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(9665/FM01) Unit FP1 Pure Mathematics

Mark scheme

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236XFM01/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}}$	M1	Accept any expression of the form $ax^{-\frac{1}{2}}$ for a non-zero a
	$= 0.1$ when $x = 25$	A1	**PI**
	$\delta y \approx \frac{\mathrm{d}y}{\mathrm{d}x} \times \delta x$	M1	**PI**
	$= 0.1 \times 0.4$ or 0.04	A1	
	Estimate $= 5 +$ their 0.04	M1	**PI**
	5.04	A1	**oe** eg $\frac{126}{25}$

	Question 1 Total	**6**	

Q	Answer	Marks	Comments
2(a)	Because the upper limit is infinite.	**B1**	**oe**
		1	

Q	Answer	Marks	Comments
2(b)	$I = \lim_{N\to\infty}\int_4^N x^{-3}\mathrm{d}x$	**M1**	Limiting process seen in the solution
	$= \lim_{N\to\infty}\left[\frac{x^{-2}}{-2}\right]_4^N$	**m1**	Correct integration with limiting process seen **PI**
	$= \lim_{N\to\infty}\left(-\frac{1}{2N^2}-\left(-\frac{1}{2\times 4^2}\right)\right)$ $= 0-\left(-\frac{1}{32}\right)$		
	$= \frac{1}{32}$	**A1**	Shows limits correctly substituted, leading to the correct answer **SC1** for correct integration and correct answer without use of limiting process **NMS** 1/3
		3	

	Question 2 Total	**4**	

Q	Answer	Marks	Comments
3(a)	$(x+1)^3-(x-1)^3$ $=x^3+3x^2+3x+1-\left(x^3-3x^2+3x-1\right)$ $=6x^2+2$	B1	Expands both brackets and correctly simplifies **AG**
		1	

Q	Answer	Marks	Comments
3(b)	$\sum_{r=15}^{n}\left(6r^2+2\right)=\sum_{r=15}^{n}\left\{(r+1)^3-(r-1)^3\right\}$	M1	Uses result from part (a) and evaluates at least one value of r
	$=\cancel{16^3}-14^3$ $+\cancel{17^3}-15^3$ $+\cancel{18^3}-\cancel{16^3}$ $+\ldots$	M1	At least the first 3 (or last 3) values of r used
	$+\cancel{(n-1)^3}-\cancel{(n-3)^3}$ $+\quad n^3\quad-\cancel{(n-2)^3}$ $+(n+1)^3-\cancel{(n-1)^3}$	M1	Removes all cancelling terms to leave a cubic expression in n
	$=n^3+(n+1)^3-14^3-15^3$ $\left[=n^3+(n+1)^3-6119\right]$	A1	**isw**
		4	

	Question 3 Total	**5**	

Q	Answer	Marks	Comments
4(a)	$\frac{x}{3}+\frac{\pi}{6}=2n\pi-\frac{\pi}{4}$ or $2n\pi-\frac{3\pi}{4}$	B1	**oe** must have both parts
	Going from $\left(\frac{x}{3}+\frac{\pi}{6}\right)$ to x	M1	Including multiplication of all terms by 3
	$x=6n\pi-\frac{5\pi}{4}$ or $x=6n\pi-\frac{11\pi}{4}$	A1 A1	**oe eg** $x=3n\pi-\frac{\pi}{2}+(-1)^{n+1}\left(\frac{3\pi}{4}\right)$
		4	

Q	Answer	Marks	Comments
4(b)	$n=1:\quad x=\left(6-\frac{5}{4}\right)\pi\quad\left[=\frac{13}{4}\pi\right]$ $x=\left(6-\frac{11}{4}\right)\pi\quad\left[=\frac{19}{4}\pi\right]$	M1	Any two of the first four positive terms **ft** their part **(a)**
	$n=2:\quad x=\left(12-\frac{5}{4}\right)\pi\quad\left[=\frac{37}{4}\pi\right]$ $x=\left(12-\frac{11}{4}\right)\pi\quad\left[=\frac{43}{4}\pi\right]$	M1	Any three of the first four positive terms **ft** their part **(a)**
	Total $=28\pi$	A1	
		3	

