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FM02 Further Pure Mathematics Unit FPSM1

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Mark	Comments

Q	Answer	Mark	Total	Comments
1	$hf(x,y) = 0.05\sqrt{5 + 2 \times 7.2}$ $= 0.220227$	M1	5	PI
	$y_2 = 7.2 + 0.220227 = 7.420227$	A1ft		7.2 + their value of $hf(x,y)$
	y_3 $= 7.420227 + 0.05\sqrt{5.05 + 2 \times 7.420227}$ $(= 7.64322)$	m1 A1ft		
	7.6432	A1		CAO
Total	**5**			

Q	Answer	Marks	Total	Comments
2(a)	$\log_{10} y = \log_{10} a + \log_{10} b^x$ $Y = \log_{10} a + x\log_{10} b$ This is a linear relationship between Y and x.	M1 A1 E1	3	
2(b)	$(1.43136 =) \log_{10} 27 = \log_{10} a + 3\log_{10} b$ and $(1.07918 =) \log_{10} 12 = \log_{10} a + 8\log_{10} b$ $0.35218 = -5\log_{10} b$ $b = 0.850$ $a = 43.9$	M1A1 M1 A1 A1	5	For forming two equations For eliminating 1 unknown A1A0 for b = awrt 0.85 and a = awrt 44
Total 8				

Q	Answer	Mark	Comments

3(a)(i)	Stretch parallel to y-axis Scale factor 2	B1 B1	2	
3(ii)	Enlargement Scale factor 4	B1 B1	2	
3(iii)	reflection In the line $y = x$	B1 B1	2	
3(b)	$\mathbf{M} = \mathbf{BA}$ $\begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix}$	B1 B1	2	PI
3(c)	Det($\mathbf{M}$) = 32 Det($\mathbf{M}$) × 1.5 48	M1 M1 A1ft	3	
Total 11				

Q	Answer	Marks	Total	Comments
4(a)	f(4.3) = -5.95 and f(4.4) = 11.69 Change of sign and f is continuous on the interval (4.3, 4.4) so α is in the interval.	M1 A1	2	
4(b)	f(4.35) = 2.63; positive so α < 4.35 f(4.325) = -1.72; negative so α > 4.325 α lies between 4.325 and 4.350	M1 A1 A1	3	
4(c)	$f'(x) = 4x^3 - 34x - 5$ $x_2 = 4.3 - \frac{-5.95}{166.828}$ $x_2 = 4.336$ $x_3 = 4.335$	B1 M1 M1 A1 A1	5	PI M1 for numerator in correct form, possibly unsimplified (PI) M1 for denominator in correct form, possibly unsimplified (PI) 4.335664876 4.334972043
Total 10				

Q	Answer	Mark	Comments
5(a)	$\mathbf{D^T} = \begin{pmatrix} -4 & 2 & -3 \\ 3 & 10 & -15 \\ -1 & -11 & 5 \end{pmatrix}$	M1	
	$\mathbf{CD^T} = \begin{pmatrix} -23 & 0 & 0 \\ 0 & -23 & 0 \\ 0 & 0 & -23 \end{pmatrix}$	M1	3
	= -23 $\mathbf{I}$	A1	-23 $\mathbf{I}$ without sight of $\mathbf{D^T}$ scores 1 mark
5(b)	$\mathbf{CD^T} = (\mathbf{CD^T})^\mathbf{T}$ (as any values that would change position equal zero.)	E1	3
	$(\mathbf{CD^T})^\mathbf{T} = (\mathbf{D^T})^\mathbf{T}\mathbf{C^T}$	E1	for using $(\mathbf{AB})^\mathbf{T} = \mathbf{B^TA^T}$
	$= \mathbf{DC^T}$	E1	
Total 6			

Q	Answer	Mark	Comments
6	P(J) = $\frac{1}{5}\times\frac{2}{5}+\frac{1}{5}\times\frac{3}{10}+\frac{3}{5}\times\frac{7}{10}$	M1	Correct attempt to find P(J) Implied by sight of 0.56 or $\frac{14}{25}$
	P(C|J) = $\frac{P(J\mid C)P(C)}{P(J\mid Y)P(Y)+P(J\mid C)P(C)+P(J\mid T)P(T)}$ = $\frac{\frac{1}{5}\times\frac{2}{5}}{\frac{1}{5}\times\frac{2}{5}+\frac{1}{5}\times\frac{3}{10}+\frac{3}{5}\times\frac{7}{10}}$	M1	Apply Bayes Theorem
	= $\frac{1}{7}$ oe	A1	Do not accept decimal unless it is $0.\dot{1}42857\dot{}$
	Total	**3**	

