

OXFORD

INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL AS FURTHER MATHEMATICS

FM02

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

June 2022

Version 1.0 Final



2 2 6 X F M 0 2 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from oxfordaqaexams.org.uk

Copyright information

OxfordAQA retains the copyright on all its publications. However, registered schools/colleges for OxfordAQA are permitted to copy material from this booklet for their own internal use, with the following important exception: OxfordAQA cannot give permission to schools/colleges to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Copyright © 2022 Oxford International AQA Examinations and its licensors. All rights reserved.

Key to mark scheme abbreviations

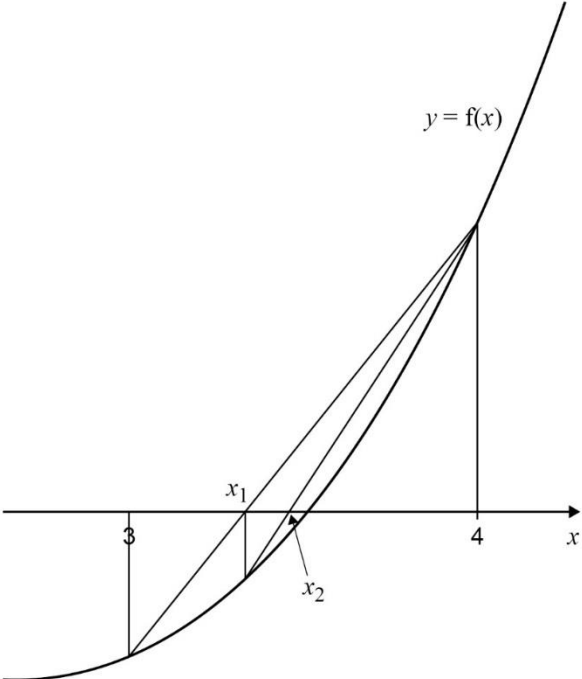
M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
-x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1	$hf(x, y) = 0.2 \left(\frac{\sqrt{3+2}}{4.8 \times (3+1)} \right)$ $[= 0.023292375]$ $y_2 = 4.8 + 0.023\dots$ $y_2 = 4.823292375$ $y_3 = 4.823292375 + 0.2 \left(\frac{\sqrt{3.2+2}}{4.823292375 \times (3.2+1)} \right)$ $y_3 = 4.846$	<p>M1</p> <p>m1</p> <p>A1</p> <p>m1</p> <p>A1</p>	<p>PI</p> <p>4.8 + their $hf(x, y)$</p> <p>AWRT 4.82</p> <p>ft their y_2</p> <p>CAO</p>
	Total	5	

Q	Answer	Marks	Comments
2(a)	Rotation about O 60° clockwise	B1 B1	Allow 'rotate' oe
		2	
2(b)	Shear	B1	Ignore additional information about shears and no other transformations mentioned
		1	
2(c)	$\mathbf{AB} = \begin{bmatrix} \frac{1}{2} + \frac{\sqrt{3}}{2}p & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} + \frac{1}{2}p & \frac{1}{2} \end{bmatrix}$ $\mathbf{BA} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2}p - \frac{\sqrt{3}}{2} & \frac{1}{2} + \frac{\sqrt{3}}{2}p \end{bmatrix}$ $\mathbf{AB} - \mathbf{BA} = \begin{bmatrix} \frac{1}{2} + \frac{\sqrt{3}}{2}p & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} + \frac{1}{2}p & \frac{1}{2} \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2}p - \frac{\sqrt{3}}{2} & \frac{1}{2} + \frac{\sqrt{3}}{2}p \end{bmatrix}$ $\mathbf{AB} - \mathbf{BA} = \frac{\sqrt{3}}{2}p \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	B1 B1 M1 A1	$\left[\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ p & 1 \end{bmatrix} \right]$ $\left[\begin{bmatrix} 1 & 0 \\ p & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \right]$ ft their AB and BA Allow $\begin{bmatrix} \frac{\sqrt{3}}{2}p & 0 \\ 0 & -\frac{\sqrt{3}}{2}p \end{bmatrix}$ and $k = \frac{\sqrt{3}}{2}p$ CSO
		4	
2(d)	det C = -1 det A = det B = 1 and det CAB = (det A)(det B)(det C) det CAB = 1 × 1 × -1 = -1	B1 M1 A1	or det AB = 1 and det CAB = (det AB)(det C) CSO
		3	

