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FURTHER MATHEMATICS

FM01

(9665/FM01) Unit FP1 Pure Mathematics

Mark scheme

June 2022

Version 1.0 Final

226XFM01/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$(6+h)^3 - 4(6+h)^2$ $= 6^3 + 3(36)h + 3(6)h^2 + h^3$ $-4(36 + 12h + h^2)$	M1	
	$= 72 + 60h + 14h^2 + h^3$	M1	PI
	Gradient $= \dfrac{72 + 60h + 14h^2 + h^3 - 72}{h}$	M1	PI
	$= 60 + 14h + h^2$	A1	
		4	
1(b)	Gradient of curve $= \lim\limits_{h \to 0}[60 + 14h + h^2]$	M1	
	$= 60$	A1F	SC1 for using $h = 0$ leading to gradient = *their* 60
		2	
	Total	6	

Q	Answer	Marks	Comments
2	$z = \dfrac{a+4\text{i}}{7+b\text{i}} \times \dfrac{7-b\text{i}}{7-b\text{i}}$	M1	Alternative method, if using z = x +iy: M1 for forming simultaneous equations M1 for solving their simultaneous equations
	$= \dfrac{7a + 4b + 28\text{i} - ab\text{i}}{49 + b^2}$	M1	Condone one error
	Re$(z) = \dfrac{7a + 4b}{49 + b^2}$	A1	
	Im$(z) = \dfrac{28 - ab}{49 + b^2}$	A1	Must not include i
	Total	4	

Q	Answer	Marks	Comments
3(a)	$3x-\frac{\pi}{6}=\frac{\pi}{4}+n\pi$	B1	**oe**
	Going from $\left(3x-\frac{\pi}{6}\right)$ to x	M1	including division of all terms by 3
	$x=\frac{5\pi}{36}+\frac{n\pi}{3}$	A1	**oe** eg $\frac{\pi}{36}(12n+5)$
		3	
3(b)	Also include solutions to $\tan\left(3x-\frac{\pi}{6}\right)=-1$	M1	
	$3x-\frac{\pi}{6}=-\frac{\pi}{4}+n\pi$	M1	**PI**
	$x=\frac{5\pi}{36}+\frac{n\pi}{3},-\frac{\pi}{36}+\frac{n\pi}{3}$	A1	**oe** eg $x=\frac{5\pi}{36}+\frac{n\pi}{3},\frac{11\pi}{36}+\frac{n\pi}{3}$ **or** $x=\frac{\pi}{36}(6n+5)$ (complete solution)
		3	
	Total	**6**	

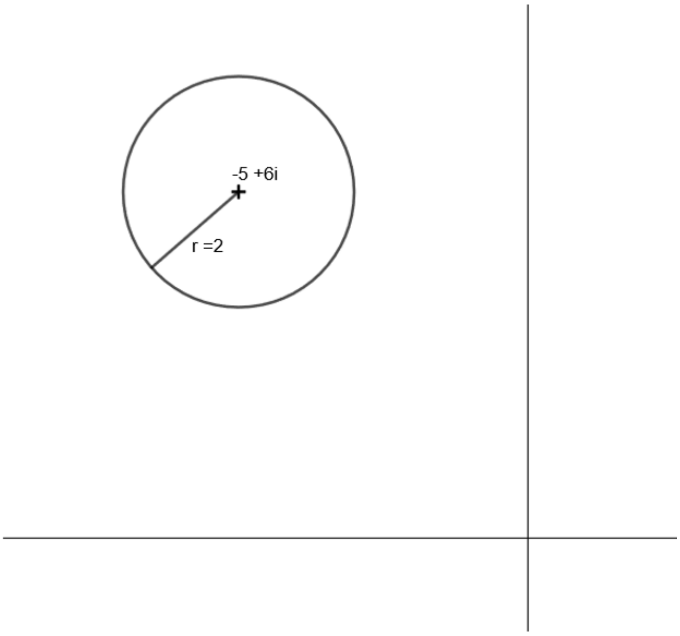
Q	Answer	Marks	Comments
4(a)	At $x=0$, which is a limit of the integral, the integrand $x^{-\frac{1}{4}}$ is not defined	E1	**oe**
		1	
4(b)	$\int_0^{16} x^{-\frac{1}{4}}\,dx=\lim_{h\to 0}\int_h^{16} x^{-\frac{1}{4}}\,dx$ $=\lim_{h\to 0}\left[\frac{4x^{\frac{3}{4}}}{3}\right]_h^{16}$	M1	For integrating
	$=\lim_{h\to 0}\left(\frac{4\left(16^{\frac{3}{4}}\right)}{3}-\frac{4h^{\frac{3}{4}}}{3}\right)$	M1	For correct use of limit
	$=\frac{32}{3}$	A1	
		3	
	Total	**4**	

