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FM04

(9665/FM04) Unit FS2 Statistics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
-x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1	$H_0: \sigma = 1$ $H_1: \sigma > 1$ $v = 100$ $\frac{(n-1)s^2}{\sigma^2} = \frac{(101-1) \times 1.2^2}{1^2}$ $= 144$ $\chi_{100}^2(0.99) = 135.807$ $144 > 135.807$ <p>Reject H_0. Evidence to suggest that the standard deviation of the diameters of metal discs produced by the machine has increased</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>A1ft</p> <p>E1ft</p>	<p>Both hypotheses, oe</p> <p>PI by correct critical value</p> <p>PI Condone one slip</p> <p>Finds correct critical value or correct probability, 0.0026</p> <p>Correctly compares their χ^2 test statistic and their critical value or their probability and 0.01</p> <p>States that null hypothesis is rejected and gives a conclusion in context Conclusion must not be definite ft their comparison using a χ^2 model</p>
	Total	7	

Q	Answer	Marks	Comments
2(a)	$z = 1.8808$	B1	AWRT 1.88
	$1.8808 \times \frac{1.6}{\sqrt{n}} = 0.2$	M1	Set up an equation with their $z \times \frac{1.6}{\sqrt{n}}$ oe
	$n = 226$	A1	226 or 227
		3	
2(b)	$\bar{x} = 10.18$	B1	AWRT
	10.18 ± 0.2	M1	Follow through their \bar{x}
	(9.98, 10.38)	A1	AWRT
		3	
	Total	6	

Q	Answer	Marks	Comments									
3	<p>H₀: There is not an association between gender and survey response H₁: There is an association between gender and survey response</p> <table border="1" data-bbox="252 539 730 647"> <thead> <tr> <th>Expected</th> <th>S</th> <th>N</th> </tr> </thead> <tbody> <tr> <td>M</td> <td>24</td> <td>16</td> </tr> <tr> <td>F</td> <td>24</td> <td>16</td> </tr> </tbody> </table> $\sum \frac{(O - E - 0.5)^2}{E} = \frac{(21 - 24 - 0.5)^2}{24} + \frac{(27 - 24 - 0.5)^2}{24} + \frac{(19 - 16 - 0.5)^2}{16} + \frac{(13 - 16 - 0.5)^2}{16}$ <p>= 1.30 ν = 1 Critical value = 2.71 1.30 < 2.71</p> <p>Accept H₀. Evidence to suggest that there is not an association between gender and survey response</p>	Expected	S	N	M	24	16	F	24	16	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>A1ft</p> <p>E1ft</p>	<p>Both hypotheses, variables must be stated in at least the null hypothesis oe</p> <p>At least two expected values correct</p> <p>All correct</p> <p>Attempts to calculate test statistic</p> <p>Condone use of $\sum \frac{(O - E)^2}{E}$</p> <p>AWRT 1.3</p> <p>PI by correct critical value</p> <p>AWRT 2.7</p> <p>Correctly compares their X² test statistic and their critical value</p> <p>States that null hypothesis is not rejected and gives a conclusion in context Conclusion must not be definite ft their comparison using a X² model</p>
Expected	S	N										
M	24	16										
F	24	16										
	Total	9										

Q	Answer	Marks	Comments
4(a)(i)	$P(X \leq 1) = 0.0266$ [< 0.03] $P(X \leq 2) = 0.0884$ [> 0.03] $P(X \geq 11) = 0.0253$ [< 0.03] $P(X \geq 10) = 0.0538$ [> 0.03] Accept if $2 \leq X \leq 10$ $X \sim \text{Po}(5)$ Type II probability = $P(2 \leq X \leq 10)$ $= P(X \leq 10) - P(X \leq 2)$ $= 0.9863 - 0.0404$ $= 0.946$	M1 M1 A1 M1 M1 A1	Both seen Both seen or $P(X \leq 10) = 0.9747$ and $P(X \leq 9) = 0.9462$ oe, PI Use of Follow through their region AWRT
		6	
4(a)(ii)	Power = $1 - 0.946$ $= 0.054$	M1 A1ft	Applies power formula with their Type II probability AWRT ft their probability of Type II error
		2	
4(b)	$X \leq 1$ or $X \geq 11$	B1ft	ft their acceptance region in part (a)
		1	
	Total	9	

