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FURTHER MATHEMATICS

FM04

(9665/FM04) Unit FS2 Statistics

Mark scheme

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231XFM04/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$\chi^2 = \frac{s^2}{\sigma_0^2} \times (n-1) = \frac{100}{\sigma_0^2} \times 9$	**M1**	Use of correct statistic. **PI** Allow n for $n - 1$
	$\chi_9^2(0.975) = 19.023$	**B1**	Finds critical value
	$\sigma_0^2 > \frac{900}{19.023}$ [= 47.3111]	**M1**	Allow either >, ≥ or = **oe**
	$\sigma_0 = 6.878[31...]$	**A1**	Must show answer at least 4 sf or explicitly state as 6.88 to 3 sf **AG**
		4	

Q	Answer	Marks	Comments
1(b)	$\sigma_0^2 < \frac{900}{\chi_9^2(0.025)} = \frac{900}{2.700}$ [= 333.333]	**M1**	Allow either <, ≤ or = **oe** **ft** their degrees of freedom in **(a)**
	$\sigma_0 = 18.3$	**A1**	**AWRT** Allow truncation to 18.2
		2	

	Question 1 Total	**6**	

Q	Answer	Marks	Comments
2(a)	$3 \times 0.7^2 \times 0.3$ or 0.3^3	**M1**	**PI** or one value (of 0.441 or 0.027) correct
	<table><tr><td>v</td><td>15</td><td>**60**</td><td>**105**</td><td>150</td></tr><tr><td>P(V=v)</td><td>0.343</td><td>**0.441**</td><td>0.189</td><td>**0.027**</td></tr></table>	**B1** **A1**	Both 60 and 105 needed Both 0.441 and 0.027 needed
		3	

Q	Answer	Marks	Comments
2(b)(i)	$0.343 + 0.441[= 0.784]$ **or** $0.189 + 0.027[= 0.216]$	**M1**	**PI** **ft** their 0.441 or 0.027
	<table><tr><td>m</td><td>5</td><td>50</td></tr><tr><td>P($M = m$)</td><td>**0.784**</td><td>**0.216**</td></tr></table>	**A1**	
		2	

Q	Answer	Marks	Comments
2(b)(ii)	$\mathrm{E}(M) = 0.784 \times 5 + 0.216 \times 50[= 14.72]$ **or** $\mathrm{E}(M^2) = 0.784 \times 5^2 + 0.216 \times 50^2[= 559.6]$	**M1**	**PI** **ft** their **(b)(i)**
	$\text{Var}(M) = 559.6 - 14.72^2$	**M1**	Use of $\text{Var}(M) = \mathrm{E}(M^2) - (\mathrm{E}(M))^2$ **PI**
	$\text{Var}(M) = 343$	**A1**	**AWRT**
		3	

	Question 2 Total	**8**	

Q	Answer	Marks	Comments
3(a)	$\frac{27.8+30.4}{2} = 29.1$	B1	
		1	

Q	Answer	Marks	Comments
3(b)	Critical value $z=(\pm)1.96(00)$	B1	**AWRT** 1.96
	$30.4-27.8=2.6=2\times1.96\times\frac{\sqrt{6.6}}{\sqrt{n}}$	M1	Use of $\frac{\sqrt{6.6}}{\sqrt{n}}$ in an equation to find n
	$n=\frac{6.6\times1.96^2}{1.3^2}=15.0027$ so 15	A1	**AG CSO** Either value for n given to at least three significant figures or calculation for n with correct substitution must be seen
		3	

Q	Answer	Marks	Comments
3(c)	30 is in the confidence interval	B1	Condone use of "it" for 30
	Evidence that the **target** (of mean conference call of 30 minutes) has been met	E1	Must be in context
		2	

Q	Answer	Marks	Comments
3(d)	It is a **normal** distribution with **known** [population] variance	B2	1 mark for each feature (normal distribution, known variance)
		2	

	Question 3 Total	**8**	

Q	Answer	Marks	Comments
4(a)	$M_Z'(t) = t\,e^{\frac{1}{2}t^2}$	M1	Allow $ate^{\frac{1}{2}t^2}$
	$M_Z'(0) = 0 \times e^0 = 0$	A1	
	$M_Z''(t) = (1+t^2)e^{\frac{1}{2}t^2}$	M1	Of form $(a+bt^2)\,e^{\frac{1}{2}t^2}$ **oe**
	$\sigma^2 = M_Z''(0) - \mu^2$	M1	
	$= 1 - 0 = 1$	A1	
		5	

Q	Answer	Marks	Comments
4(b)	$M_X(t) = e^{at} \times e^{\frac{1}{2}(bt)^2}$	M1	Use of $M_X(t) = e^{at} \times M_Z(bt)$
	$M_X(t) = e^{at + \frac{1}{2}b^2t^2}$	A1	
		2	

