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(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

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231XFM02/MS

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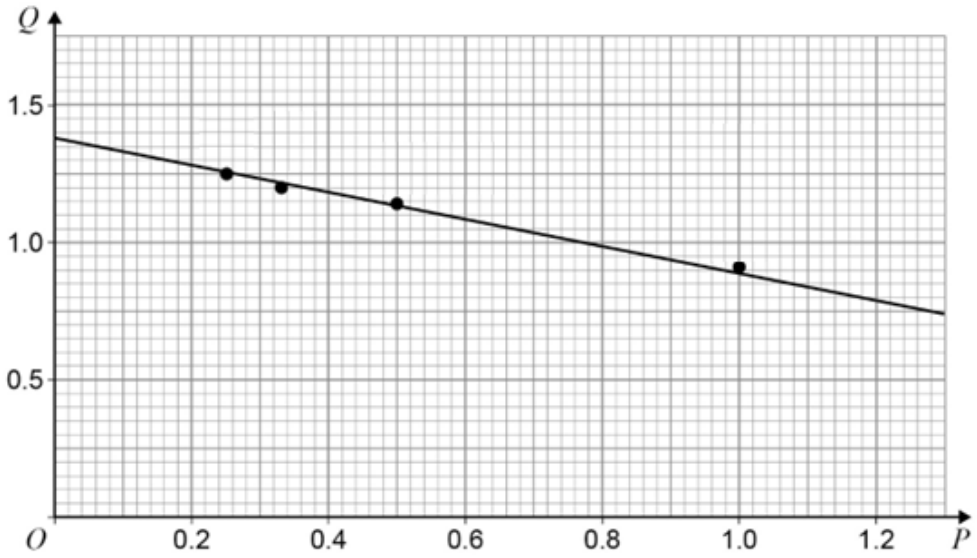
Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1	$hf(x,y) = 0.1\times\left(3\times 1+\frac{2\times 1^3}{-1}\right)$ $= 0.1$ $y_2 = -1+0.1$ $= -0.9$ $y_3 = -0.9+0.1\times\left(3\times 1.1+\frac{2\times 1.1^3}{-0.9}\right)$ $= -0.9+\frac{77}{2250}$ $= -0.9+0.03422$ $= -0.8657778$ $= -0.8658$	**M1** **A1** **M1** **A1**	 correct use of formula **PI** by values to at least 5 decimal places or equivalent eg $y_3 = \frac{-974}{1125}$ **CAO**
		4	

	Question 1 Total	**4**	

Q	Answer	Marks	Comments
2(a)	P | 1 | 0.5 | 0.33 | 0.25 Q | **0.91** | **1.14** | **1.20** | **1.25**	B1	**CAO** Condone 1.2 for 1.20
		1	

Q	Answer	Marks	Comments
2(b)	Their points plotted correctly Line of best fit drawn	B1ft B1ft	All their points plotted ± 0.5 squares Their line of best fit drawn
			
		2	

Q	Answer	Marks	Comments
2(c)(i)	b = their intercept b = 1.4 a = their gradient a = –0.45	B1ft B1ft	Intercept must be from a suitable line of best fit for their points Gradient must be from a suitable line of best fit for their points
		2	

Q	Answer	Marks	Comments
2(c)(ii)	$\frac{1}{y} = \frac{-0.45}{x} + 1.4$	**B1ft**	alternative forms accepted eg $y = \frac{x}{1.4x - 0.45}$ **ft** their values of a and b
		1	

Q	Answer	Marks	Comments
2(d)	$y = \frac{1.6}{1.4 \times 1.6 - 0.45}$ $= 0.89$	**B1ft**	**ft** their values of a and b
		1	