Q	Answer	Marks	Comments
5(a)	$\sum_{r=2}^{n}\left(\frac{1}{(r-1)^2}-\frac{1}{(r+1)^2}\right)$ $=\frac{1}{1^2}-\frac{1}{3^2}$ $+\frac{1}{2^2}-\frac{1}{4^2}$ $+\frac{1}{3^2}-\frac{1}{5^2}$ $+\cdots$	M1	Must use method of differences to gain any marks
	$+\frac{1}{(n-3)^2}-\frac{1}{(n-1)^2}$ $+\frac{1}{(n-2)^2}-\frac{1}{n^2}$ $+\frac{1}{(n-1)^2}-\frac{1}{(n+1)^2}$	M1	
	$=\frac{1}{1^2}+\frac{1}{2^2}-\frac{1}{n^2}-\frac{1}{(n+1)^2}$	M1	
	$=\frac{5}{4}-\frac{(n+1)^2+n^2}{n^2(n+1)^2}$ or $\frac{5}{4}-\frac{2n^2+2n+1}{n^2(n+1)^2}$ as required	A1	Or equivalent for f(n)
		4	
5(b)	Let $S_\infty=\lim_{n\to\infty}(S_n)$	M1	For taking a limit **PI**
	Then $S_\infty=\frac{5}{4}$	A1ft	
	$\sum_{r=11}^{\infty}\left(\frac{1}{(r-1)^2}-\frac{1}{(r+1)^2}\right)=S_\infty-S_{10}$	M1	**oe**
	$=\frac{11^2+10^2}{11^2\times10^2}=\frac{221}{12100}$	A1ft	
		4	
	Total	**8**	

Q	Answer	Marks	Comments
6(a)	$(x-2)(x-5)-2x=0$ $x^2-9x+10=0$	**M1**	
	$\alpha+\beta=9$	**A1**	
	$\alpha\beta=10$	**A1**	
		3	
6(b)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$	**M1**	
	$=9^2-2\times 10=61$	**A1**	
		2	
6(c)	Sum of roots $=\alpha+\beta-\frac{1}{\alpha^2}-\frac{1}{\beta^2}$		**PI**
	$=\alpha+\beta-\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$	**M1**	
	$=9-\frac{61}{100}=\frac{839}{100}$	**A1**	
	Product of roots $=\left(\alpha-\frac{1}{\beta^2}\right)\left(\beta-\frac{1}{\alpha^2}\right)$		
	$=\alpha\beta-\frac{1}{\alpha}-\frac{1}{\beta}+\frac{1}{\alpha^2\beta^2}$	**M1**	
	$=\alpha\beta-\frac{\alpha+\beta}{\alpha\beta}+\frac{1}{\alpha^2\beta^2}$	**M1**	
	$=10-\frac{9}{10}+\frac{1}{100}=\frac{911}{100}$	**A1**	
	$100x^2-839x+911=0$	**A1**	**oe** (integer coefficients) **cao**
		6	
	Total	**11**	

Q	Answer	Marks	Comments
7(a)	$V = 2h^2$	**B1**	
		1	
7(b)	$\frac{\mathrm{d}V}{\mathrm{d}h} = 4h$	**M1**	
	$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}t}$	**M1**	**PI**
	$-0.006 = 4h \times \frac{\mathrm{d}h}{\mathrm{d}t}$	**M1**	Condone omission of minus sign
	$h = 0.75$ so $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{-0.006}{4 \times 0.75}$	**M1**	For substituting $h = 0.75$ in their expression for $\frac{\mathrm{d}V}{\mathrm{d}h}$ or $\frac{\mathrm{d}h}{\mathrm{d}t}$
	Rate of decrease = 0.002 (metres per minute)	**A1ft**	Condone inclusion of minus sign Follow through from their $V = kh^2$ in part **(a)**
		5	
	Total	**6**	

