
INTERNATIONAL A-LEVEL FURTHER MATHEMATICS **FM05**

(9665/FM05) Unit FM2 Mechanics

Mark scheme

June 2022

Version 1.0 Final



Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from [oxfordaqaexams.org.uk](https://www.oxfordaqaexams.org.uk)

Copyright information

OxfordAQA retains the copyright on all its publications. However, registered schools/colleges for OxfordAQA are permitted to copy material from this booklet for their own internal use, with the following important exception: OxfordAQA cannot give permission to schools/colleges to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Copyright © 2022 Oxford International AQA Examinations and its licensors. All rights reserved.

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$-5\mathbf{i} - 4\mathbf{j} = 3(2\mathbf{i} - 3\mathbf{j}) - 3\mathbf{w}$ $\mathbf{w} = \frac{11\mathbf{i} - 5\mathbf{j}}{3}$	M1 A1 A1	Forms equation based on the impulse equation. Condone sign errors. Correct equation. Correct velocity. Solutions may be seen using column vectors.
		3	
1(b)	$2\mathbf{v} + 3\left(\frac{11\mathbf{i} - 5\mathbf{j}}{3}\right) = 5(2\mathbf{i} - 3\mathbf{j})$ $\mathbf{v} = \frac{-1\mathbf{i} - 10\mathbf{j}}{2}$	M1 A1ft A1	Forms equation based on the impulse equation or conservation of momentum, with their $\mathbf{w}$ from part (a). Correct equation, with their $\mathbf{w}$ from part (a). Correct velocity. Solutions may be seen using column vectors.
		3	
	Total	6	

Q	Answer	Marks	Comments
2(a)	$WD = \int_4^9 5\sqrt{x} \, dx$ $= \left[\frac{10}{3} x^{\frac{3}{2}} \right]_4^9$ $= 90 - \frac{80}{3}$ $= \frac{190}{3} \text{ [J]}$	M1 A1 A1	Forms correct integral. Condone incorrect / missing limits. Correct integration, with correct limits. Correct WD. Accept 63 J Condone missing units.
		3	
2(b)	$\frac{1}{2} \times 6 \times 4^2 + \frac{190}{3} = \frac{1}{2} \times 6 \times v^2$ $3v^2 = 48 + \frac{190}{3}$ $v = 6.09 \text{ [m s}^{-1}\text{]}$	M1 A1ft A1	Forms three term energy equation, with correct terms and any signs. Allow their work done from part (a). Correct equation. Allow their work done from part (a). Correct speed. Condone missing units.
		3	
	Total	6	

Q	Answer	Marks	Comments
3(a)	$21 = \omega^2(a^2 - 0.2^2)$ $9 = \omega^2(a^2 - 0.4^2)$ $\frac{(a^2 - 0.4^2)}{9} = \frac{(a^2 - 0.2^2)}{21}$ $7a^2 - 1.12 = 3a^2 - 0.12$ $a^2 = 0.25$ $a = 0.5 \text{ [metres]}$	M1 A1 M1 A1	Forms two equations connecting a and ω Both equations correct. Eliminates ω Obtains correct amplitude, must be positive. Condone missing units.
		4	
3(b)	$9 = \omega^2(0.5^2 - 0.4^2)$ $\omega^2 = \frac{9}{(0.5^2 - 0.4^2)}$ $\omega = 10$ $\text{Period} = \frac{2\pi}{10} = \frac{\pi}{5} \text{ [seconds]}$	M1 A1 A1	Uses an equation to find ω with their a . Correct ω Correct period. Accept 0.63
		3	
3(c)	$\text{Max Speed} = 0.5 \times 10 = 5 \text{ m s}^{-1}$	B1	Obtains correct speed with the correct units.
		1	
	Total	8	