	Question 2 Total	**7**	

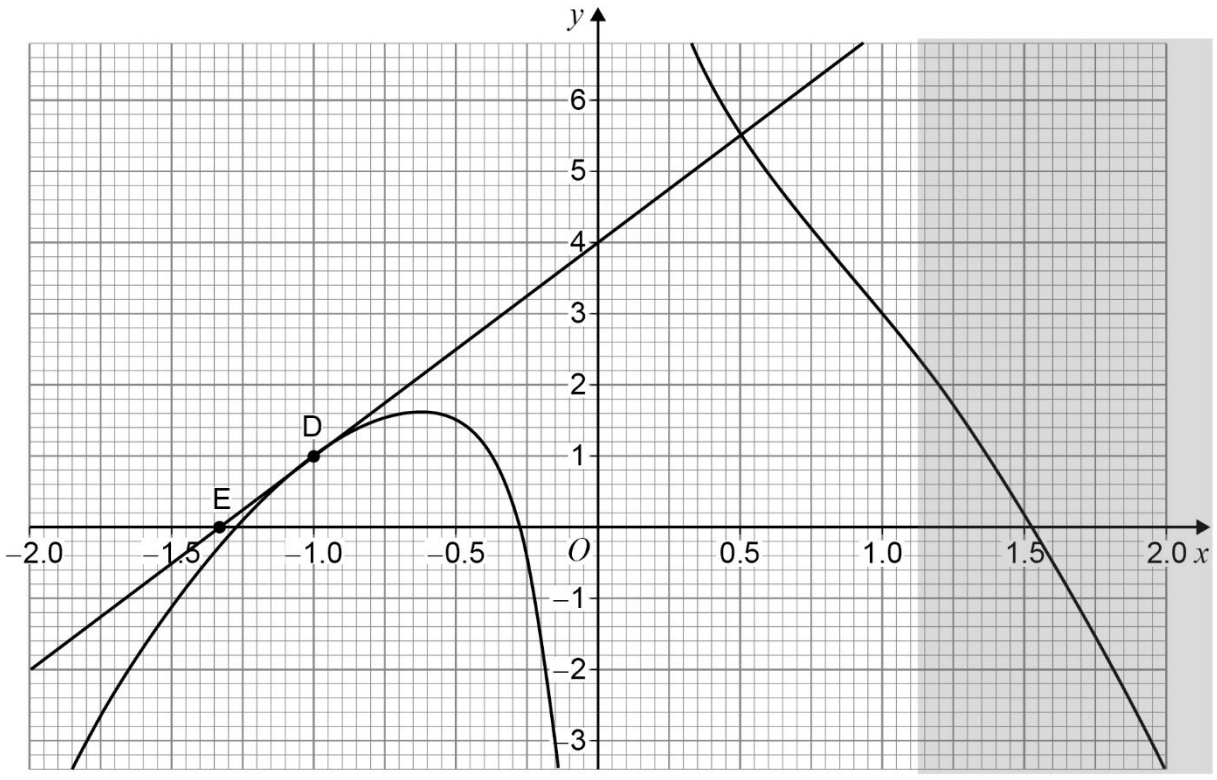
Q	Answer	Marks	Comments
3(a)	$\begin{bmatrix} 4+4k & 24k & -3+12k \\ 0 & 33 & 12 \\ 4 & 6 & 9 \end{bmatrix} = \begin{bmatrix} 20k & 24k & 0 \\ 0 & 33 & 12 \\ 4 & 6 & 9 \end{bmatrix}$	**M1**	equating their obtained matrix with the **A**+4**B** and using at least one expression to evaluate k
	$4 = 16k$ or $-3+12k = 0$ $k = \frac{1}{4}$	**A1**	**CSO**
		2	

Q	Answer	Marks	Comments
3(b)(i)	$\begin{bmatrix} 4 & 0 & -3 \\ -2 & 1 & -2 \\ 4 & -2 & 5 \end{bmatrix}\begin{bmatrix} 0.25 & 1.5 & 0.75 \\ 0.5 & 8 & 3.5 \\ 0 & 2 & 1 \end{bmatrix}$ $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	**M1** **A1ft**	showing correct multiplication allowing for **ft** with their value of k **ft** their value of k
		2	

Q	Answer	Marks	Comments
3(b)(ii)	$\mathbf{C}\begin{bmatrix} 1 & 6 & 3 \\ 2 & 32 & 14 \\ 0 & 8 & 4 \end{bmatrix} = \mathbf{AB}$ $\mathbf{C}(4\mathbf{B}) = \mathbf{AB}$ $\mathbf{C} = \frac{1}{4}\mathbf{A}$ $= \begin{bmatrix} 1 & 0 & -0.75 \\ -0.5 & 0.25 & -0.5 \\ 1 & -0.5 & 1.25 \end{bmatrix}$	**M1** **A1**	equating $\begin{bmatrix} 1 & 6 & 3 \\ 2 & 32 & 14 \\ 4 & 6 & 9 \end{bmatrix} = 4\mathbf{B}$ **or** $\mathbf{C}\begin{bmatrix} 1 & 6 & 3 \\ 2 & 32 & 14 \\ 4 & 6 & 9 \end{bmatrix} =$ their **AB** correct Matrix for **C**
		2	

	Question 3 Total	**6**	

Q	Answer	Marks	Comments
4(a)	$f(1) = 3$ $f(2) = -3.5$ As there has been a change of sign between $x = 1$ and $x = 2$, and as the curve is continuous [on this interval], then there is a root $1 < \gamma < 2$	**M1** **A1**	Correct evaluation of a suitable interval Must state that there is a change of sign and that the curve is continuous (condone unbroken) and concludes a root is present in the interval
		2	

Q	Answer	Marks	Comments
4(b)	Tangent drawn at $x = -1$ The next approximation for a root is closer to α rather than β [see diagram below]	**B1** **E1**	 Gives justification that using $x_1 = -1$ will converge to the root α where $-2 < \alpha < -1$
			