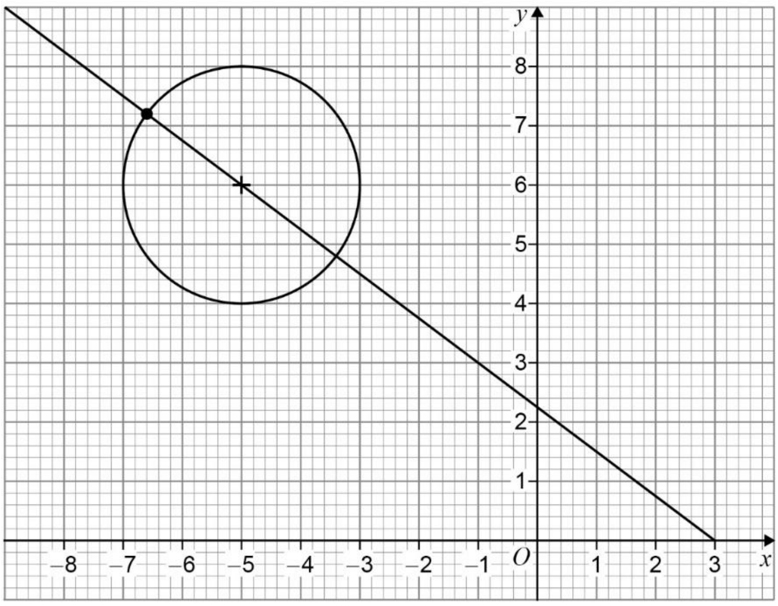
Q	Answer	Marks	Comments
8(a)	Circle with centre -5 + 6i, radius = 2	B1 B1	Allow Cartesian coordinates or values shown on axes



		2	
8(b)	$-5+6\text{i}-3=-8+6\text{i}$ $\lvert -8+6\text{i}\rvert = 10$ $\frac{2}{10}\times(-8+6\text{i}) = -1.6+1.2\text{i}$ $z_1 = -5+6\text{i}+-1.6+1.2\text{i}$ $z_1 = -6.6+7.2\text{i}$	M1 M1 M1 M1 A1	
		5	
	Total	7	

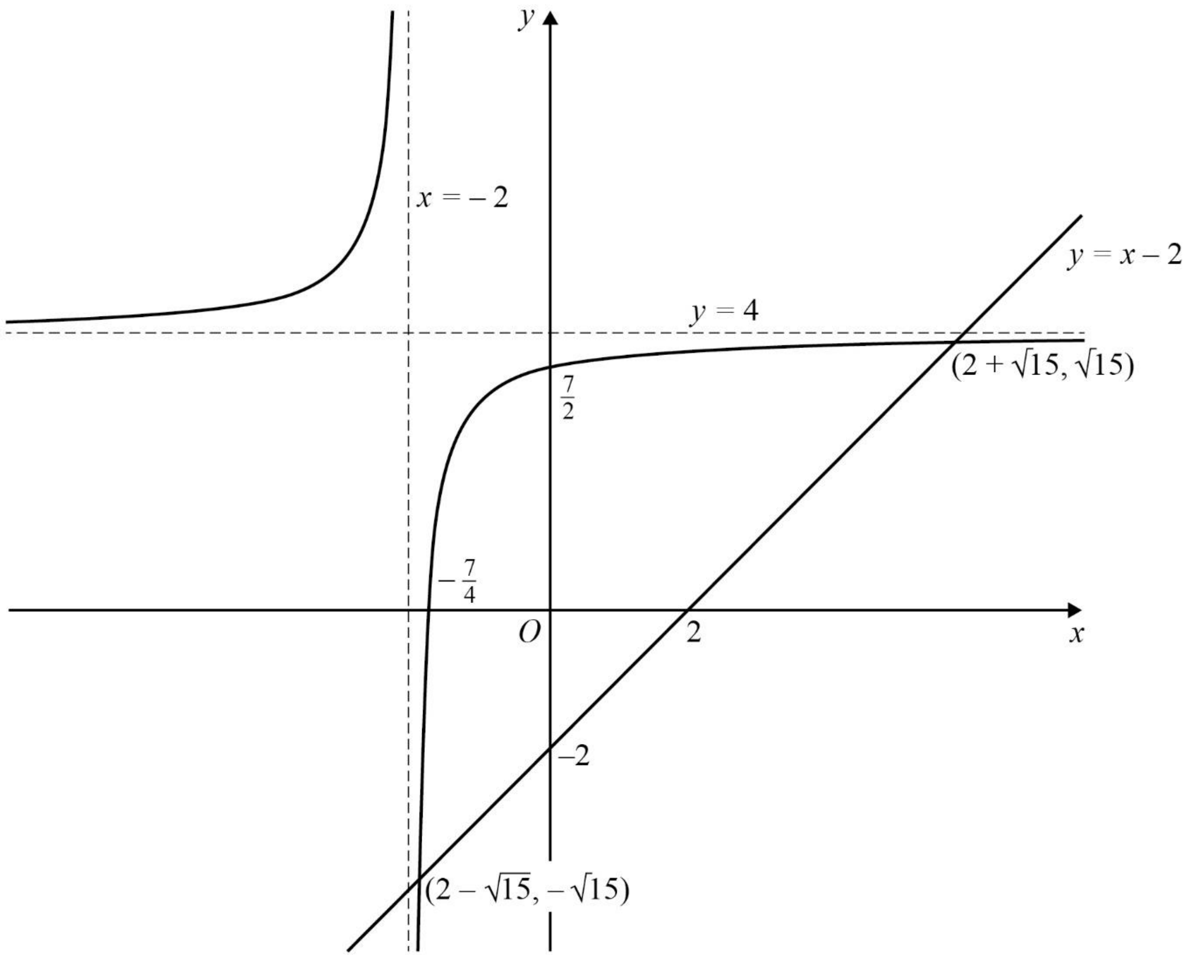
		2	
8(b) Alt 1	$\lvert z_1-3\rvert = 12$ $-5+6\text{i}-3=-8+6\text{i}$ $[z_1-3=]\quad 12(-0.8+0.6\text{i})$ $z_1-3=-9.6+7.2\text{i}$ $z_1=-6.6+7.2\text{i}$	M1 M1 M1 M1 A1	 oe
		5	

