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(9665/FM01) Unit FP1 Pure Mathematics

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$2\times\left(-\frac{5}{4}+h\right)^2+3\times\left(-\frac{5}{4}+h\right)$ $=2\left(\frac{25}{16}-\frac{5}{2}h+h^2\right)-\frac{15}{4}+3h$	M1	Substitutes $x=-\frac{5}{4}+h$ into the curve equation
	$=-\frac{5}{8}-2h+2h^2$	A1	Correct expansion and simplification **PI**
	$2\times\left(-\frac{5}{4}\right)^2+3\times\left(-\frac{5}{4}\right)=-\frac{5}{8}$ Gradient $=\dfrac{\left(-\frac{5}{8}-2h+2h^2\right)-\left(-\frac{5}{8}\right)}{\left(-\frac{5}{4}+h\right)-\left(-\frac{5}{4}\right)}$	M1	Correctly applies the gradient formula to find an expression for the gradient
	$=\dfrac{-2h+2h^2}{h}$ $=-2+2h$	A1	Accept $2h-2$
		4	

Q	Answer	Marks	Comments
1(b)	Gradient of curve $=\lim\limits_{h\to 0}(-2+2h)$	M1	
	$=-2$	A1ft	Follow through a linear answer in terms of h from **part (a)** **SC1** for using $h=0$ leading to gradient = their –2
		2	

	Question 1 Total	**6**	

Q	Answer	Marks	Comments
2	Other root = $-4-7\text{i}$	M1	Identifies the conjugate as the 2nd root
	Sum of roots = -8 Product of roots $=(-4+7\text{i})(-4-7\text{i})$	M1	Forms an equation using the sum or the product of the roots. Allow one sign error.
	$b=8$	A1	Correct value for b or c Accept a quadratic equation of the form $x^2+8x+c=0$ or $x^2+bx+65=0$
	$c=65$	A1	Correct values for b and c Accept $x^2+8x+65=0$
		4	

Q	Answer	Marks	Comments
2 ALT	$(-4+7\text{i})^2+b(-4+7\text{i})+c=0$	M1	Substitutes the given root into the equation or Equates the given root to $\dfrac{-b\pm\sqrt{b^2-4c}}{2}$
	Real part: $-33-4b+c=0$ Imaginary part: $-56+7b=0$	M1	Compares real or imaginary parts of an appropriate equation in b and c
	$b=8$	A1	Correct value for b or c Accept a quadratic equation of the form $x^2+8x+c=0$ or $x^2+bx+65=0$
	$c=65$	A1	Correct values for b and c Accept $x^2+8x+65=0$
		4	

	Question 2 Total	**4**	

Q	Answer	Marks	Comments
3(a)	$\lim\limits_{N\to\infty}\int_3^N x^{-\frac{1}{5}}\,dx$	B1	Replaces upper limit with a variable and considers it tending to infinity
	$=\lim\limits_{N\to\infty}\left[\frac{5}{4}x^{\frac{4}{5}}\right]_3^N$	B1	Integrates to $\frac{5}{4}x^{\frac{4}{5}}$
	But $\lim\limits_{N\to\infty}\left(N^{\frac{4}{5}}\right)$ is not defined, so the integral has no finite value.	E1	Explains that $N^{\frac{4}{5}}$ has no finite value as N tends to infinity Allow $\lim\limits_{N\to\infty}\left(N^{\frac{4}{5}}\right)=\infty$ but do not allow $N=\infty$ and/or $N^{\frac{4}{5}}=\infty$
		3	

Q	Answer	Marks	Comments
3(b)	$\lim\limits_{N\to\infty}\int_3^N x^{-2}\,dx$	M1	Replaces upper limit with a variable **PI**
	$=\lim\limits_{N\to\infty}\left[-\frac{1}{x}\right]_3^N$	M1	Correct integration with the upper limit replaced with a variable and limit notation used
	$=\frac{1}{3}$	A1	**NMS** scores 1 mark out of 3 **SC2** for correct integration and correct answer without use of limiting process Do not condone substitution of $N=\infty$
		3	