Q	Answer	Marks	Comments
4(c)	$\mathrm{E}(X) = a$ **and** $\mathrm{Var}(X) = b^2$	B1	Both $\mathrm{E}(X)$ and $\mathrm{Var}(X)$ required
		1	

Q	Answer	Marks	Comments
4(d)	$e^{\mu t + \dots}$ or $e^{\dots + \frac{1}{2}\sigma^2 t^2}$	M1	
	$e^{\mu t + \frac{1}{2}\sigma^2 t^2}$	A1	
		2	

	Question 4 Total	**10**	

Q	Answer	Marks	Comments
5(a)	$\mathrm{E}\left(\overline{X}\right)=\frac{n\lambda}{n}=\lambda$ and $\mathrm{E}\left(\overline{Y}\right)=\frac{n\times 2\lambda}{n}=2\lambda$	B1	Both. **PI**
	$\mathrm{E}(S)=\frac{\lambda+2\lambda}{3}=\lambda$ or $\mathrm{E}(T)=2\lambda-\lambda=\lambda$	M1	Either found
	$\mathrm{E}(S)=\lambda$ and $\mathrm{E}(T)=\lambda$ so estimators are unbiased	A1	Statement and **both** estimators correct
		3	

Q	Answer	Marks	Comments
5(b)	$\text{Var}(S)=\left(\frac{1}{3}\right)^2\text{Var}\left(\overline{X}\right)+\left(\frac{1}{3}\right)^2\text{Var}\left(\overline{Y}\right)$	M1	Correct expression for Var(S) or Var(T) May be seen in **(c)**
	$\text{Var}(T)=\text{Var}\left(\overline{Y}\right)+\text{Var}\left(\overline{X}\right)$		
	$\text{Var}(S)=\frac{1}{9}\times\frac{\lambda}{n}+\frac{1}{9}\times\frac{2\lambda}{n}=\frac{\lambda}{3n}$	A1	**PI** May be seen in **(c)**
	$\text{Var}(T)=\frac{\lambda}{n}+\frac{2\lambda}{n}=\frac{3\lambda}{n}$	A1	**PI** May be seen in **(c)**
	Relative Efficiency $=\dfrac{\frac{1}{\text{Var}(S)}}{\frac{1}{\text{Var}(T)}}=\dfrac{\frac{3n}{\lambda}}{\frac{n}{3\lambda}}$	M1	**ft** their Var(S) and Var(T) **oe**
	[Relative Efficiency] = 9 [which is not a function of n, so the efficiency is independent of n]	A1	Answer of 9 is sufficient for award of mark
		5	

Q	Answer	Marks	Comments
5(c)	Var$(S) \to 0$ or Var$(T) \to 0$ as $n \to \infty$ so estimators are consistent	**M1** **A1**	Either may be shown from a function of n that tends to zero Conclusion required **CSO**
		2	

	Question 5 Total	**10**	

Q	Answer	Marks	Comments
6(a)	$\int_{100}^{t} -\frac{\pi}{200}\sin\left(\frac{\pi x}{100}\right)\mathrm{d}x$	M1	Must have correct limits
	$=\left[\frac{1}{2}\cos\left(\frac{\pi x}{100}\right)\right]_{100}^{t}$	M1	Integrand of form $a\cos\left(\frac{\pi x}{100}\right)$ **oe**
	$=\frac{1}{2}\cos\left(\frac{\pi t}{100}\right)-\frac{1}{2}\cos\left(\frac{100\pi}{100}\right)$		
	$\mathrm{F}(t)=\begin{cases}0 & t<100\\ \frac{1}{2}\cos\left(\frac{\pi t}{100}\right)+\frac{1}{2} & 100\le t\le 200\\ 1 & t>200\end{cases}$	A1	**AG** must see intermediate line with values substituted into integrand Limits for t need to be shown
		3	

Q	Answer	Marks	Comments
6(b)(i)	F(160) – F(140), F(180) – F(160) **or** F(200) – F(180) seen	M1	PI
	<table><tr><td>Interval</td><td>100-120</td><td>120-140</td><td>140-160</td><td>160-180</td><td>180-200</td></tr><tr><td>Sprints</td><td>164</td><td>430</td><td>**532**</td><td>**430**</td><td>**164**</td></tr></table>		
	Either of 430 or 164 seen All 532, 430, 164	A1 A1	Allow +/– 1 for both **A** marks
		3	

Q	Answer	Marks	Comments
6(b)(ii)	H_0 : Reaction times have the same distribution as T H_1 : Reaction times do not have the same distribution as T	**B1**	**oe**, eg H_0 : Suggested model is appropriate, Athletics trainer's claim is valid (condone true), Data fits given distribution Both hypotheses
	$\sum\frac{(O-E)^2}{E}=\frac{(145-164)^2}{164}+\frac{(390-430)^2}{430}$ $+\frac{(561-"532")^2}{"532"}+\frac{(470-"430")^2}{"430"}+\frac{(154-"164")^2}{"164"}$	**M1**	
	$=11.8$	**A1ft**	**ft** their **(b)(i)** given to 1 decimal place
	$v=5-1=\mathbf{4}$	**B1**	
	$\chi^2(0.99)=13.277$	**B1**	
	$11.8<13.277$, Do not reject $\mathbf{H_0}$	**A1ft**	
	Sufficient evidence to support the athletics trainer's claim	**E1ft**	Must not be definite; consistent with conclusion on $\mathbf{H_0}$
		7	