Q	Answer	Marks	Comments																		
5	<p> $H_0: \mu_{2008} = \mu_{2018}$ $H_1: \mu_{2008} \neq \mu_{2018}$ </p> <table border="1" data-bbox="252 477 754 689"> <thead> <tr> <th>Household</th> <th colspan="2">Difference</th> </tr> </thead> <tbody> <tr> <td>A</td> <td>10</td> <td>-10</td> </tr> <tr> <td>B</td> <td>3</td> <td>-3</td> </tr> <tr> <td>C</td> <td>0</td> <td>0</td> </tr> <tr> <td>D</td> <td>-2</td> <td>2</td> </tr> <tr> <td>E</td> <td>0</td> <td>0</td> </tr> </tbody> </table> <p> $\bar{x} = \frac{11}{5}$ or $-\frac{11}{5}$ </p> <p> $s^2 = \frac{1}{5-1} \left(113 - \frac{11^2}{5} \right)$ $= 22.2$ </p> <p> $t = \frac{2.2}{\sqrt{\frac{22.2}{5}}} \text{ or } \frac{-2.2}{\sqrt{\frac{22.2}{5}}}$ $= 1.04 \text{ or } -1.04$ </p> <p> $v = 4$ </p> <p> t_4 critical value = 2.13 or -2.13 </p> <p> $1.04 < 2.13$ </p> <p> Accept H_0. Evidence to suggest that average weekly household income has not changed between 2008 and 2018 </p>	Household	Difference		A	10	-10	B	3	-3	C	0	0	D	-2	2	E	0	0	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>A1ft</p> <p>E1ft</p>	<p>Both hypotheses</p> <p>All differences</p> <p>Sight of 2.2 or -2.2 Must be consistent with their differences</p> <p>Attempt at variance formula Allow one slip PI</p> <p>Accept $s =$ AWRT 4.71</p> <p>Using their mean and variance Condone use of $\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{S_p^2 \left(\frac{1}{n_x} + \frac{1}{n_y} \right)}}$</p> <p>AWRT Must be consistent with their differences</p> <p>PI by correct critical value</p> <p>AWRT Must be consistent with their differences unless changes -1.04 to 1.04 first</p> <p>Correctly compares their t test statistic and their critical value</p> <p>States that null hypothesis is not rejected and gives a conclusion in context Conclusion must not be definite ft their comparison using either a t or z test statistic</p>
Household	Difference																				
A	10	-10																			
B	3	-3																			
C	0	0																			
D	-2	2																			
E	0	0																			
	Total	11																			

Q	Answer	Marks	Comments
6	$H_0: \mu_X = \mu_Y$ $H_1: \mu_X > \mu_Y$ $\bar{x} = \frac{65760}{400}$ $\bar{y} = \frac{49197}{300}$ $s_X^2 = \frac{1}{400-1} \left(10814020 - \frac{65760^2}{400} \right)$ <p>or</p> $s_Y^2 = \frac{1}{300-1} \left(8070022 - \frac{49197^2}{300} \right)$ $s_X^2 = 7.71$ $s_Y^2 = 7.38$ $z = \frac{164.4 - 163.99}{\sqrt{\frac{7.7\dots}{400} + \frac{7.3\dots}{300}}}$ $= 1.96$ <p>z critical value = 1.64</p> $1.96 > 1.64$ <p>Reject H_0. Evidence to suggest that the average height of women who play basketball is larger than the average height of women who don't play basketball</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>A1ft</p> <p>E1ft</p>	<p>Both hypotheses</p> <p>Implied by sight of 164.4</p> <p>Implied by sight of 163.99</p> <p>Attempt at one of the variance formulae Allow one slip Implied by one correct answer</p> <p>AWRT</p> <p>AWRT</p> <p>Correct numerator for their means, subtracted either way round</p> <p>Correct denominator for their variances</p> <p>AWRT</p> <p>Allow 1.645 AWRT</p> <p>Correctly compares their z or t test statistic and critical value</p> <p>States that null hypothesis is rejected and gives a conclusion in context Conclusion must not be definite ft their comparison using z or t test statistic</p>
	Total	12	

Q	Answer	Marks	Comments
7(a)	$E(R) = \frac{1}{n} E(B_1) = \frac{np}{n}$ $= p \text{ therefore unbiased}$	<p>M1</p> <p>A1</p>	<p>Finds $E(R)$ in terms of n and p</p> <p>Must see p and conclusion n must not be assigned a value</p>
		2	
7(b)	$\text{Var}(T) = \text{Var} \left(\frac{\sum_{i=1}^k B_i}{kn} \right) = \frac{kn p(1-p)}{k^2 n^2}$ $= \frac{p(1-p)}{kn} \text{ or } \frac{kp(1-p)}{k^2 n}$ <p>As $n \rightarrow \infty$, $\text{Var}(T) \rightarrow 0$</p> <p>Therefore consistent estimator</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>Finds $\text{Var}(T)$ in terms of k, n and p, condoning slips</p> <p>May be seen in part (c) if not attempted in this part</p> <p>Correct simplification</p> <p>May be seen in part (c) if not attempted in this part</p> <p>Applies limiting process to their $\text{Var}(T)$</p> <p>CSO</p>
		4	
7(c)	$\text{Var}(R) = \text{Var} \left(\frac{B_1}{n} \right) = \frac{np(1-p)}{n^2}$ $= \frac{p(1-p)}{n}$ $\text{Relative Efficiency} = \frac{\frac{1}{\text{Var}(T)}}{\frac{1}{\text{Var}(R)}} = \frac{\frac{kn}{p(1-p)}}{\frac{n}{p(1-p)}}$ $= k$ <p>Estimator T is more efficient than estimator R</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>E1</p>	<p>Finds $\text{Var}(R)$ in terms of n and p, condoning slips</p> <p>PI by correct simplified $\text{Var}(R)$ or relative efficiency</p> <p>Correct simplification</p> <p>PI by correct relative efficiency</p> <p>Applies formula with either the correct $\text{Var}(T)$ or their $\text{Var}(T)$ from part (b) and their $\text{Var}(R)$</p> <p>Correct simplification</p> <p>CSO</p>
		5	
	Total	11	

