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(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1	$h\mathrm{f}(2,1) = 0.2 \times \frac{1}{1+\sqrt{2}}$	M1	Correct substitution into RHS of this expression
	$= 0.0828427$	A1	**AWRT** 0.0828 **PI**
	$y_2 = 1 + 0.0828427 = 1.0828427$	m1	1 + their value of $h\mathrm{f}(2,1)$
	$y_3 = 1.0828427 + 0.2 \times \frac{1}{1.0828427 + \sqrt{2.2}}$	M1	Correct substitution using their y_2 into the second term.
	$[= 1.0828427 + 0.2 \times 0.3896991 = 1.1607825]$		
	1.161	A1	Correct answer given to 3 dp
		5	

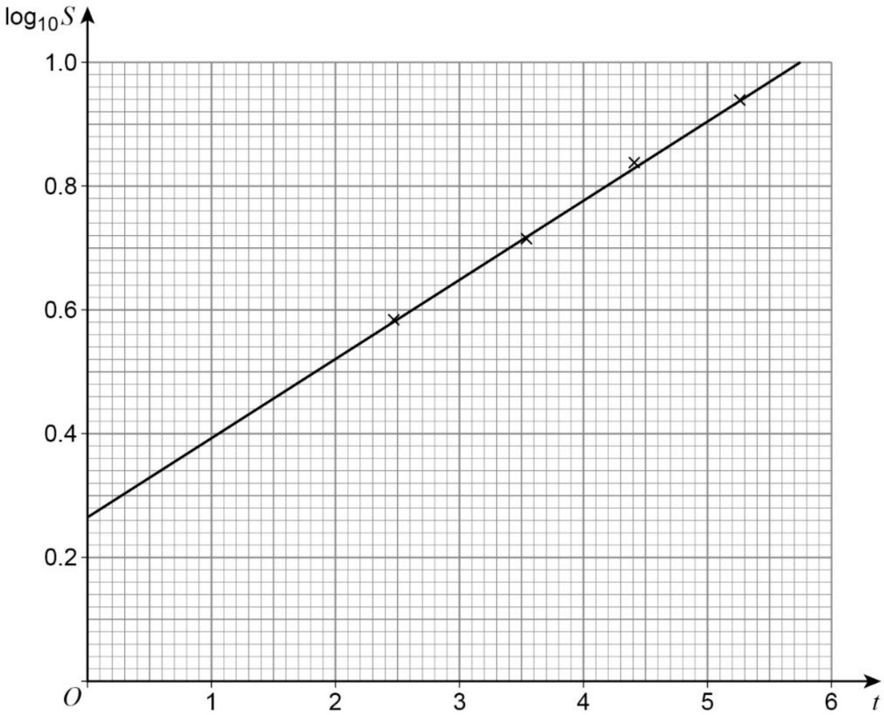
	Question 1 Total	**5**	

Q	Answer	Marks	Comments
2(a)	$=\begin{bmatrix} 2p & -1 \\ -2 & 3p \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \end{bmatrix}=\begin{bmatrix} 2p\times3+(-1)\times(-2) \\ (-2)\times3+3p\times(-2) \\ 3\times3+0\times(-2) \end{bmatrix}$	M1	transpose **A** *and* correct calculation for at least one row **PI** by correct answer Condone multiplying by $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ as a misread
	$=\begin{bmatrix} 6p+2 \\ -6-6p \\ 9 \end{bmatrix}$	A1	
		2	

Q	Answer	Marks	Comments
2(b)(i)	$\mathbf{C}^2=\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$	M1	Finds $\mathbf{C}^2$ May be seen within incorrect working
	$k=-2$	A1	Clearly stated
		2	

Q	Answer	Marks	Comments
2(b)(ii)	$\mathbf{C}^{12}=(-2)^6\mathbf{I}$ or $\mathbf{C}^{12}=\begin{bmatrix} 64 & 0 \\ 0 & 64 \end{bmatrix}$	M1	Use of $\mathbf{C}^2=k\mathbf{I}$ **PI** by $\mathbf{C}^{13}=\begin{bmatrix} 64 & 0 \\ 0 & 64 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix}$, $\mathbf{C}^{13}=\begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 64 & 0 \\ 0 & 64 \end{bmatrix}$ or $\mathbf{C}^{13}=64\begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix}$
	so $\mathbf{C}^{13}=\begin{bmatrix} 0 & 128 \\ -64 & 0 \end{bmatrix}$	A1ft	**ACF** **ft** from their k **NMS** scores zero
		2	