Q	Answer	Marks	Comments
4(a)	$m \frac{dv}{dt} = -kv^2$ $\frac{1}{v^2} \frac{dv}{dt} = -\frac{k}{m}$ $\int \frac{1}{v^2} dv = \int -\frac{k}{m} dt$ $-v^{-1} = -\frac{k}{m}t + c$ $t = 0, v = U \Rightarrow c = -\frac{1}{U}$ $-\frac{1}{v} = -\frac{1}{U} - \frac{kt}{m}$ $\frac{1}{v} = \frac{m + ktU}{mU}$ $v = \frac{mU}{m + ktU}$	M1 M1 A1 M1 A1	States correct differential equation. Separates variables and integrates. Correct integration. Finds constant of integration or uses correct limits of integration. Correct constant and correct final answer from correct working.
		5	
4(b)	$0.9U = \frac{mU}{m + ktU}$ $0.9mU + 0.9U^2kt = mU$ $0.9Ukt = 0.1m$ $t = \frac{m}{9kU}$	M1 M1 A1	Substitutes $0.9U$ Solves for t Obtains correct result.
		3	
	Total	8	

Q	Answer	Marks	Comments
5(a)	$0.04 \times 2.45 \ddot{\theta} = -0.04 \times 9.8 \sin \theta$ $\sin \theta \approx \theta$ $\ddot{\theta} \approx -4\theta$ [Angular] acceleration is proportional to the [angular] displacement and in the opposite direction [so therefore SHM].	M1 B1 A1 E1	Forms differential equation. Small angle approximation seen or used. Obtains $\ddot{\theta} \approx -4\theta$ or $\ddot{\theta} \approx -\frac{g}{l}\theta$ Concludes that motion is SHM from correct working. Condone use of = instead of $\approx$
		4	
5(b)	$\text{Period} = \frac{2\pi}{\sqrt{4}} = \pi \text{ [seconds]}$	M1 A1ft	Finds period using the value of their ω Correct period. Allow 3.1
		2	
5(c)	$\theta = \frac{\pi}{12} \cos(2t)$ $\frac{\pi}{24} = \frac{\pi}{12} \cos(2t_1) \Rightarrow t_1 = 0.5236$ $\frac{\pi}{36} = \frac{\pi}{12} \cos(2t_2) \Rightarrow t_2 = 0.6155$ $0.6155 - 0.5236 = 0.092 \text{ seconds}$	M1 A1 A1 A1	States or uses a formula for angle based on their ω Correct time for one angle. Correct time for other angle. Correct difference from correct working. AWRT 0.09
		4	
	Total	10	

Q	Answer	Marks	Comments
6(a)	$0^2 = (25 \sin 40^\circ)^2 + 2 \times s \times (-9.8 \cos 20^\circ)$ $s = \frac{(25 \sin 40^\circ)^2}{2 \times 9.8 \cos 20^\circ} = 14 \text{ metres}$	M1 A1 M1 A1	Forms equation based on motion perpendicular to the slope to find the maximum distance from the plane. Correct equation. Solves for max height. Correct height.
		4	
6(b)	$0 = 25 \sin 40^\circ t - 4.9 \cos 20^\circ t^2$ $t = 0 \text{ or } t = \frac{25 \sin 40^\circ}{4.9 \cos 20^\circ} = 3.49$ $v_x = 25 \cos 40^\circ - 9.8 \sin 20^\circ \left(\frac{25 \sin 40^\circ}{4.9 \cos 20^\circ} \right) = 7.453$ $v_y = 25 \sin 40^\circ - 9.8 \cos 20^\circ \left(\frac{25 \sin 40^\circ}{4.9 \cos 20^\circ} \right) = -16.07$ $\tan \alpha = \frac{16.07}{7.453}$ $\alpha = 65^\circ$	M1 A1 A1 M1 A1 M1 A1	Equation to find time of flight. Correct equation. Correct time. Finding one correct component of velocity on impact. Both components correct. Finding angle. Correct angle. Allow $\pm 65^\circ$ Final answer must be to the nearest degree.
		7	
	Total	11	