		2	

Q	Answer	Marks	Comments
4(c)	$f(x) = \frac{1}{x} - 2x^2 + 4$ $f'(x) = \frac{-1}{x^2} - 4x$	**M1**	correct derivative
	$f'(-1) = \frac{-1}{(-1.3)^2} - 4(-1.3)$ $= 4.60828402$	**A1**	**PI** **AWRT** 4.608
	$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ $x_2 = -1.3 - \frac{\frac{1}{-1.3} - 2(-1.3)^2 + 4}{\frac{-1}{(-1.3)^2} - 4(-1.3)}$ $= -1.3 - \frac{\frac{1}{-1.3} - 2(-1.3)^2 + 4}{4.60828402}$ $= -1.3 + 0.0323832$ $= -1.267616846$	**M1**	correctly substituting into formula or obtaining correct value for $\frac{f(x)}{f'(x)}$
	$= -1.2676$	**A1**	correct answer to 4 dp
		4	

	Question 4 Total	**8**	

Q	Answer	Marks	Comments
5(a)(i)	$\begin{bmatrix}\cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta\end{bmatrix}\begin{bmatrix}2\\0\end{bmatrix}=\begin{bmatrix}-1\\p\end{bmatrix}$ $\Rightarrow 2\cos 2\theta = -1$ and $2\sin 2\theta = p$ $\Rightarrow \cos 2\theta = -\frac{1}{2}$ $\cos^2 2\theta + \sin^2 2\theta = 1$ $\Rightarrow \left(\frac{-1}{2}\right)^2 + \left(\frac{p}{2}\right)^2 = 1$ $\Rightarrow \left(\frac{p}{2}\right)^2 = \frac{3}{4}$ $\Rightarrow p = \pm\sqrt{3}$ but $p < 0$ $\therefore p = -\sqrt{3}$	**M1** **M1** **A1**	Obtains correct expressions containing $p, \cos 2\theta$ and $\sin 2\theta$ **PI** Eliminates θ to gain an equation to find p or finds $2\theta = 240°$ **oe** **CSO**
		3	

Q	Answer	Marks	Comments
5(a)(ii)	$\mathbf{M} = \begin{bmatrix}-\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix}$	**B1ft**	**ft** their value of p or θ
		1	

Q	Answer	Marks	Comments
5(a)(iii)	$\cos 2\theta = -\frac{1}{2}$ and $\sin 2\theta = -\frac{\sqrt{3}}{2}$ $\tan 2\theta = \sqrt{3}$ $2\theta = 240^\circ$ $\theta = 120^\circ$	**M1**	**PI** attempts to find the angle for the equation of the line of reflection
	$y = \tan 120^\circ x$ or $y = -\sqrt{3}x$	**A1ft**	**ft** their values from **(a)(ii)**
	Reflection in the line $y = -\sqrt{3}x$	**A1**	describing fully with equation of the line of reflection **CSO**
		3	

Q	Answer	Marks	Comments
5(b)(i)	$\begin{bmatrix} c & d \\ d & -c \end{bmatrix}\begin{bmatrix} -1 \\ -\sqrt{3} \end{bmatrix} = \begin{bmatrix} -1 \\ -\sqrt{3} \end{bmatrix}$	**M1**	**PI** obtains correct matrix equation could be in terms of p **ft** their value of p
	$\Rightarrow -c - \sqrt{3}d = -1$ and $\Rightarrow -d + \sqrt{3}c = -\sqrt{3}$	**A1ft**	**PI** obtains both correct simultaneous equations could be in terms of p **ft** their value of p
	$d = \frac{\sqrt{3}}{2}$ **and** $c = -\frac{1}{2}$	**A1**	
		3	

Q	Answer	Marks	Comments
5(b)(ii)	$\mathbf{NM} = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$	M1	using correct order of multiplication **NM** using their c and d and their **M**
	$= \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$	A1	**CSO**
		2	

Q	Answer	Marks	Comments
5(b)(iii)	$\Rightarrow \cos\ \theta = -\frac{1}{2}$ and $\sin\ \theta = -\frac{\sqrt{3}}{2}$ $\theta = 240^{\circ}$	M1	Correct method to find θ
	Single transformation is a rotation about the origin of 120° clockwise	B1 A1	Identifies transformation as rotation Full description **oe** such as 240° [anticlockwise]
		3	