	Question 3 Total	**6**	

Q	Answer	Marks	Comments
4(a)	$5x - \frac{\pi}{12} = 2n\pi + \frac{\pi}{6}$ or $5x - \frac{\pi}{12} = 2n\pi + \frac{5\pi}{6}$ [where $n \in \mathbb{Z}$]	B1	**oe**
	$x = \frac{1}{5}\left(2n\pi + \frac{\pi}{6} + \frac{\pi}{12}\right)$ or $x = \frac{1}{5}\left(2n\pi + \frac{5\pi}{6} + \frac{\pi}{12}\right)$	M1	Add $\frac{\pi}{12}$ and then divide by 5 **oe**
	$x = \frac{2n\pi}{5} + \frac{\pi}{20}$ or $x = \frac{2n\pi}{5} + \frac{11\pi}{60}$	A1 A1	**oe** eg $x = \frac{n\pi}{5} + \frac{\pi}{60} + (-1)^n \frac{\pi}{30}$
		4	

Q	Answer	Marks	Comments
4(b)	$\left[\frac{27\pi}{60}, \frac{51\pi}{60}, \frac{75\pi}{60},\right] \frac{99\pi}{60}$ $\left[\frac{35\pi}{60}, \frac{59\pi}{60},\right] \frac{83\pi}{60} \left[, \frac{107\pi}{60}\right]$	M1	$\frac{33\pi}{20}$ or $\frac{83\pi}{60}$ or at least two solutions from their solution sets or Solves their linear expression in n equal to $\frac{3\pi}{2}$
	Closest to $\frac{90\pi}{60}$	M1	Investigates their solution sets and compares them to $\frac{3\pi}{2}$ or selects $\frac{99\pi}{60}$ or $\frac{83\pi}{60}$ Accept rounded decimals for the method marks only
	$\frac{83\pi}{60}$	A1	**CAO**
		3	

	Question 4 Total	**7**	

Q	Answer	Marks	Comments
5(a)	$\frac{1}{r!}-\frac{1}{(r+1)!} = \frac{r+1}{(r+1)!}-\frac{1}{(r+1)!}$ $= \frac{r}{(r+1)!}$	B1	
		1	

Q	Answer	Marks	Comments
5(b)	$\sum_{r=1}^{n}\frac{r}{(r+1)!}=\sum_{r=1}^{n}\left(\frac{1}{r!}-\frac{1}{(r+1)!}\right)$	B1	Writes as a sum of $\frac{1}{r!}-\frac{1}{(r+1)!}$ **PI**
	$=\frac{1}{1!}-\frac{1}{2!}$ $+\frac{1}{2!}-\frac{1}{3!}$	M1	Writes at least two pairs of fractions of the form $\frac{1}{r!}-\frac{1}{(r+1)!}$
	$+\ldots$ $+\frac{1}{(n-1)!}-\frac{1}{n!}$ $+\frac{1}{n!}-\frac{1}{(n+1)!}$	M1	Writes at least three pairs of fractions of the form $\frac{1}{r!}-\frac{1}{(r+1)!}$ including the first pair, the last pair, and at least one other pair
	$=1-\frac{1}{(n+1)!}$	A1	**ISW, ACF**
		4	

Q	Answer	Marks	Comments
5(c)	$\sum_{r=5}^{\infty}\frac{r}{(r+1)!}=\sum_{r=1}^{\infty}\frac{r}{(r+1)!}-\sum_{r=1}^{4}\frac{r}{(r+1)!}$ $\lim_{n\to\infty}\left(1-\frac{1}{(n+1)!}\right)-\left(1-\frac{1}{5!}\right)$ or $\lim_{n\to\infty}\left(\frac{1}{5!}-\frac{1}{(n+1)!}\right)$	M1	Indicates that the required sum is the difference $\sum_{1}^{\infty}...-\sum_{1}^{4}...$ or Substitutes 4 into their **part (b)** answer or Indicates that $\frac{1}{(n+1)!}$ approaches 0 as n tends to infinity or Replaces first term in series from **part (b)** with $\frac{1}{5!}$ **PI**
	$\frac{1}{120}$	A1	**CAO**
		2	