Q	Answer	Marks	Comments
2(d) ALT	<p>CAB =</p> $\begin{bmatrix} \left(\frac{1}{2} + \frac{\sqrt{3}}{2}p\right)\cos 2\theta + \left(\frac{1}{2}p - \frac{\sqrt{3}}{2}\right)\sin 2\theta & \frac{\sqrt{3}}{2}\cos 2\theta + \frac{1}{2}\sin 2\theta \\ \left(\frac{\sqrt{3}}{2} - \frac{1}{2}p\right)\cos 2\theta + \left(\frac{1}{2} + \frac{\sqrt{3}}{2}p\right)\sin 2\theta & -\frac{1}{2}\cos 2\theta + \frac{\sqrt{3}}{2}\sin 2\theta \end{bmatrix}$ <p>det CAB =</p> $\left(\left(\frac{1}{2} + \frac{\sqrt{3}}{2}p\right)\cos 2\theta + \left(\frac{1}{2}p - \frac{\sqrt{3}}{2}\right)\sin 2\theta\right)\left(-\frac{1}{2}\cos 2\theta + \frac{\sqrt{3}}{2}\sin 2\theta\right)$ $- \left(\left(\frac{\sqrt{3}}{2} - \frac{1}{2}p\right)\cos 2\theta + \left(\frac{1}{2} + \frac{\sqrt{3}}{2}p\right)\sin 2\theta\right)\left(\frac{\sqrt{3}}{2}\cos 2\theta + \frac{1}{2}\sin 2\theta\right)$ <p>det CAB = $-\frac{1}{4}(\cos^2 2\theta + \sin^2 2\theta) - \frac{3}{4}(\cos^2 2\theta + \sin^2 2\theta)$</p> <p>det CAB = -1</p>	<p>B1</p> <p>M1</p> <p>A1</p>	<p>CSO</p>
		3	
	Total	10	

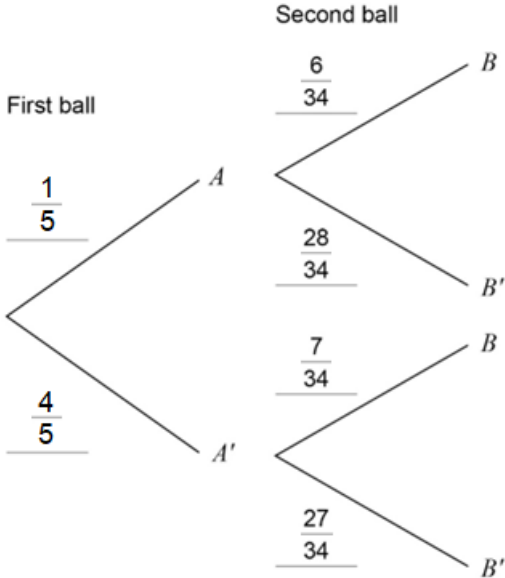
Q	Answer	Marks	Comments												
3(a)	$\log_{10} y = n \log_{10} x + b$ $\log_{10} y = \log_{10} x^n + \log_{10} 10^b$ $\log_{10} y = \log_{10} 10^b x^n$ $y = 10^b x^n$	<p>M1</p> <p>A1</p>	<p>Apply at least one law of logarithms correctly</p> <p>CSO</p>												
		2													
3(b)	<table border="1" data-bbox="280 678 879 757"> <tr> <td><i>X</i></td> <td>0.30</td> <td>0.60</td> <td>1</td> <td>1.30</td> <td>1.60</td> </tr> <tr> <td><i>Y</i></td> <td>0.66</td> <td>0.48</td> <td>0.26</td> <td>0.08</td> <td>-0.10</td> </tr> </table>	<i>X</i>	0.30	0.60	1	1.30	1.60	<i>Y</i>	0.66	0.48	0.26	0.08	-0.10	<p>B1</p> <p>B1</p>	<p><i>X</i> values – at least 1 dp or exact value</p> <p><i>Y</i> values – at least 2 dp</p>
<i>X</i>	0.30	0.60	1	1.30	1.60										
<i>Y</i>	0.66	0.48	0.26	0.08	-0.10										
		2													
3(c)(i)	<p>All their points plotted ± 1 square</p> <p>Line of best fit drawn</p>	<p>B1</p> <p>B1</p>	<p>Must have linear scales</p>												
		2													
3(c)(ii)	<p>$a = 10$(their <i>y</i>-intercept)</p> <p>$a = 6.8$</p> <p>$n =$ their gradient</p> <p>$n = -0.58$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>PI</p> <p>Between 6 and 8</p> <p>Between -0.7 and -0.5</p>												
		4													
	Total	10													