	Question 2 Total	**6**	

Q	Answer	Marks	Comments
3(a)(i)		B1	line of best fit seen
			
		1	

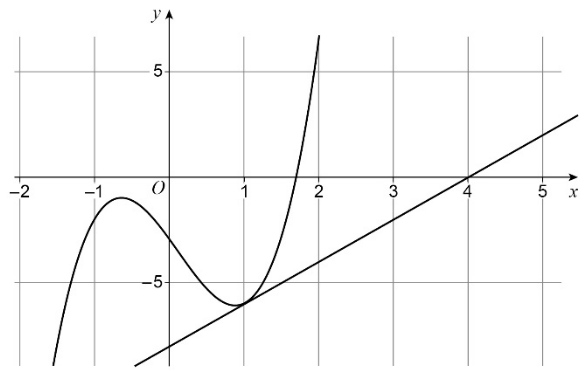
Q	Answer	Marks	Comments
3(a)(ii)	$\log_{10} S = \log_{10} a + t\log_{10} b$	M1	Takes logs of both sides to reduce to linear form **PI**
	y - intercept $= 0.27$ and gradient $= 0.13$	M1	Sight of y–intercept and gradient values Must be from a line of best fit
	$0.27 = \log_{10} a$ or $0.13 = \log_{10} b$ $\left[a = 10^{0.27} \quad \text{or} \quad b = 10^{0.13}\right]$	M1	At least one logarithm for a or b and set equal to their y-intercept or gradient
	$a = 1.9$ $b = 1.3$	A1	**AWFW** [1.8, 2.0] for a **AWFW** [1.3, 1.4] for b All **M** marks must be scored
		4	

Q	Answer	Marks	Comments
3(b)	$[11.2 = ab^t]$ $t = \dfrac{\log_{10}\left(\dfrac{11.2}{a}\right)}{\log_{10} b}$	M1	with their values of a and b **oe**, **eg** $t = \log_b\left(\dfrac{11.2}{a}\right)$ or $t = \dfrac{\log_{10} 11.2 - \text{y-intercept}}{\text{gradient}}$
	$t = 6.8$	A1ft	**ft** their a and b or their y-intercept and gradient
		2	

	Question 3 Total	**7**	

Q	Answer	Marks	Comments
4(a)	$\mathrm{f}(1) = -6$ and $\mathrm{f}(2) = 7$	**M1**	Correct evaluation of a suitable interval
	sign change **&** continuous function, so the root α lies in the interval $1 < x < 2$	**A1**	Must state that there is a change of sign and that the curve is continuous (condone unbroken) and concludes a root is present in the interval
		2	

Q	Answer	Marks	Comments
4(b)	$\mathrm{f}(1.5) = -2.625$; negative so $1.5 < \alpha \left[< 2\right]$	**M1**	range **PI** by subsequent calculation of $\mathrm{f}(1.75)$
	$\mathrm{f}(1.75) = 1.265625$	**m1**	
	so $1.5 < \alpha < 1.75$	**A1**	**CSO** but accept rounded or truncated values of $\mathrm{f}(1.5) = -2.625$ and $\mathrm{f}(1.75) = 1.265625$
		3	

Q	Answer	Marks	Comments
4(c)(i)		**B1**	tangent at $x = 1$ drawn, crossing x-axis
	tangent meets x-axis further from root [than $x = 1$]	**E1**	may include reference to root in interval $1.5 < x < 1.75$ from part (b)
		2	

Q	Answer	Marks	Comments
4(c)(ii)	$\mathrm{f}'(x) = 9x^2 - 2x - 5$	B1	Correct first derivative **PI**
	$\mathrm{f}'(1.75) = 9(1.75)^2 - 2(1.75) - 5$ $[= 19.0625...]$	M1	Substitution of $x = 1.75$ into their first derivative **PI**
	$1.75 - \frac{1.265625}{19.0625}$	m1	Use of Newton-Raphson formula **PI**
	$= 1.6836$	A1	**CAO**, must be 4 dp
		4	

	Question 4 Total	11	

Q	Answer	Marks	Comments
5(a)	$\det(\mathbf{A}) = (1-0.4k)(1+0.4k) - (-0.8k)(0.2k)$	M1	Correct expression for determinant
	$= 1 - 0.16k^2 + 0.16k^2 = 1 \neq 0$ So non-singular for all values of k	A1	Finds determinant to be 1 and non-zero and gives conclusion
		2	

Q	Answer	Marks	Comments
5(b)(i)	$\begin{bmatrix} 1.4 & 0.8 \\ -0.2 & 0.6 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$	M1	LHS correct and attempt at multiplication resulting in a 2×1 vector
	$(3,1)$	A1	**CAO** must be coordinates
		2	