	Question 6 Total	**13**	

Q	Answer	Marks	Comments
7(a)(i)	The test is a two-tailed test.	**B1**	
		1	

Q	Answer	Marks	Comments
7(a)(ii)	$z = \dfrac{53.4 - 45 - 10}{\sqrt{\left(\dfrac{6^2}{60} + \dfrac{4^2}{80}\right)}}$	**M1** **M1**	Correct numerator Correct denominator
	$= -1.788(85..)$	**A1**	**AWRT** –1.79 $p = 0.0736$
	$z_{\text{crit}} = +/-1.9600$	**B1**	**AWRT** 1.96
	–1.7889 > –1.9600 Do not reject H_0	**A1ft**	Follow through their z and z_{crit}
	Sufficient evidence to suggest that the mean length of Galapagos penguins is 10 cm more than that of Fairy penguins	**E1**	Gives a conclusion in context based on a comparison of the correct test statistic and correct critical value Condone definite conclusion
		6	

Q	Answer	Marks	Comments
7(b)	The result is **valid** as the sample is sufficiently **large** to use a normal approximation for the mean (Central Limit Theorem)	**E1**	**oe** must clearly state validity with reason Condone “can use” **oe**
		1	

	Question 7 Total	**8**	

Q	Answer	Marks	Comments
8	$z = 1.6449$	**B1**	**AWRT** 1.645
	$\overline{X_c} = 100 + 1.6449 \times \frac{10}{\sqrt{30}}$ $\left[= 103.00316 \quad \Rightarrow \quad \text{Acceptance region: } \bar{X} < 103\right]$	**M1**	
	$P\left(\overline{X} < 103 \mid \mu\right) \le 0.05$	**m1**	**PI** Condone $<$ or = [μ is the population mean.]
	$103 < \mu - 1.6449 \times \frac{10}{\sqrt{30}}$	**m1**	Condone = Dependent on all previous method marks
	$\mu > 106.0(031)$	**A1**	**AG** Strict inequality sign required
		5	

	Question 8 Total	**5**	

Q	Answer	Marks	Comments
9(a)	Computer 1, 2, 3, 4, 5 — Difference –2.2, +8.1, **a–113.5**, –6.6, –2.5 and Computer 6, 7, 8, 9, 10 — Difference –7.6, +0.1, +4.0, –4.2, +1.2	M1	Attempt differences; allow 1 mistake, allow negative of table values PI
	$\bar{d} = \dfrac{-123.2 + a}{10} = 0.1a - 12.32$	B1	Allow negative, 12.32 – 0.1a
	$\sum d^2 = 213.11 + (113.5 - a)^2$ $(= a^2 - 227a + 13095.36)$ $s^2 = \dfrac{1}{10-1}\left(\sum d^2 - 10\bar{d}^2\right)$	M1	Allow $a^2 - ba + c$, with b and c positive values
	$= \dfrac{1}{9}\left(11577.536 - 202.36a + 0.9a^2\right)$		
	$= 0.1a^2 - 22.48\dot{4}a + 1286.392\dot{8}$	A1	**oe**
	$t = \dfrac{\bar{d}}{\left(\dfrac{s}{\sqrt{10}}\right)} = \dfrac{0.1a - 12.32}{\sqrt{\dfrac{0.1a^2 - 22.48\dot{4}a + 1286.392\dot{8}}{10}}}$	M1	**ft** with their mean and variance Allow $-t$
	$t = \dfrac{\sqrt{10}\,(0.1a - 12.32)}{\sqrt{0.1a^2 - 22.48a + 1286}}$	A1	**AG** Must be convincingly shown
		6	

Q	Answer	Marks	Comments
9(b)	$H_0: \mu_{new} = \mu_{old}$ $H_1: \mu_{new} < \mu_{old}$	**B1**	**oe**
	$t = -1.23(1\ldots.)$	**M1**	Correct substitution of $a = 91.8$ into formula Condone 1.23
	$\nu = 9$	**B1**	**PI**
	Critical value $t_9 = 1.383$	**B1**	
	$-1.23 > -1.383$, Do not reject H_0	**A1ft**	Allow $1.23 < 1.383$ **ft** their t and critical value
	Insufficient evidence to support the reduction in start-up times	**E1**	Gives a conclusion in context based on a comparison of the correct test statistic and correct critical value Condone definite conclusion
		6	

	Question 9 Total	**12**	