	Question 4 Total	**7**	

Q	Answer	Marks	Comments
5	$(2+\mathrm{i})^2 - a(2+\mathrm{i}) + (b+\mathrm{i}) = 0$	**M1**	Substitutes the given root into the equation
	$3+4\mathrm{i}-2a-a\mathrm{i}+b+\mathrm{i}=0$	**A1**	Correct expansion of $(2+\mathrm{i})^2$
	Equating imaginary parts: $4-a+1=0$ $a=5$	**M1**	Accept equating of real parts
	Sum of roots $=5$ Second root $=5-(2+\mathrm{i})$	**M1**	Forms an equation in the second root using their values of a and/or b. If using $p+q\mathrm{i}$ then must correctly proceed to an equation in p only and an equation in q only.
	Second root $=3-\mathrm{i}$	**A1**	
		5	

Q	Answer	Marks	Comments
ALT	Let other root $= p+q\mathrm{i}$ (p and q real) $(1)\quad 2+\mathrm{i}+p+\mathrm{i}q=a$ $(2)\quad (p+q\mathrm{i})(2+\mathrm{i})=b+\mathrm{i}$	**M1** **A1**	Uses the sum (or product) of roots with $p+q\mathrm{i}$ in place of the unknown root Uses sum **and** product to form two correct equations in a, b, p and q
	Equating imaginary parts in (1): $1+q=0 \Rightarrow q=-1$	**M1**	Equates imaginary parts to form an equation in p and/or q
	Equating imaginary parts in (2): $2q+p=1 \Rightarrow p=1-2q\,[=3]$	**M1**	Forms two correct equations in p and q
	Second root $=3-\mathrm{i}$	**A1**	
		5	

	Question 5 Total	**5**	

Q	Answer	Marks	Comments
6(a)	When $x = 7$, $y = 49p - 21$ When $x = 7 + h$, $y = p(7+h)^2 - 3(7+h)$ $= 49p + 14hp + h^2 p - 21 - 3h$	**M1**	Calculates the y-coordinate when $x = 7 + h$
	Gradient $= \dfrac{49p + 14hp + h^2 p - 21 - 3h - (49p - 21)}{h}$	**M1**	Correct method for gradient of line
	$= 14p + hp - 3$	**A1**	
		3	

Q	Answer	Marks	Comments
6(b)	Gradient of curve $= \lim_{h \to 0}[14p + hp - 3] = 14p - 3$	**M1**	Replaces each h term with 0
	$14p - 3 = 0$ $p = \dfrac{3}{14}$	**A1ft**	Condone no limiting process seen Condone h=0 seen **ft** a linear expression in p
		2	

	Question 6 Total	**5**	

Q	Answer	Marks	Comments
7(a)	$\alpha+\beta=\dfrac{2}{3}$	B1	
	$\alpha\beta=3$	B1	
		2	

Q	Answer	Marks	Comments
7(b)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$	M1	Seen or implied
	$=\dfrac{4}{9}-6=-\dfrac{50}{9}$	A1	**AG**
		2	

Q	Answer	Marks	Comments
7(c)	Sum of roots $=\alpha^4+\beta^4=\left(\alpha^2+\beta^2\right)^2-2\alpha^2\beta^2$ $=\left(-\dfrac{50}{9}\right)^2-2\times 9$	M1	Correctly expresses the new sum in terms of $\alpha^2+\beta^2$, $\alpha+\beta$ and/or $\alpha\beta$ **PI**
	$=\dfrac{1042}{81}$	A1	Correct new sum **PI**
	Product of roots $=\alpha^4\beta^4$ $=3^4=81$	B1	Correct new product **PI**
	$81x^2-1042x+6561=0$	B1ft	**oe** (integer coefficients) **ft** their new sum and product of roots Must be an equation
		4	

