OXFORD

INTERNATIONAL
AQA EXAMINATIONS

International A-level

FURTHER MATHEMATICS

FM04

Mark scheme

June 2019

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Mark	Comments
1	75 + 2.3263$\sqrt{\frac{4^2}{50}}$	M1	Attempts to find value at edge of critical region
	= 76.3 AWRT	A1	
	P($\overline{X}$ ≥ 76.3 | μ = 77) =P$\left(Z \geq \frac{76.3-77}{\frac{4}{\sqrt{50}}}\right)$	M1	Standardises correct probability for their 76.3 Implied by correct answer May do equivalent working for P(Type II error) instead by doing ≤ instead of ≥
	= P(Z ≥ -1.24) = P(Z ≤ 1.24)	m1	Rearranges to probability that can be found in standard table Implied by correct answer
	= 0.88 to 0.89	A1	AWFW
	Total	**5**	

Q	Answer	Mark	Comments
2	H_0: σ^2 = 2.56 or H_0: σ = 1.6 H_1: σ^2 ≠ 2.56 H_1: σ ≠ 1.6	B1	Both hypotheses Condone sd or var =
	d.o.f. v = 20 – 1 = 19	B1	
	$\frac{(n-1)s^2}{\sigma^2} = \frac{(20-1)\times 4}{1.6^2}$	M1	Attempts to calculate value for test
	= 29.6875	A1	AWRT 29.7
	X^2_{19}(0.025) = 8.907< 29.6875 < X^2_{19} (0.975) = 32.852	M1	Their 29.7 compared to both critical values
	Accept H_0 Evidence to suggest that standard deviation is 1.6g or the business' claim is true	E1ft	Accepts null hypothesis and makes comment in context ft their test value Must not be definite
	Total	**6**	

Q	Answer	Mark	Comments
3 (a)	$M_{X_i}(t) = E(e^{tx_i}) = \sum_{x_i=0}^{\infty} e^{tx_i} \frac{e^{-\lambda_i}\lambda_i^{x_i}}{x_i!}$	M1	Expresses mgf as infinite summation Accept $\sum_{x}$ or $\sum$ provided no finite series appears in working Condone $\sum_{x_i=1}^{\infty}$
	$= e^{-\lambda_i} \sum_{x_i=0}^{\infty} \frac{(\lambda_i e^t)^{x_i}}{x_i!}$	m1	Rearranges summation into the form of the Maclaurin series of e^x
	$= e^{-\lambda_i} e^{\lambda_i e^t}$	m1	Recognises Maclaurin series
	$= e^{\lambda_i (e^t - 1)}$	A1	cso
3(b)	$M_{X_1+X_2}(t) = M_{X_1}(t) M_{X_2}(t)$ $= e^{2(e^t-1)} e^{3(e^t-1)}$	M1	Multiplies together the mgfs of X_1 and X_2
	$= e^{5(e^t-1)}$	A1	
3(c)	Poisson distribution with mean/parameter 5	B1ft	ft their (b)
	Total	**7**	

Q	Answer	Mark	Comments
4 (a)	$\mu = 23.5$	B1	
4(b)(i)	(10, 10), (10, 20), (10, 50), (20, 20), (20, 50), (50, 50)	B1	Finds all possible combinations Ignore order
	(10, 10), (10, 20), (20, 10), (10, 50), (50, 10), (20, 20), (20, 50), (50, 20), (50, 50)	B1	All 9 possible samples
4(b)(ii)	$\bar{x}$ = 10, 15, 20, 30, 35, 50	B1	All 6 possible means identified
	$P(\bar{X} = 10) = 0.25^2$ or $P(\bar{X} = 20) = 0.55^2$ or $P(\bar{X} = 50) = 0.2^2$	M1	Method to find correct probability for $\bar{x}$ = 10, 20 or 50 or for combinations (10, 10), (20, 20) or (50, 50)
	$P(\bar{X} = 15) = 2 \times 0.25 \times 0.55$ or $P(\bar{X} = 30) = 2 \times 0.25 \times 0.2$ or $P(\bar{X} = 35) = 2 \times 0.55 \times 0.2$	M1	Method to find correct probability for $\bar{x}$ = 15, 30 or 35 or for combinations (10, 20), (10, 50) or (20, 50) for each order
	$P(\bar{X} = 10) = 0.0625$ $P(\bar{X} = 15) = 0.275$ $P(\bar{X} = 20) = 0.3025$ $P(\bar{X} = 30) = 0.1$ $P(\bar{X} = 35) = 0.22$ $P(\bar{X} = 50) = 0.04$	A2,1	Fully specified distribution Function or table A2 all correct A1 for 3 correct
4(b)(iii)	0.0625 x 10 + 0.275 x 15 + 0.3025 x 20 + 0.1 x 30 + 0.22 x 35 + 0.04 x 50	M1	ft on their probability distribution
	= 23.5	A1	CAO NMS scores 2/2
	Total	**10**	