	Question 5 Total	**15**	

Q	Answer	Marks	Comments
6(a)	$G_X(t) = 0.008 + 0.096t + 0.384t^2 + 0.512t^3$	M1	expands $G_X(t)$ or uses chain rule to differentiate to $k(0.2 + 0.8t)^2$
	$G'_X(t) = 0.096 + 0.768t + 1.536t^2$	A1	Obtains correct $G'_X(t)$ **oe**, chain rule gives $2.4(0.2 + 0.8t)^2$
	$G'_X(1) = 0.096 + 0.768(1) + 1.536(1)^2$	M1	attempts to find $G'_X(1)$
	$E(X) = G'_X(1) = 2.4$	A1	
		4	

Q	Answer	Marks	Comments
6(b)	$P(X \geq 2) = 0.384 + 0.512 = 0.896$	B1ft	**ft** their expanded $G_X(t)$, their 0.384 + their 0.512 **oe**
		1	

	Question 6 Total	**5**	

Q	Answer	Marks	Comments
7(a)	$\frac{1-p}{p^2} = 3.75$	M1	forms correct equation
	$3.75p^2 + p - 1 = 0$	M1	rearranges to three term quadratic = 0 **oe** **PI** by correct final answer
	$p = 0.4$	A1	if –2/3 seen, it must be rejected
		3	

Q	Answer	Marks	Comments
7(b)	$E(X) = 2.5$	B1ft	**ft** their $\frac{1}{p}$
		1	

Q	Answer	Marks	Comments
7(c)	$P(X \leq 5) = 1 - (1 - 0.4)^5$	M1	attempts to calculate (1 – their p)^5 **oe** **PI**
	$= 0.92224$	A1	**oe**
		2	

	Question 7 Total	**6**	

Q	Answer	Marks	Comments
8(a)	$E(X) = 4$	B1	oe, PI
	$E(Y) = 5.5$	B1	oe, PI
	$E(4X - 3Y) = 4E(X) - 3E(Y)$	M1	applies formula
	$E(4X - 3Y) = -0.5$	A1	
		4	

Q	Answer	Marks	Comments
8(b)	$Var(X) = 4$	B1	oe
	$Var(Y) = 8.25$	B1	oe
	$Cov(X, Y) = \sqrt{4 \times 8.25}\rho$	M1	express covariance in terms of ρ PI
	$10 = 4 + 8.25 + 2\sqrt{4 \times 8.25}\rho$	M1	form correct equation to find ρ oe
	$\rho = -0.196$	A1	AWRT
		5	

	Question 8 Total	**9**	

Q	Answer	Marks	Comments
9	$MLT^{-2} = [k]\left(LT^{-1}\right)^n$	M1 A1	**M1**: dimensions equation with at least one side correct Condone consistent use of units **A1**: correct dimensions equation Condone consistent use of units
	$MLT^{-2} = [k]L^nT^{-n}$ $[k] = ML^{1-n}T^{n-2}$	A1	correct dimensions for k
		3	

	Question 9 Total	3	

Q	Answer	Marks	Comments
10(a)	$I = 0.3\times 4 - 0.3\times(-6)$	M1	uses impulse equation condone sign errors
	$I = 3$ N s	A1	obtains correct magnitude must include units
		2	

Q	Answer	Marks	Comments
10(b)	$\frac{1}{2}\times 75T = 3$	M1	finds impulse in terms of T from graph and sets equal to their impulse
	$T = \frac{6}{75} = 0.08$	A1	correct T
		2	

	Question 10 Total	4	

Q	Answer	Marks	Comments
11	$-20+2t=t+80$	**M1**	forms equation to find the time when they meet **oe** condone sign errors
	$t=100$	**A1**	correct time
	$450-0.5\times100=150+100U$	**M1**	forms equation to find U **oe**
	$U=2.5$	**A1**	correct U
		4	

	Question 11 Total	**4**	

Q	Answer	Marks	Comments
12(a)	$7 \times 0.8 = 5.6$	B1	finds speed after collision with the wall
	$0.3 \times 2 + 0.5 \times (-5.6) = 0.3v_P + 0.5v_Q$ $-22 = 3v_P + 5v_Q$	M1 A1	equation for conservation of momentum condone sign errors correct equation
	$v_P - v_Q = -0.5(2 - (-5.6))$ $v_P - v_Q = -3.8$	M1	applies equation for restitution condone sign errors
	$v_P = -5.125$	A1	correct velocity for *P* or *Q* note: $v_Q = -1.325$
	$I = 0.3 \times (-5.125) - 0.3 \times (2)$ $= -2.1375$ N s	M1	applies impulse equation with their velocities condone sign errors
	$\lvert I \rvert = 2.14$ to 3 sf	A1	correct magnitude
		7	

Q	Answer	Marks	Comments
12(b)	$2.1375 = 0.075F$	M1	uses $I = Ft$ with their impulse
	$F = \dfrac{2.1375}{0.075} = 29$ N (to 2 sf)	A1	**AWRT** 29 N
		2	

	Question 12 Total	**9**	