	Question 7 Total	**8**	

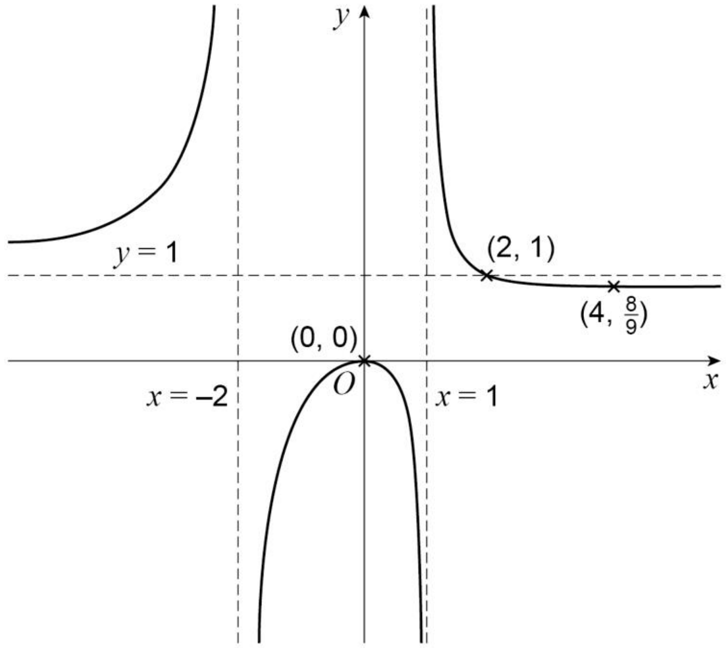
Q	Answer	Marks	Comments
8(a)	$x=-2$ and $x=1$	**B1**	
	$y=1$	**B1**	
		2	

Q	Answer	Marks	Comments
8(b)	$k=\dfrac{x^2}{(x-1)(x+2)}$ $k\left(x^2+x-2\right)=x^2$ $(k-1)x^2+kx-2k=0$	**M1**	Equating to k and rearranging into a 3-term quadratic in x
	For real roots $k^2-4(k-1)(-2k)\geq 0$	**m1**	Discriminant conditions for real roots being applied
	$9k^2-8k\geq 0$ $k\leq 0$ or $k\geq\dfrac{8}{9}$	**A1**	
		3	

Q	Answer	Marks	Comments
8(c)	$y=0\Rightarrow x=0$ One stationary point is $(0,0)$	**B1**	
	$y=\dfrac{8}{9}\Rightarrow\left(\dfrac{8}{9}-1\right)x^2+\dfrac{8}{9}x-\dfrac{16}{9}=0$ $x^2-8x+16=0$	**M1**	**ft** their $k\geq\dfrac{8}{9}$
	$x=4$ The other stationary point is $\left(4,\dfrac{8}{9}\right)$	**A1**	Accept $x=4$ and $y=\dfrac{8}{9}$
		3	

Q	Answer	Marks	Comments
8(d)	$y=1 \Rightarrow 1=\dfrac{x^2}{x^2+x-2}$ $x^2+x-2=x^2$ $x=2$	M1	**ft** their asymptote
	The point is $(2,1)$	A1	Accept $x=2$ and $y=1$
		2	

Q	Answer	Marks	Comments
8(e)	Graph correct for $x<-2$	B1ft	
	Graph correct for $-2<x<1$	B1ft	Accept $(0,0)$ missing if their graph clearly has a maximum at the origin
	Graph correct for $x>1$	B1ft	Must include a clear minimum point with coordinates. Must clearly approach the horizontal asymptote from below. **ft** their asymptotes and coordinates for all three marks
		3	



	Question 8 Total	**13**	

Q	Answer	Marks	Comments
9(a)	$\sum_{r=1}^{n}\left(r^3+r^2\right)=\sum_{r=1}^{n}r^3+\sum_{r=1}^{n}r^2$	M1	Writes as the sum of two summations **PI**
	$=\frac{1}{4}n^2\left(n+1\right)^2+\frac{1}{6}n\left(n+1\right)\left(2n+1\right)$	M1	Forms a correct expression in terms of n
	$=\frac{1}{12}n\left(n+1\right)\left\{3n\left(n+1\right)+2\left(2n+1\right)\right\}$	M1	Identifies n and $\left(n+1\right)$ as common factors
	$=\frac{1}{12}n\left(n+1\right)\left\{3n^2+7n+2\right\}$		
	$=\frac{1}{12}n\left(n+1\right)\left(n+2\right)\left(3n+1\right)$	A1	
		4	