Q	Answer	Marks	Comments
5(b)(ii)	$x' = 1.4x + 0.8(mx+c)$ $y' = -0.2x + 0.6(mx+c)$	M1	Valid attempt to find x', y' Condone $x = 1.4x + 0.8(mx+c)$ and $mx + c = -0.2x + 0.6(mx+c)$
	$-0.2x + 0.6(mx+c)$ $= m(1.4x + 0.8(mx+c)) + c$	m1	ft their y' = m(their x') + c
	$0.8m^2 + 0.8m + 0.2 = 0$ $0.6c - 0.8mc - c = 0$	m1	Attempt to find m and c by comparing coefficients or setting coefficients = 0
	$m = -\frac{1}{2}$	B1	correct value of m
	lines are $y = -\frac{1}{2}x + c$ [where c is real]	A1	no restrictions on c
		5	

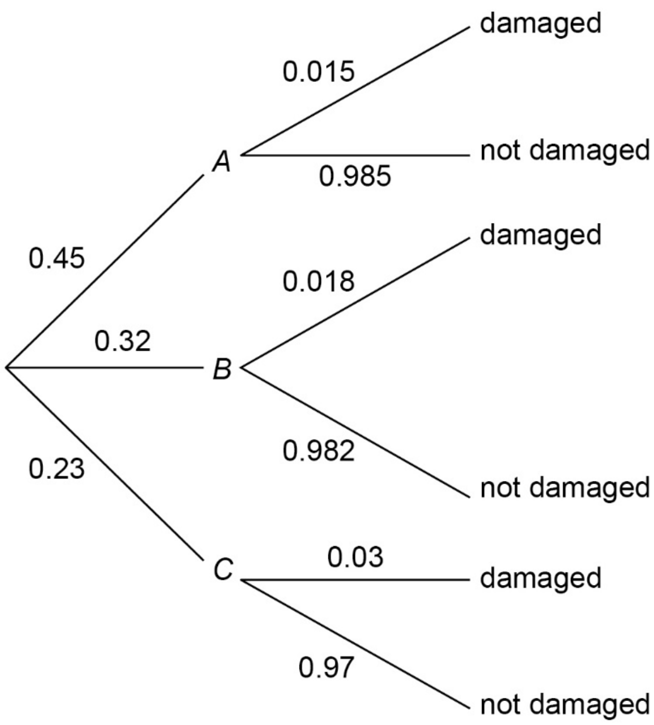
Q	Answer	Marks	Comments
5(b)(iii)	The determinant of **A** is equal to 1	E1	
	All the invariant lines of **A** are parallel	E1	
		2	

	Question 5 Total	**11**	

Q	Answer	Marks	Comments
6(a)	$G'_W(t) = p + 3(0.9 - p)t^2$	B1	oe
		1	

Q	Answer	Marks	Comments
6(b)	$G'_W(1) = p + 3(0.9 - p) = 2.5$	M1	Forms an equation in p using $G'_W(1) = 2.5$
	p = 0.1	A1	
	$P(W \leq 1) = 0.1 + p$	M1	PI
	$P(W \leq 1) = 0.2$	A1	
		4	

	Question 6 Total	5	

Q	Answer	Marks	Comments
7(a)		**M1** **A1**	Correct structure – branches and events Allow use of any letter for damaged or not damaged Fully correct
		2	

Q	Answer	Marks	Comments
7(b)	$\text{P}(C \mid \text{not damaged}) = \dfrac{0.23\times0.97}{0.45\times0.985+0.32\times0.982+0.23\times0.97}$ $= 0.2275[160873]$ $= 0.228$	**M1** **M1** **A1**	Numerator calculation seen Denominator calculation seen **AG** Must see an answer given to four or more significant figures before final answer
		3	

	Question 7 Total	**5**	

Q	Answer	Marks	Comments
8(a)	$\frac{n+1}{2} = 15 \Rightarrow n = 29$	M1	Forms correct equation and attempts to solve to find n Condone slips on rearranging to find n
	Var(X) $= \frac{29^2 - 1}{12} = 70$	A1	
		2	

Q	Answer	Marks	Comments
8(b)(i)	$p(1 - p)$ = 0.1824	M1	Forms correct equation
	p = 0.76 and p = 0.24	A1	Finds both solutions **oe**
	p = 0.24 $\Rightarrow$ E(Y) = $\frac{1}{0.24}$ or p = 0.76 $\Rightarrow$ E(Y) = $\frac{1}{0.76}$	M1	Attempts to find E(Y) for one of their values of p where $0 < p < 1$
	Var(Y) = $\frac{1-0.24}{0.24^2}$ or Var(Y) = $\frac{1-0.76}{0.76^2}$	M1	Attempts to find Var(Y) for one of their values of p where $0 < p < 1$
	p = 0.24 gives E(Y) > 2 but p = 0.76 gives E(Y) < 2 so p = 0.24 so Var(Y) = $\frac{1-0.24}{0.24^2} = \frac{475}{36}$	A1	**AG** States that p = 0.24 gives E(Y) > 2 but p = 0.76 gives E(Y) < 2 so p = 0.24 and so Var(Y) = $\frac{1-0.24}{0.24^2} = \frac{475}{36}$
		5	

