dfrac{dH}{dt}=-\dfrac{24\left(2\cos\left(\frac{t}{2}\right)-\frac{1}{2}\sin\left(\frac{t}{2}\right)\right)}{\left(4\sin\left(\frac{t}{2}\right)+\cos\left(\frac{t}{2}\right)+3\right)^2}=0\Rightarrow\tan\left(\frac{t}{2}\right)=4\Rightarrow H=\dfrac{24}{3+\cos(1.326)+4\sin(1.326)}$

A1 3.37 (metres) Assume metres unless otherwise stated, but 3.37 cm is A0. Accept 337 cm as long as the units are stated, but do not accept $\dfrac{24}{3+\sqrt{17}}$ and do not isw if incorrect units are given following 3.37.

(c)

M1 Attempts to use their answer to part (a) (including their R and their α) AND proceeds to

$\cos\left(\beta t\pm"1.326"\right)=k,\quad -1<k<1$ and $\beta=1$ or $\frac{1}{2}$

A1 $\cos\left(\beta t\pm"1.326"\right)=\dfrac{-0.6}{\sqrt{17}}$ or awrt -0.146 where $\beta=1$ or $\frac{1}{2}$

M1 Full method to make t the subject from an equation of the form $\cos\left(\frac{1}{2}t\pm"1.326"\right)=k,\quad -1<k<1$

Look for $2\times\left(\text{their }\arccos(k)\pm\text{their }\alpha\right)$

A1 awrt $t=6.09$ (Ignore any extra solutions outside the domain, but A0 if extras inside are given.)

Question Number	Scheme	Marks
8(i)(a)	$g'(x) = 3e^{3x}\sec 2x + 2e^{3x}\sec 2x\tan 2x$	M1 A1
		(2)
(b)	$g'(x) = 0 \Rightarrow\ e^{3x}\sec 2x\left(3 + 2\tan 2x\right) = 0$	M1
	$\tan 2x = -1.5 \Rightarrow x = -0.491$	dM1 A1
		(3)
(ii)	$x = \ln(\sin y) \Rightarrow \dfrac{dx}{dy} = \dfrac{1}{\sin y} \times \cos y$	B1
	Attempts $\cos y = \sqrt{1-\sin^2 y} = \sqrt{1-e^{2x}}$ or $\dfrac{dy}{dx} = \dfrac{\sin y}{\cos y}$	M1
	Hence $\dfrac{dy}{dx} = \dfrac{\sin y}{\cos y} = \dfrac{e^x}{\sqrt{1-e^{2x}}}$	dM1 A1
		(4)
		(9 marks)

(i)(a)

M1 Correct attempt at the product rule $g'(x) = Ae^{3x}\sec 2x + Be^{3x}\sec 2x\tan 2x$

For use of the quotient rule look for $\dfrac{A\cos 2x\, e^{3x} - Be^{3x}\sin 2x}{\cos^2 2x}$

A1 $g'(x) = 3e^{3x}\sec 2x + 2e^{3x}\sec 2x\tan 2x$ Allow in any form and then isw

For use of the quotient rule look for $\dfrac{3\cos 2x\, e^{3x} + 2e^{3x}\sin 2x}{\cos^2 2x}$

(i)(b)

M1 Sets $g'(x) = 0 \Rightarrow$ and takes out / factorises out $e^{3x}\sec 2x$ to identify a linear factor in tan2x

For the quotient rule they should be factorising out $\dfrac{e^{3x}}{\cos^2 2x}$ to leave a linear factor in $\cos 2x$ and $\sin 2x$

dM1 Correct order of operations to x = ...

For quotient rule approach they must use $\tan 2x = \dfrac{\sin 2x}{\cos 2x}$ (oe correct work)

You may need to check their answer if no method is shown. Accept awrt 2.s.f. for their tan 2x = …. in either radians or degrees.

A1 $x = awrt\ -0.491$ only in the range. If extra solutions arise from trying to solve sec2x = 0 then A0.

(ii)

B1 $\dfrac{dx}{dy} = \dfrac{1}{\sin y} \times \cos y$ OR $e^{x} = \cos y \dfrac{dy}{dx}$ via $e^{x} = \sin y$

M1 For one of the two operations needed to complete the proof

- Either an attempt to get $\cos y$ in terms of e^{x} look for an attempt using $\sin^2 y + \cos^2 y = 1$ with $\sin y$ being replaced by e^{x}. Alternatively allow use of arcsin
- Or taking the reciprocal and making $\dfrac{dy}{dx}$ the subject (variables must be consistent)

dM1 Applies both operations to obtain $\dfrac{dy}{dx}$ in terms of just e^{x}

A1 $\dfrac{dy}{dx} = \dfrac{e^{x}}{\sqrt{1-e^{2x}}}$ or $\dfrac{e^{x}}{\cos\left(\arcsin e^{x}\right)}$ Allow $\dfrac{1}{\sqrt{1-\left(e^{x}\right)^2}}$ or states $f(x) = \sqrt{1-e^{2x}}$ following a correct expression for $\dfrac{dy}{dx} = \dfrac{e^{x}}{\cos y}$ or similar.