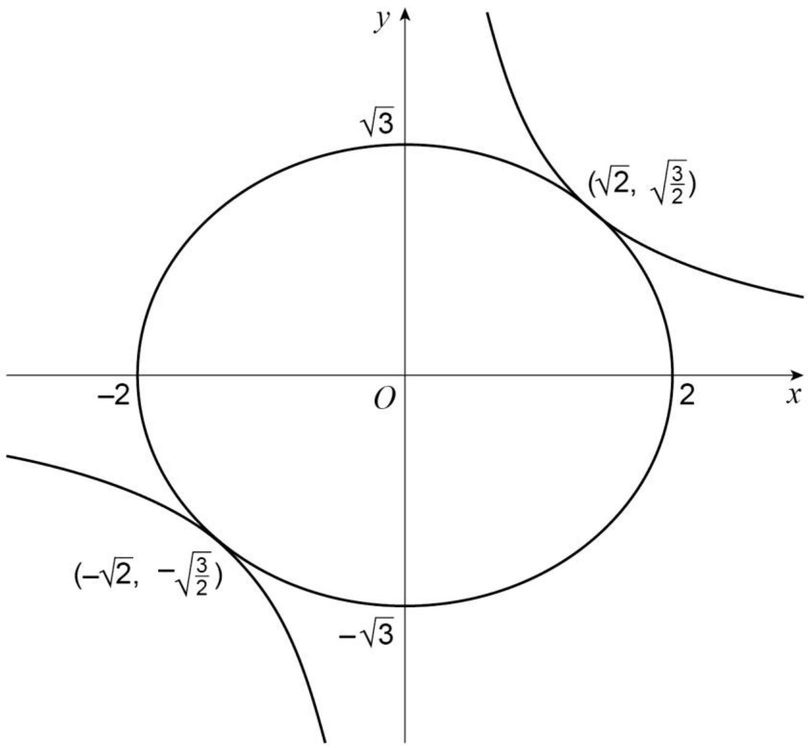
Q	Answer	Marks	Comments
9(b)	$n=37$ and $n=36$	B1	**B1** for any two correct answers and no more than six answers in total
	$n=35$ and $n=12$	B1	**B2** for any four correct answers and no more than six answers in total
	$n=49$	B1	**B3** for all five correct answers and no extras
		3	

	Question 9 Total	**7**	

Q	Answer	Marks	Comments
10(a)	Given $P(x, y)$ $\sqrt{(x-1)^2 + y^2} = \frac{1}{2}\lvert x-4\rvert$	M1	Forms a correct equation Condone modulus not considered for all three marks
	$4\left\{(x-1)^2 + y^2\right\} = (x-4)^2$ $4x^2 - 8x + 4 + 4y^2 = x^2 - 8x + 16$	m1	Removes roots and expands.
	$3x^2 + 4y^2 = 12$ $\frac{x^2}{4} + \frac{y^2}{3} = 1$	A1	**AG** Must be convincingly shown
		3	