Q	Answer	Mark	Comments
7	$\rho_{XY} = \dfrac{\text{cov}(X,Y)}{\sqrt{\text{var}(X)\,\text{var}(Y)}}$ $0.78 = \dfrac{\text{cov}(X,Y)}{\sqrt{2\times 8}}$	M1	Substitute values into formula for ρ_{XY}
	cov (X, Y) = 3.12 or $\frac{78}{25}$ oe	A1	PI. Find cov(X, Y)
	Var (X + Y) = Var (X) + Var (Y) + 2 cov (X, Y) = 2 + 8 + 2 × 3.12	M1	Substitute values into formula for Var (X + Y)
	= 16.24 or $\frac{406}{25}$ oe	A1	
	Total	**4**	

Q	Answer	Mark	Comments
8(a)	$\frac{1-p}{p^2} = 6$	M1	Forms equation using formula for var (X)
	$0 = 6p^2 + p - 1$ $0 = (2p + 1)(3p - 1)$	M1	Solve quadratic Implied by p = 1/3 or -1/2
	$p = \frac{1}{3}$	A1	Loses mark if $p = -\frac{1}{2}$ not discounted
8(b)	$\frac{1}{\left(\frac{1}{3}\right)} = 3$	B1ft	ft their p from (a) if $0 < p < 1$ Loses mark if second answer given
8(c)	$P(X \leq 2) = 1 - \left(1-\frac{1}{3}\right)^2$ or $\frac{1}{3}+\frac{2}{9}$	M1	Applies formula for cdf of geometric with their p from (a) if $0 < p < 1$ or adds P(X = 1) and P(X = 2)
	$= \frac{5}{9}$ oe	A1	Accept 0.556 AWRT
	Total	**6**	

Q	Answer	Mark	Comments
9(a)	$\frac{5}{8}+\frac{3}{16}$	M1	Identifies and attempts to add correct probabilities
	0.8125 or $\frac{13}{16}$ oe	A1	
9(b)(i)	$G'_Y(t) = \frac{1}{16}+\frac{10}{8}t+\frac{9}{16}t^2$	M1	Correct expression for $G'_Y(t)$
	Mean = $G'_Y(1)$ = 1.875 or $\frac{15}{8}$ oe	A1	Correct mean
9(b)(ii)	$G''_Y(t) = \frac{10}{8}+\frac{18}{16}t$	B1ft	ft their $G'_Y(t)$
	Variance = $G''_Y(1) + \mu - \mu^2$ $= \frac{10}{8}+\frac{18}{16} + 1.875 - 1.875^2$	M1	Substitutes values into formula for variance
	= 0.734375 or $\frac{47}{64}$ oe	A1	
	Total	**7**	

Q	Answer	Marks	Total	Comments

10	$\boldsymbol{r_A}=(2\mathbf{i}+3\mathbf{j})t$ $\boldsymbol{r_B}=(\mathbf{3i}-\mathbf{4j})t+z\mathbf{j}$ $\boldsymbol{r_B}-\boldsymbol{r_A}=\mathrm{t}\boldsymbol{i}+(z-7t)\mathbf{j}$ $s^2=t^2+(z-7t)^2$ $s^2=50t^2-14zt+z^2$ $\frac{d(s^2)}{dt}=100t-14z$ For min distance $t=\frac{7z}{50}$ $\frac{7z}{50}=28$ $z=200$	M1 A1 M1 A1 A1	5	
		Total	5	

Q	Answer	Marks	Total	Comments
11	$[v]=[M]^a\times[r]^b\times[G]^c$ $LT^{-1}=M^aL^bL^{3c}M^{-c}T^{-2c}$ $0=a-c$ $1=b+3c$ $-1=-2c$ $c=\frac{1}{2},b=-\frac{1}{2},a=\frac{1}{2}$	M1A1 M1 A1 A1	5	
		Total	5	

Q	Answer	Marks	Total	Comments
12(a)	$F(0.05)=\frac{k}{160000}$	M1A1	2	

12(b)	$I = \int_0^{0.1} kt^2(t-0.1)^2\,dt$ $= k\left[\frac{t^5}{5} - \frac{t^4}{20} + \frac{t^3}{300}\right]_0^{0.1}$ $= \frac{k}{3000000}$	M1 M1A1 A1	4	
12(c)	$v = 0.6 \times 4.8 = 2.88$ $\frac{k}{3000000} =$ $0.25 \times 2.88 - 0.25 \times (-4.8)$ $k = 1.92 \times 37500 = 5760000$ or 5800000	M1 M1A1 A1	4	Condone 1 wrong sign for M1
		Total	10	