Alt:

B1 $x = \ln(\sin y) \Rightarrow y = \arcsin\left(e^{x}\right)$

M1 $\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-\left(e^{x}\right)^2}} \times \ldots$ or $\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-(\ldots)^2}} \times e^{x}$

dM1 both of these

A1 $\dfrac{e^{x}}{\sqrt{1-\left(e^{x}\right)^2}}$

Question Number	Scheme	Marks
9 (a)	$$x^2-x-12\,\overline{\smash{)}\,x^4-x^3-10x^2+3x-9}$$ with quotient x^2+2 $\underline{x^4-x^3-12x^2}$ $2x^2+3x-9$ $\underline{2x^2-2x-24}$ $5x+15$	M1A1
	$$\frac{x^4-x^3-10x^2+3x-9}{x^2+x-12}\equiv x^2+2+\frac{5(x+3)}{(x-4)(x+3)}$$	M1
	$\equiv x^2+2+\dfrac{5}{(x-4)}$ or stating $P=2, Q=5$	A1
		(4)
(b)	$$\mathrm{g}'(x)=2x-\frac{5}{(x-4)^2}$$	M1A1
	Subs $x=2$ into $\mathrm{g}'(2)=2\times 2-\dfrac{5}{(2-4)^2}=\dfrac{11}{4}$	M1
	Uses $m=\mathrm{g}'(2)=\left(\dfrac{11}{4}\right)$ with $(2, \mathrm{g}(2))=\left(2,\dfrac{7}{2}\right)$ to form equation of tangent $$y-\frac{7}{2}=\frac{11}{4}(x-2)\Rightarrow y=\frac{11}{4}x-2$$	dM1A1
		(5)
(c)	$$\int x^2+2+\frac{5}{(x-4)}\,\mathrm{d}x=\frac{1}{3}x^3+2x,+5\ln\lvert x-4\rvert$$	M1 A1ft
	Area $R=\left[\dfrac{1}{3}x^3+2x+5\ln\lvert x-4\rvert\right]_0^2=\left(\dfrac{8}{3}+4+5\ln 2\right)-\left(0+0+5\ln 4\right)$	dM1
	$$=\frac{20}{3}+5\ln 2-5\ln 4=\frac{20}{3}-5\ln 2$$	ddM1 A1
		(5) **(14 marks)**
Alt (a)	$x^4-x^3-10x^2+3x-9\equiv(x^2+P)(x^2-x-12)+Q(x+3)$	M1
	Compare terms (OR sub in values) and solve simultaneously to find P and/or Q ie $x^2\Rightarrow P-12=-10,\quad x\Rightarrow -P+Q=3,\quad \text{const}\Rightarrow -12P+3Q=-9$ $\Rightarrow P=\ldots$ or $Q=\ldots$	M1
	$$P=2, Q=5\,*$$	A1,A1
	Award in the order shown here on ePEN.	

(a)

M1: Divides to obtain a quadratic quotient and a linear remainder. May divide by $(x-4)$ and then by $(x+3)$ or vice versa to reach these but must be a full process.

FYI: By $x+3$ first gives x^3-4x^2+2x-3 as quotient, followed by $x^2+2+\dfrac{5}{x-4}$

By $x-4$ first gives $x^3+3x^2+2x+11+\frac{35}{x-4}$ then $x^2+2+\frac{5}{x+3}+\frac{35}{(x+3)(x-4)} \to x^2+2+\frac{5}{x-4}$

A1: Obtains a quotient of x^2+2 and a remainder of $5x+15$

M1: Writes the given expression in the required form using $x^2-x-12=(x-4)(x+3)$ or divides $x+3$ into their remainder term.

A1: Correct answer. $(P=2, Q=5)$ May be awarded following an incorrect "ax^2+2" quadratic factor.

Alt:

M1: Multiplies though completely by the denominator and cancels the $x-4$ term.

M1: Complete process of comparing coefficients or substituting values to find a value for either P or Q

A1: Either $P=2$ or for **showing** $Q=5$* (must have seen a correct equation for Q)

A1: Both $P=2$ and **showing** $Q=5$* Note that $Q=5$ is given so it must be shown from correct work, not just stated.

Note M0M1A1A0 is possible if Q is assumed and factorisation of x^2-x-12 is never seen.

(b)

M1: For $\frac{Q}{x-4} \to \frac{\dots}{(x-4)^2}$

A1: For $g'(x)=2x-\frac{5}{(x-4)^2}$. Note that this can be scored from an incorrect P.

M1: Attempts the gradient of C at the point where $x=2$

dM1: Depends on previous M. A complete method of finding the equation of the tangent. If $y=mx+c$ is used, they must proceed as far as finding c.

A1: $y=2.75x-2$ or exact equivalent and isw.

Note: This may be attempted from the original function $g(x)=\frac{x^4-x^3-10x^2+3x-9}{x^2+x-12}$

M1: Scored for an attempt at the quotient rule A1 if correct and so on.

$$\to \frac{(ax^3+bx^2+cx+d)(x^2+x-12)-(x^4-x^3-10x^2+3x-9)(px+q)}{(x^2+x-12)^2}$$

(c)

M1: Attempts to integrate with $\int \frac{..}{(x-4)}\,dx \to ..\ln|x-4|$ Condone $\ln(x-4)$

A1ft: $\int x^2+P+\frac{5}{(x-4)}\,dx=\frac{1}{3}x^3+Px+5\ln|x-4|$ following through on their P.

dM1: Dependent on first M mark. Attempt the area of R using the limits 0 and 2 in their integrated function and subtracting the correct way round (or recovered).

ddM1: Depends on both previous M's. Scored for attempting to combine two log terms using correct log work

Allow the method and final accuracy if $\ln(-2)-\ln(-4) \to \ln\left(\frac{-2}{-4}\right)=-\ln 2$ is used (bod that modulus is meant)

Do not allow if $\ln(-a) \to -\ln a$ is used.

A1: cao $\frac{20}{3} - 5\ln 2$

Note: If a candidate gives correct values of m and c in (b) and of a and b in (c) but has not stated the answer in correct form, then penalise only the first instance.

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