Q	Answer	Marks	Comments
8(b)(ii)	Var(X) + 6^2Var(Y) = 70 + $6^2 \times \frac{475}{36}$	M1	Substitutes their value of Var(X) and $\frac{475}{36}$ into Var(X) + 6^2Var(Y)
	= 545	A1ft	**ft** their Var(X) from part **(a)** Implied by Cov(X, Y) = –0.5
	X and Y are dependent as Var (X – 6Y) ≠ Var(X) + 6^2Var(Y)	E1	**CSO**, **oe** eg Cov(X, Y) is non-zero Conclusion with justification
		3	

	Question 8 Total	**10**	

Q	Answer	Marks	Comments
9(a)	$[I]=[Ft]$ $= MLT^{-2}\times T$	**M1**	Uses the formula $I = Ft$ [or $I = mv - mu$] and attempts to find the dimensions of impulse Condone use of units
	$= MLT^{-1}$	**A1**	Correct dimensions of impulse.
		2	

Q	Answer	Marks	Comments
9(b)	$v = eu$ $[e]=\dfrac{LT^{-1}}{LT^{-1}}=L^0T^0=1$ [$\therefore$ dimensionless]	**B1**	Correct argument Condone only seeing L^0T^0
		1	

Q	Answer	Marks	Comments
9(c)	$[mu(1+e)] = M\times LT^{-1}\times 1$ $= MLT^{-1}$	**M1**	Finds dimensions of RHS Condone use of units
	$[I]=MLT^{-1}$	**A1**	Correct dimensions of RHS
	$\therefore$ dimensionally consistent	**A1**	Compares with dimensions of impulse and reaches correct conclusion.
		3	

	Question 9 Total	**6**	

Q	Answer	Marks	Comments
10(a)	$I = 6\times 0.08$ $= 0.48$ N s	B1	Correct impulse. Condone omission of units.
		1	

Q	Answer	Marks	Comments
10(b)	$-0.48 = 0.1\times v_P - 0.1\times 12$	M1	Forms an equation using $I = mv - mu$ Condone sign errors
	$v_P = \dfrac{1.2-0.48}{0.1} \Rightarrow v_P = 7.2 \text{ m s}^{-1}$	A1	**AG** Correct speed, with intermediate working shown.
		2	

Q	Answer	Marks	Comments
10(c)	$0.48 = 0.4\times v_Q - 0.4\times 8$ or $0.1\times 12 + 0.4\times 8 = 0.1\times 7.2 + 0.4\times v_Q$ $\Rightarrow v_Q = 9.2 \text{ m s}^{-1}$	M1	Attempts to find velocity of Q after the collision Condone sign errors
	$7.2 - 9.2 = -e(12-8)$	M1	Uses Newton's Experimental Law with their velocities Condone sign errors
	$e = \dfrac{2}{4} = 0.5$	A1	Correct coefficient of restitution.
		3	

	Question 10 Total	**6**	

Q	Answer	Marks	Comments
11(a)	$\mathbf{r}_A = \begin{bmatrix} 80 \\ 100 \end{bmatrix} + t\begin{bmatrix} 5 \\ p \end{bmatrix}$ $\mathbf{r}_B = \begin{bmatrix} 200 \\ 40 \end{bmatrix} + t\begin{bmatrix} 4 \\ 5 \end{bmatrix}$	B1	Forms correct expressions for position vectors of *A* and *B* **PI**
	$80 + 5t = 200 + 4t$	M1	Forms a correct equation to find the time of interception **oe**
	$t = 120$	A1	Correct time for interception.
	$100 + 120p = 40 + 5 \times 120$ $p = \frac{540}{120} = \frac{9}{2} = 4.5$	A1	Correct value for p
		4	

Q	Answer	Marks	Comments
11(b)	$s^2 = (t-120)^2 + (60-t)^2$ $[= 2t^2 - 360t + 18000]$	**M1**	Finds [square of] distance between A and B
	$\frac{\mathrm{d}}{\mathrm{d}t}(s^2) = 4t - 360$ $0 = 4t - 360$ $t = \frac{360}{4} = 90$	**M1**	Uses differentiation to find the time for the minimum distance or completes the square $s^2 = 2(t-90)^2 + 1800$ **PI** by correct final answer
	$s = \sqrt{30^2 + (-30)^2}$	**M1**	Uses time to find the shortest distance **PI** by correct final answer
	$= 42$ m	**A1**	Correct minimum distance. **CAO** Condone omission of unit
		4	

	Question 11 Total	**8**	