Q	Answer	Marks	Comments
7(a)	$mg \cos 30^\circ = \frac{mv^2}{r}$ $v^2 = \frac{\sqrt{3} gr}{2}$ $\frac{1}{2}mv^2 = \frac{1}{2}mU^2 + mgr(1 - \cos 30^\circ)$ $v^2 = U^2 + gr(2 - \sqrt{3})$ $\frac{\sqrt{3} gr}{2} = U^2 + gr(2 - \sqrt{3})$ $U^2 = gr\left(\frac{3\sqrt{3}}{2} - 2\right)$ $U = \sqrt{gr\left(\frac{3\sqrt{3}}{2} - 2\right)}$	M1 A1 A1 M1 A1 A1 m1 A1	Applying Newton's second Law at the point where the particle leaves the hemisphere. Must include a trig term. Correct equation. Correct expression for v or v^2 Applying conservation of energy. Correct equation. Correct expression for v or v^2 Eliminating v Correct expression for U
		8	
7(b)	As U is independent of mass, so U does not change.	B1 E1	States no change and provides a reason. Correct reason.
		2	
7(c)	U would decrease as it is directly proportional to the square root of the radius, so U does change.	B1 E1	States that there is a change and provides a reason. Correct reason.
		2	
	Total	12	

Q	Answer	Marks	Comments
8	Before Collision along line of centres: $u_P = 5 \cos 60^\circ = 2.5$ $u_Q = -4$ $3 \times 2.5 + 2 \times (-4) = 3v_P + 2v_Q$ $-0.5 = 3v_P + 2v_Q$ $v_P - v_Q = -\frac{2}{5}(2.5 - (-4))$ $v_P - v_Q = -2.6$ $v_P = -\frac{57}{50}$ $v_Q = \frac{73}{50}$ Speed of Q = $\frac{73}{50} = 1.46 \text{ m s}^{-1}$ $\text{Speed of P} = \sqrt{\left(\frac{57}{50}\right)^2 + (5 \sin 60^\circ)^2}$ $= \sqrt{\left(\frac{57}{50}\right)^2 + \left(\frac{75}{4}\right)}$ $= 4.48 \text{ m s}^{-1}$	B1 M1 A1 M1 A1 M1 A1 A1 M1 A1	Correct components along line of centres seen. Conservation of momentum along the line of centres. Correct equation. Applies coefficient of restitution along lines of centres. Correct equation. Solves their simultaneous equations. Correct velocity component for P. Recognises that Q has no perpendicular component and states the speed of Q Uses component perpendicular to the line of centres. Correct speed for P
	Total	10	

Q	Answer	Marks	Comments
9	At max speed: $T + 2T \cos 30^\circ = mg$ $T = \frac{mg}{1 + \sqrt{3}}$ $e = \frac{2}{\sin 30^\circ} - 3 = 1$ $\frac{\lambda}{3} e = \frac{mg}{1 + \sqrt{3}}$ $\lambda = \frac{3mg}{1 + \sqrt{3}}$ At Lowest point Let x be the distance of the particle below O Length of string attached to A/B = $\sqrt{x^2 + 4}$ Length of string attached to C = $x + 4 - 2\sqrt{3}$ $mgx = 2 \times \frac{3mg}{6(1 + \sqrt{3})} \times (\sqrt{x^2 + 4} - 3)^2 + \frac{3mg}{6(1 + \sqrt{3})} (x + 1 - 2\sqrt{3})^2$ $2(\sqrt{x^2 + 4} - 3)^2 + (x + 1 - 2\sqrt{3})^2 - 2x(1 + \sqrt{3}) = 0$ Substituting $x = 5.85$ gives -0.244 Substituting $x = 5.94$ gives 0.980 $\therefore x = 5.9$ to 2 sf.	M1 A1 M1 A1 B1 B1 M1 A1 A1	Uses resultant force is zero to find tension. Correct tension. Finds extension. Correct modulus of elasticity or stiffness $k = \frac{mg}{1 + \sqrt{3}}$ Maximum length of angled ropes. Maximum length of vertical rope. Equation for conservation of energy with at least two terms correct. Must be based on EPE for three strings and one GPE for the particle and no other terms. Correct equation. Value of 5.9 justified.
	Total	9	