		2	
8(b) Alt 2	Equation of circle: $(x+5)^2+(y-6)^2=4$	**B1**	
	Equation of straight line: $y=-\frac{3}{4}x+\frac{9}{4}$	**B1**	
	$(x+5)^2+\left(-\frac{3}{4}x+\frac{9}{4}-6\right)^2=4$ $25x^2+250x+561=0$	**M1**	For substituting straight line equation into circle equation and forming a quadratic equation in x or y
	$x=-6.6$	**M1**	For solving their quadratic equation
	$z_1=-6.6+7.2\text{i}$	**A1**	
		5	



Q	Answer	Marks	Comments
9(a)	$x = -2$	B1	
	$y = 4$	B1	
		2	
9(b)	$x - 2 = \frac{4x + 7}{x + 2}$	M1	
	$(x - 2)(x + 2) = 4x + 7$		
	$x^2 - 4 - 4x - 7 = 0$		
	$x^2 - 4x - 11 = 0$	M1	
	$x = 2 \pm \sqrt{15}$	A1	
	$(2 + \sqrt{15}, \sqrt{15})$		
	and $\left(2 - \sqrt{15}, -\sqrt{15}\right)$	A1	
		4	

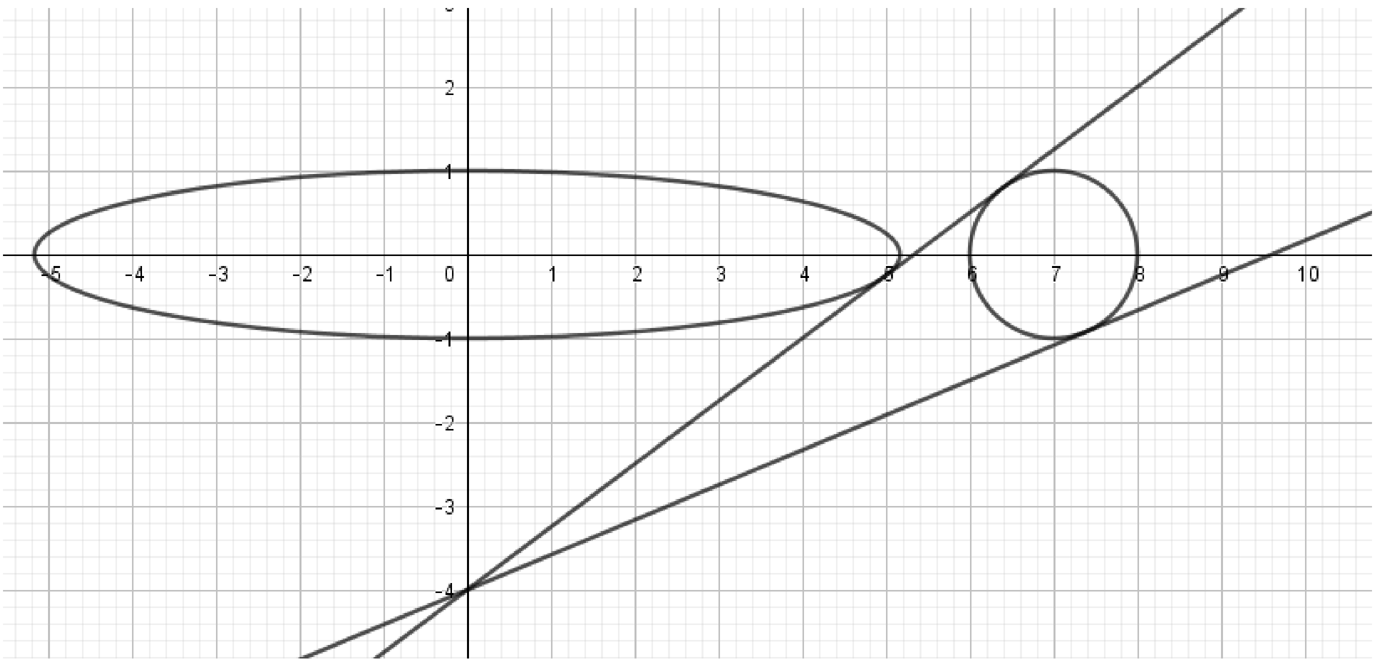
9(c)	Graph of $y = \mathrm{f}(x)$, correct shape	**B1**	
	Asymptotes shown and graph approaches asymptotes	**B1**	
	Values at axes intercepts shown	**B1**	
	Line drawn correctly	**B1**	



		4	
9(d)	$-2 < x \leq 2 - \sqrt{15}, x \geq 2 + \sqrt{15}$	**B1B1**	B1 for either subset (condone use of strict inequality) B2 for exactly both and no more
		2	
	Total	**12**	