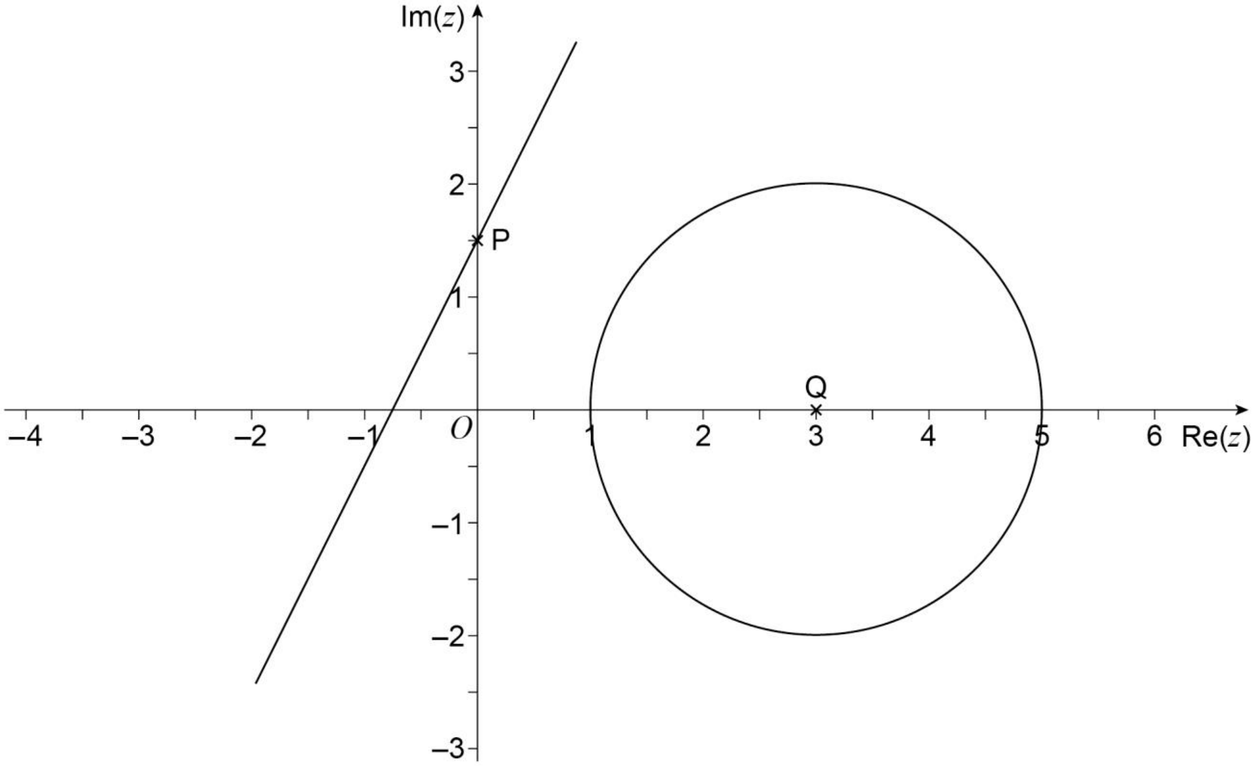
Q	Answer	Marks	Comments
10(b)	$xy = \sqrt{3} \Rightarrow y^2 = \frac{3}{x^2}$	B1	Forms a correct equation in x only (or y)
	$3x^2 + 4\left(\frac{3}{x^2}\right) = 12$ $3x^2 - 12 + \frac{12}{x^2} = 0$	M1	Rearranges into a 3-term polynomial **PI**
	$x^4 - 4x^2 + 4 = 0$ $(x^2 - 2)^2 = 0$ $x = \pm\sqrt{2}$	M1	Solves their 3-term polynomial
	The only points of intersection are $\left(\sqrt{2}, \sqrt{\frac{3}{2}}\right)$ and $\left(-\sqrt{2}, -\sqrt{\frac{3}{2}}\right)$	A1	**oe** Accept coordinates written separately
		4	

Q	Answer	Marks	Comments
10(c)	*E* drawn correctly	**M1**	Attempt at symmetry about the axes
	Axis intercepts of *E* shown correctly	**A1**	
	H drawn correctly with correct asymptotic behaviour	**M1**	Attempt at symmetry about the axes
	Points of intersection of *H* and *E* shown correctly	**A1**	Dependent on exactly two intersection points
		4	