<table>
<tr><th>Q</th><th>Answer</th><th>Mark</th><th>Comments</th></tr>
<tr><td>5</td><td>H_0: Accidents per day have a Poisson distribution
H_1: Accidents per day don't have a Poisson distribution</td><td>B1</td><td>Both hypotheses
Variable must be mentioned in at least the null hypothesis</td></tr>
<tr><td></td><td>$\hat{\lambda} = 1.46$</td><td>B1</td><td>Estimates λ (oe)</td></tr>
<tr><td></td><td>$100 \times \dfrac{e^{-1.46} \times 1.46^x}{x!}$ for x = 0, 1, 2 or 3</td><td>M1</td><td>Method to calculate at least one expected value for 0, 1, 2 or 3 using their $\hat{\lambda}$
Implied by one correct expected value</td></tr>
<tr><td></td><td><table><tr><th>Accidents per day</th><th>Expected</th></tr><tr><td>0</td><td>23.2</td></tr><tr><td>1</td><td>33.9</td></tr><tr><td>2</td><td>24.8</td></tr><tr><td>3</td><td>12.0</td></tr><tr><td>4 or more</td><td>6.1</td></tr></table></td><td>A1</td><td>AWRT but accept 12.05 for 12.0</td></tr>
<tr><td></td><td>$\sum \dfrac{(O-E)^2}{E} = \dfrac{(22-23.2)^2}{23.2} + \dfrac{(30-33.9)^2}{33.9} + \dfrac{(33-24.8)^2}{24.8} + \dfrac{(13-12)^2}{12} + \dfrac{(2-6.1)^2}{6.1}$</td><td>m1</td><td>Attempts to calculates χ^2-test statistic with their expected values
At least three terms required or implied by correct answer</td></tr>
<tr><td></td><td>= 6.06 to 6.07</td><td>A1</td><td>AWFW</td></tr>
<tr><td></td><td>d.o.f. v = 3</td><td>M1</td><td>Attempts to find d.o.f. as 3 or 4 (PI)</td></tr>
<tr><td></td><td>$X^2_{5\%} = 7.815$</td><td>A1</td><td></td></tr>
<tr><td></td><td>7.815 > 6.06</td><td>M1</td><td>Compares their test statistic with their critical value
or compare p = 0.1087 with 0.05</td></tr>
<tr><td></td><td>Accept H_0 so there is some evidence to suggest/support that Devon's claim, that the number of accidents per day follows a Poisson distribution is true</td><td>E1ft</td><td>Accepts null hypothesis and makes comment in context
ft their test value and critical value
Must not be definite</td></tr>
<tr><td></td><td>Total</td><td>10</td><td></td></tr>
</table>

Q	Answer	Mark	Comments
6 (a)	Var$\left(\frac{\sum_{i=1}^{n} X_i}{n}+\frac{1}{n}\right)$ = Var($\overline{X}$)	M1	Attempts to find variance
	= $\frac{\sigma^2}{n}$	A1	
	$\frac{\sigma^2}{n}\to 0$ as n$\to\infty$	m1	Attempts to find variance as n tends to infinity
	So estimator is consistent	A1	cso Conclusion required
6(b)(i)	E$\left(X_i^2\right)$ = Var $\left(X_i^2\right)$ + (E$\left(X_i\right)$)^2	M1	Attempts to find E$\left(X_i^2\right)$ Must see Var $\left(X_i^2\right)$ and E$\left(X_i\right)$
	= $\sigma^2+\mu^2$	A1	
6(b)(ii)	E$\left(\overline{X}^2\right)$ = Var $\left(\overline{X}\right)$ + (E$\left(\overline{X}\right)$)^2	M1	Attempts to find E$\left(\overline{X}^2\right)$ Condone E$^2\left(\overline{X}\right)$ for (E$\left(\overline{X}\right)$)^2
	= $\frac{\sigma^2}{n}+\mu^2$	A1	
	E$\left(\frac{\sum_{i=1}^{n} X_i^2}{n}-\overline{X}^2\right)$ = $\frac{1}{n}$E$\left(\sum_{i=1}^{n} X_i^2\right)$ − E$\left(\overline{X}^2\right)$	M1	Attempts to find E$\left(\frac{\sum_{i=1}^{n} X_i^2}{n}-\overline{X}^2\right)$
	= $\frac{n}{n}\left(\sigma^2+\mu^2\right)-\left(\frac{\sigma^2}{n}+\mu^2\right)$	m1	Substitutes expressions for E$\left(X_i^2\right)$ and E$\left(\overline{X}^2\right)$
	= $\sigma^2-\frac{\sigma^2}{n}$ or $\frac{n-1}{n}\sigma^2$ ≠ σ^2 so biased	A1	Obtains expression in terms of σ and n and concludes biased as not equal to σ^2
	Total	**11**	