Q	Answer	Marks	Comments
4(a)	$f(3) = -3$ and $f(4) = 6$ Change of sign and f is continuous on the interval so α is in the interval (3, 4)	M1 A1	Correct evaluation of a suitable interval Must state that there is a change of sign and that the curve is continuous (condone unbroken) and concludes a root is present in the interval
		2	
4(b)	Vertical lines to curve at 3 and 4 Correct points on graph joined by line that meets x -axis at x_1 Vertical line to curve at their x_1 Correct points on graph joined by line that meets x -axis at x_2	M1 A1 M1 A1	See diagram
			
		4	
4(c)	$x_1 = \frac{3 \times 4 + 6 \times 3}{3 + 6}$ $x_1 = \frac{10}{3} \text{ and } f(x_1) = -\frac{38}{27}$ $x_2 = 3.46 \text{ and } f(x_2) = -0.464664$ $x_3 = 3.50$	M1 A1 A1 A1	Uses values from (a) to find x_1 AWRT 3.33 and -1.41 AWRT 3.46 (if earlier values rounded) and -0.465 CSO, accept 3.5
		4	
	Total	10	

Q	Answer	Marks	Comments
5	$\mathbf{P}(4\mathbf{I} + \mathbf{Q}) = \mathbf{R}$ $4\mathbf{I} + \mathbf{Q} = \begin{bmatrix} -2 & 4 \\ -2 & 5 \end{bmatrix} \text{ and } \det(4\mathbf{I} + \mathbf{Q}) = -2$ $(4\mathbf{I} + \mathbf{Q})^{-1} = \begin{bmatrix} -2.5 & 2 \\ -1 & 1 \end{bmatrix}$ $\mathbf{P} = \mathbf{R}(4\mathbf{I} + \mathbf{Q})^{-1}$ $\mathbf{P} = \begin{bmatrix} -2 & 3 \\ -16 & 37 \end{bmatrix} \begin{bmatrix} -2.5 & 2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>Condone 4 for 4I if I implied by later work PI</p> <p>PI</p>
5 ALT	$\mathbf{P} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $4\mathbf{P} + \mathbf{PQ} = \begin{bmatrix} -2a - 2b & 4a + 5b \\ -2c - 2d & 4c + 5d \end{bmatrix}$ $-2a - 2b = -2$ $4a + 5b = 3$ $-2c - 2d = -16$ $4c + 5d = 37$ $\mathbf{P} = \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>PI At least two correct elements</p> <p>PI Fully correct</p> <p>Form simultaneous equations in a and b</p> <p>Form simultaneous equations in c and d</p>
	Total	5	

Q	Answer	Marks	Comments
6(a)	0.121032	B1	CAO
		1	
6(b)	$P(R > 6) = (1 - 0.18)^6$ 0.304	M1 A1	PI Applies formula (oe) or finds $P(R \leq 6)$ [= 0.6959...] AWRT
		2	
6(c)	$E(R) = 5.56$	B1	AWRT
		1	
6(d)	$\text{Var}(R) = 25.3$	B1	AWRT
		1	
6(e)	$\text{Var}(R - S) = 28.3$	B1ft	AWRT ft their $\text{Var}(R) + 3$
		1	
	Total	6	

Q	Answer	Marks	Comments
7(a)	$G_Y(t) = (0.22 + 0.78t)^2$ $G_Y(t) = 0.6084t^2 + 0.3432t + 0.0484$ $G'_Y(t) = 1.2168t + 0.3432$ $G''_Y(t) = 1.2168 \text{ or } \frac{1521}{1250}$	B1 M1 M1 A1	Applies formula for $G_Y(t)$ Multiplies out Differentiates $G'_Y(t) = 2 \times 0.78(0.22 + 0.78t)$ gains both M marks CAO
		4	
7(b)	$G'_Y(1) = 1.2168 \times 1 + 0.3432$ $\text{Var}(Y) = G''_Y(1) + G'_Y(1) - (G'_Y(1))^2$ $= 1.2168 + 1.56 - 1.56^2$ $= 0.3432 \text{ or } \frac{429}{1250}$	M1 M1 A1	PI Accept sight of 1.56 Their $G'_Y(t)$ must have non-constant terms Uses variance formula and substitutes in their values AWFW [0.34, 0.35]
		3	
7(b) ALT	$Y \sim B(2, 0.78)$ $\text{Var}(Y) = 2 \times 0.78 \times 0.22$ $= 0.3432 \text{ or } \frac{429}{1250}$	M1 M1 A1	PI Identifies binomial distribution Applies variance formula
		3	
	Total	7	