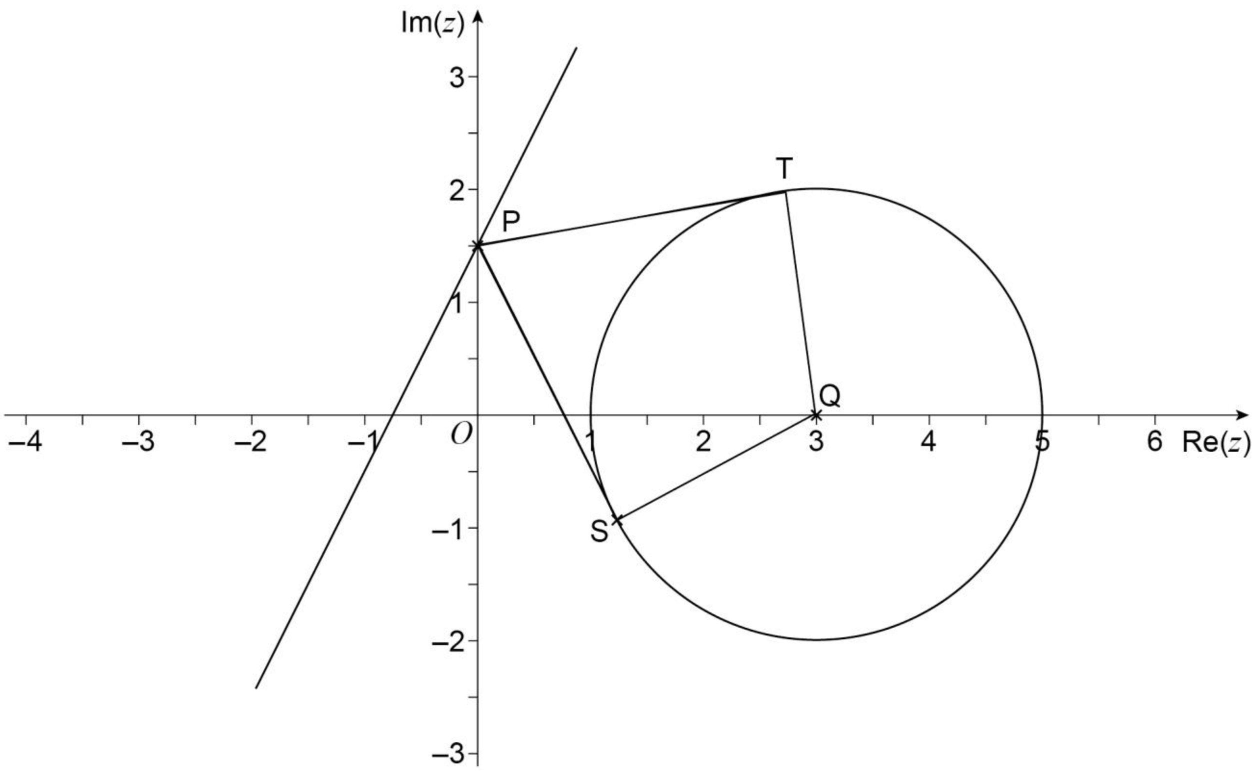
	Question 10 Total	**11**	

Q	Answer	Marks	Comments
11(a)	*C* drawn in 1st and 4th quadrants with centre on real axis	**B1**	
	Q shown correctly and axis intercepts of *C* shown correctly	**B1**	
	L drawn with positive gradient and positive intercept on imaginary axis	**B1**	
	P shown correctly and real axis intercept of *L* shown between –1 and –0.5	**B1**	See diagram below *P* is located at $\frac{3}{2}\mathrm{i}$



		4	

Q	Answer	Marks	Comments
11(b)	$PQ^2 = \left(\frac{3}{2}\right)^2 + 3^2 = \frac{45}{4}$	**B1**	Correct *PQ* or PQ^2
	angle $PTQ = 90°$	**M1**	**PI**
	$PT^2 = \frac{45}{4} - 2^2$		
	$PT = \frac{\sqrt{29}}{2}$	**A1**	Or $ST = \frac{4}{15}\sqrt{145}$
	Area of triangle *PTQ* $= \frac{1}{2} \times \frac{\sqrt{29}}{2} \times 2 = \frac{\sqrt{29}}{2}$	**M1**	Full correct method for the exact area of *PTQ* or *PSQ* or *PTQS*
	Area of quadrilateral *PTQS* $=$ (Area of triangle *PTQ*) × 2 $= \sqrt{29}$	**A1**	See diagram below
			An algebraic response gains credit if exact lengths are calculated correctly.
		5	



	Question 11 Total	**9**	