Q	Answer	Mark	Comments
7(a)	$\bar{x}$ = 899.8	B1	Accept 4499/5 oe
	z = 1.96	B1	May be implied by correct answer
	$899.8 \pm 1.96\dfrac{8}{\sqrt{5}}$	M1	Constructs confidence interval with their mean and z value Implied by correct answer
	(893, 907)	A1	AWRT
7(b)	$1.96 \times \dfrac{8}{\sqrt{n}} = 2.5$	M1	Forms correct equation or inequality for their z and σ values Accept equivalent equations
	$\sqrt{n} = \dfrac{1.96 \times 8}{2.5} (= 6.272)$	m1	Rearranges to find $\sqrt{n}$ for their equation May be unsimplified PI
	n = 39.3 or n = 40	A1	
	[40 – 5 =] 35	A1	CAO
7(c)(i)	$s^2 = \dfrac{1}{4}\left(4048993 - \dfrac{4499^2}{5}\right)$	M1	Attempts to calculate s^2 or s
	s^2 = 198.2 or s = 14.1 AWRT	A1	Accept s^2 = 991/5 oe
	t_4 = 2.776	B1	May be implied by correct answer
	$899.8 \pm 2.776\sqrt{\dfrac{198.2}{5}}$	M1	Constructs confidence interval with their mean, sample variance and t value (must be different from 1.96) Implied by correct answer
	(882, 917)	A1	AWRT
7(c)(ii)	890 is outside the confidence interval in part (a) but inside the confidence interval in part (c)	M1	Comment relating to confidence intervals found in parts (a) and (c) ft their intervals
	They don't agree or Sean rejects the null hypothesis but Millie accepts it	A1ft	Comment on Sean's and Millie's conclusions ft their intervals
	Total	**15**	

Q	Answer	Mark	Comments
8(a)	H_0: $\mu_A = \mu_B$ H_1: $\mu_A < \mu_B$	B1	Both hypotheses
	$s_p^2 = \dfrac{(5-1)\times 0.2^2 + (7-1)\times 0.24^2}{5+7-2}$	M1	Apply formula for
	s_p^2 = 0.05056 or s_p = 0.225 AWRT	A1	Accept s_p^2 = 158/3125 oe
	d.o.f. v = 10	B1	
	$\dfrac{\lvert 6-5.6\rvert}{\sqrt{0.05056\left(\frac{1}{5}+\frac{1}{7}\right)}}$	M1	Correct numerator
	$\dfrac{\lvert 6-5.6\rvert}{\sqrt{0.05056\left(\frac{1}{5}+\frac{1}{7}\right)}}$	M1	Correct denominator for their s_p^2
	= ±3.04 AWRT	A1	Sign should be consistent with t value Implied by p = 0.00625
	t_{10} = 2.764 < 3.04	M1	Compares test statistic with critical value ft their d.o.f. or compares p value 0.00625 with 0.01
	Reject H_0 Evidence to suggest that Beth's claim is true that machine B produces more chocolate per hour than machine A	E1ft	Rejects null hypothesis and makes comment in context ft their test value and critical value Must not be definite
8(b)	That the population variances for machine A and machine B are equal	E1	Accept $\sigma_A^2 = \sigma_B^2$ oe

8(c)	H_0: $\sigma_A^2 = \sigma_B^2$ or H_0: $\sigma_A = \sigma_B$ H_1: $\sigma_A^2 \neq \sigma_B^2$ H_1: $\sigma_A \neq \sigma_B$	B1	Both hypotheses
	$\frac{s_B^2}{s_A^2} = \frac{0.24^2}{0.2^2}$ or $\frac{s_A^2}{s_B^2} = \frac{0.2^2}{0.24^2}$	M1	Attempts to calculate test statistic
	= 1.44 or AWRT 0.69	A1	AWRT
	d.o.f.s $v_B = 6$ and $v_A = 4$	B1	
	$F_{6,4}$ at 95% = 6.16 > 1.44 or $F_{4,6}$ at 5% = $\frac{1}{6.16}$ = AWRT 0.16 < 0.69	M1	Compares test statistic with critical value ft their d.o.f.s or compares p value = AWRT 0.377 with 0.05) Ignore values given for the other side of the interval
	Accept H_o Evidence to suggest that population variances of machine A and machine B are equal	E1ft	Accepts null hypothesis and makes comment in context ft their probability Must not be definite
	Total	**16**	