Q	Answer	Marks	Comments
8(a)	$M_{X_i}(t) = E(e^{tX_i}) = \int_0^{\infty} e^{tx} \lambda e^{-\lambda x} dx$ $= \left[\frac{\lambda e^{(t-\lambda)x}}{t-\lambda} \right]_0^{\infty}$ $= -\frac{\lambda}{t-\lambda}$ $= \left(\frac{\lambda-t}{\lambda} \right)^{-1}$ $= \left(1 - \frac{t}{\lambda} \right)^{-1}$	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>Applies mgf formula with any or missing limits Condone spurious notation</p> <p>Integrates to $Ae^{(t-\lambda)x}$</p> <p>Applies limits to obtain a correct form oe</p> <p>Reaches correct form with no errors and at least one intermediate line Condone interchangeable x and x_i AG CSO</p>
		4	
8(b)	$M_X(t) = \left(1 - \frac{t}{\lambda} \right)^{-n}$ $M_X'(t) = \frac{n}{\lambda} \left(1 - \frac{t}{\lambda} \right)^{-n-1}$ $M_X''(t) = \frac{n(n+1)}{\lambda^2} \left(1 - \frac{t}{\lambda} \right)^{-n-2}$ $\text{Var}(X) = M_X''(0) - (M_X'(0))^2$ $= \frac{n(n+1)}{\lambda^2} \left(1 - \frac{0}{\lambda} \right)^{-n-2} - \left(\frac{n}{\lambda} \left(1 - \frac{0}{\lambda} \right)^{-n-1} \right)^2$ $= \frac{n}{\lambda^2}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>Differentiates to $A \left(1 - \frac{t}{\lambda} \right)^{-n-1}$</p> <p>$\frac{n}{\lambda} \left(1 - \frac{t}{\lambda} \right)^{-n-1}$ oe</p> <p>Differentiates to $A \left(1 - \frac{t}{\lambda} \right)^{-n-2}$</p> <p>$\frac{n(n+1)}{\lambda^2} \left(1 - \frac{t}{\lambda} \right)^{-n-2}$ oe</p> <p>Applies variance formula with their $M_X''(0)$ and $M_X'(0)$</p> <p>CAO</p>
		7	

Q	Answer	Marks	Comments
<p>8(b) ALT</p>	$M_{X_i}'(t) = \frac{1}{\lambda} \left(1 - \frac{t}{\lambda}\right)^{-2}$ $M_{X_i}''(t) = \frac{2}{\lambda^2} \left(1 - \frac{t}{\lambda}\right)^{-3}$ $M_{X_i}'(0) = \frac{1}{\lambda} \text{ and } M_{X_i}''(t) = \frac{2}{\lambda^2}$ $\text{Var}(X_i) = \frac{1}{\lambda^2}$ $\text{Var}(X) = n\text{Var}(X_i) = n \times \frac{1}{\lambda^2}$ $= \frac{n}{\lambda^2}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>Condone X for X_i oe</p> <p>Differentiates to $A \left(1 - \frac{t}{\lambda}\right)^{-3}$</p> <p>Condone X for X_i</p> <p>$\frac{2}{\lambda^2} \left(1 - \frac{t}{\lambda}\right)^{-3}$ oe</p> <p>Condone X for X_i</p> <p>X_i must be seen and used</p> <p>Must be $\text{Var}(X_i) = \frac{1}{\lambda^2}$</p> <p>not $\text{Var}(X) = \frac{1}{\lambda^2}$</p> <p>Multiplies their $\text{Var}(X_i)$ by n</p> <p>CAO</p>
		7	
<p>8(c)(i)</p>	$E(Y) = 2\lambda E(X) = 2\lambda \times \frac{n}{\lambda}$ $= 2n$	<p>M1</p> <p>A1</p>	<p>$2\lambda \times$ their $E(X)$</p> <p>CAO</p>
		2	
<p>8(c)(ii)</p>	$\text{Var}(Y) = 4\lambda^2 \text{Var}(X) = 4\lambda^2 \times \frac{n}{\lambda^2}$ $= 4n$	<p>M1</p> <p>A1</p>	<p>$4\lambda^2 \times$ their $\text{Var}(X)$</p> <p>CAO</p>
		2	
	Total	15	