Q	Answer	Marks	Comments
8(a)	$\frac{1}{5}$	B1	oe such as $\frac{7}{35}$
		1	
8(b)(i)	 <p style="text-align: center;">Second ball</p> <p style="text-align: center;">First ball</p>	<p>B1ft</p> <p>B1ft</p> <p>B1ft</p>	<p>1/5 and 4/5 oe ft their part (a)</p> <p>6/34 and 28/34 oe ft their 1/5 and 4/5</p> <p>7/34 and 27/34 oe ft their 1/5 and 4/5</p>
		3	
8(b)(ii)	$\frac{1}{5} \times \frac{28}{34}$ $\frac{1}{5} \times \frac{28}{34} + \frac{4}{5} \times \frac{27}{34}$ $\frac{14}{85} = \frac{7}{34}$	<p>M1</p> <p>M1</p> <p>A1</p>	<p>Finds numerator using Bayes Theorem Sight of 14/85 oe</p> <p>Find denominator using Bayes Theorem Sight of 4/5 oe</p> <p>oe AWRT 0.206</p>
		3	
	Total	7	

Q	Answer	Marks	Comments
9	$I = 0.3 \times 7 - 0.3 \times (-5)$	M1	Applies impulse equation. Condone sign errors for the velocities.
	$I = 3.6 \text{ N s}$	A1	If units stated, must be correct.
	Total	2	

Q	Answer	Marks	Comments
10	$(LT^{-1})^2 = [\omega]^2 L^2$	M1	Forms correct dimensional analysis equation. Condone $[\omega]^2 (L^2 - L^2)$ for RHS
	$[\omega]^2 = \frac{L^2 T^{-2}}{L^2} = T^{-2}$		
	$[\omega] = T^{-1}$	A1	
	Units of ω are s^{-1}	B1	
	Total	3	

Q	Answer	Marks	Comments
11	$v^2 = 2^2 + 6^2 - 2 \times 2 \times 6 \cos 45^\circ$	M1	Uses cosine rule for a valid triangle oe
	$v = 4.7989 \left[= 4.80 \text{ m s}^{-1} \text{ to 3sf} \right]$	A1	Finds correct resultant velocity
	$\frac{\sin \alpha}{2} = \frac{\sin 45^\circ}{4.7989} \text{ or } \frac{\sin \beta}{6} = \frac{\sin 45^\circ}{4.7989}$	M1	Uses sine rule with correct sides oe Condone 135 instead of 45
	$\alpha = 17.14 \text{ or } \beta = 62.14$	A1	AWRT 17 or 62
	Bearing = 107° to the nearest degree	A1ft	ft 90 + their α or 45 + their β
	Total	5	

Q	Answer	Marks	Comments
12(a)	$3 \times 5 = 3v_A + 2v_B$ $v_B - v_A = 0.4 \times 5$ $v_A = \frac{11}{5} = 2.2, v_B = \frac{21}{5} = 4.2$ <p>Speed for A = 2.2 m s⁻¹</p> <p>Speed for B = 4.2 m s⁻¹</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>Correct conservation of momentum equation</p> <p>Correct coefficient of restitution equation</p> <p>Attempt at solving simultaneous equations to find a value for v_A or v_B</p> <p>Correct speed for A or B</p> <p>Correct speeds for A and B including units</p>
		5	
12(b)	$2 \times 4.2 = 2w_B + w_C$ $w_C - w_B = 4.2e$ $w_B = \frac{8.4 - 4.2e}{3}$ $\frac{8.4 - 4.2e}{3} \geq 2.2$ $e \leq \frac{3}{7}$	<p>B1ft</p> <p>B1ft</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>Correct conservation of momentum equation, ft their v_B</p> <p>Correct coefficient of restitution equation, ft their v_B</p> <p>Solves simultaneous equations to find an expression for w_B</p> <p>Forms an inequality using their $w_B \geq v_A$</p> <p>Condone $w_B > v_A$</p> <p>Correct simplified inequality. Accept any equivalent fraction.</p>
		5	
	Total	10	