Q	Answer	Marks	Comments
10(a)	Translation parallel to x-axis By $\begin{bmatrix}7\\0\end{bmatrix}$	B1 B1	"parallel to x-axis" can be implied by form of vector
		2	
10(b)	$(x-7)^2+(mx-4)^2=1$ $x^2-14x+49+m^2x^2-8mx+16-1=0$ $(m^2+1)x^2-(8m+14)x+64=0$ $\Delta\geq 0$ $(8m+14)^2-4(m^2+1)(64)\geq 0$ $64m^2+224m+196-256m^2-256\geq 0$ $-192m^2+224m-60\geq 0$ or $48m^2-56m+15\leq 0$ $48m^2-56m+15=0$ has solutions 5/12 and 3/4 $\frac{5}{12}\leq m\leq\frac{3}{4}$	M1 A1 M1 M1 A1 A1	
		6	

10(c)(i)	An ellipse	**E1**	
		1	
10(c)(ii)	$\frac{x^2}{a^2} + y^2 = 1$	**B1**	
		1	
10(d)	$\frac{x^2}{a^2} + (mx - 4)^2 = 1$ $\frac{x^2}{a^2} + m^2x^2 - 8mx + 16 - 1 = 0$	**M1**	Also possible to use $m = \frac{3}{4}$ from the start
	$\left(\frac{1}{a^2} + m^2\right)x^2 - 8mx + 15 = 0$	**A1ft**	
	$\Delta \geq 0$ $64m^2 - 4\left(\frac{1}{a^2} + m^2\right)(15) \geq 0$	**M1**	Inequality must include both a and m
	$a^2 \geq \frac{15}{m^2}$	**A1**	
	The limiting case is $m = \frac{3}{4}$	**M1**	eg see diagram
	$a > 1$ so $a \geq \frac{4\sqrt{15}}{3}$	**A1**	**oe** eg $a \geq \sqrt{\frac{80}{3}}$



		6	
	Total	**16**	