	Question 5 Total	**7**	

Q	Answer	Marks	Comments
6	Let $z = x + iy$ [where $x, y \in \mathbb{R}$]	B1	or for using that real solutions will satisfy $z = z^*$
	then $3(x+iy)^2 = x - iy$		
	$3x^2 - 3y^2 + 6ixy = x - iy$ $3x^2 - 3y^2 = x$ …① and $6xy = -y$ …②	M1	Equates real parts and equates imaginary parts. Condone one error.
	② $\Rightarrow y = 0$, $x = -\frac{1}{6}$	A1	At least one of $y = 0$ or $x = -\frac{1}{6}$
	Substitutes $y = 0$ in ①: $x = 0$ or $x = \frac{1}{3}$	B1	**oe** such as $z = 0$ or $z = \frac{1}{3}$
	Substitutes $x = -\frac{1}{6}$ in ①: $\frac{1}{12} - 3y^2 = -\frac{1}{6}$	m1	Uses a real part to find an imaginary part (or vice versa)
	$y = \pm\frac{\sqrt{3}}{6}$	A1	
	$z = 0$, $z = \frac{1}{3}$, $z = -\frac{1}{6} + \frac{\sqrt{3}}{6}i$, $z = -\frac{1}{6} - \frac{\sqrt{3}}{6}i$	A1	**oe** All four correct solutions seen and no others
		7	

	Question 6 Total	7	

Q	Answer	Marks	Comments
7(a)	By similar triangles, $\frac{x}{h}=\frac{6}{10} \Rightarrow x=0.6h$	E1	oe
		1	

Q	Answer	Marks	Comments
7(b)	$V=\frac{1}{3}(0.6h)^2(h)$	M1	
	$V=0.12h^3$ [m^3]	A1	ISW, oe
		2	

Q	Answer	Marks	Comments
7(c)	$\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}$	B1	A correct chain rule connecting V, h and t seen or used
	$\frac{dV}{dh}=0.36h^2$	M1	Differentiates their answer to **part (b)** with respect to h
	$\left[\frac{dV}{dh}=\right]\ 0.36\times4^2=5.76$ [when $h=4$]	m1	**PI** Substitutes $h=4$ into their $\frac{dV}{dh}$
	$\left[\frac{dh}{dt}=\right]\ \frac{0.54}{5.76}$	M1	Full method for $\frac{dh}{dt}$ May be in terms of h, e.g. $\frac{0.54}{0.36h^2}$
	$=\frac{3}{32}$ [m min^{-1}]	A1	**oe**, such as 0.09375 [m min^{-1}]
		5	

	Question 7 Total	8	

Q	Answer	Marks	Comments
8(a)	Stretch parallel to the y-axis Stretch scale factor $\sqrt{k}$	B1 B1	
		2	

Q	Answer	Marks	Comments
8(b)(i)	$\frac{x^2}{4}+\frac{(2x+c)^2}{k}=1$	M1	Considers the intersection of E_2 and L by elimination of x or y
	$kx^2+4(2x+c)^2=4k$		
	$kx^2+4\left(4x^2+4cx+c^2\right)-4k=0$	A1	Correct quadratic equation in x in terms of k and c equal to zero [= 0 can be implied]
	$(k+16)x^2+16cx+4c^2-4k=0$		
	For real roots, $\Delta \geq 0$		
	$(16c)^2-4(k+16)\left(4c^2-4k\right)\geq 0$	M1	Correctly applies the discriminant to their quadratic in x
	$\cancel{256c^2}-16kc^2+16k^2-\cancel{256c^2}+256k\geq 0$		
	$-c^2+k+16\geq 0$	M1	Sets their discriminant ≥ 0
	$c^2-k\leq 16$ as required	A1	
		5	

Q	Answer	Marks	Comments
8(b)(ii)	Limiting value so $c^2-k=16$ $49-k=16$	M1	**ft** their A
	$k=33$	A1ft	**ft** their A
		2	

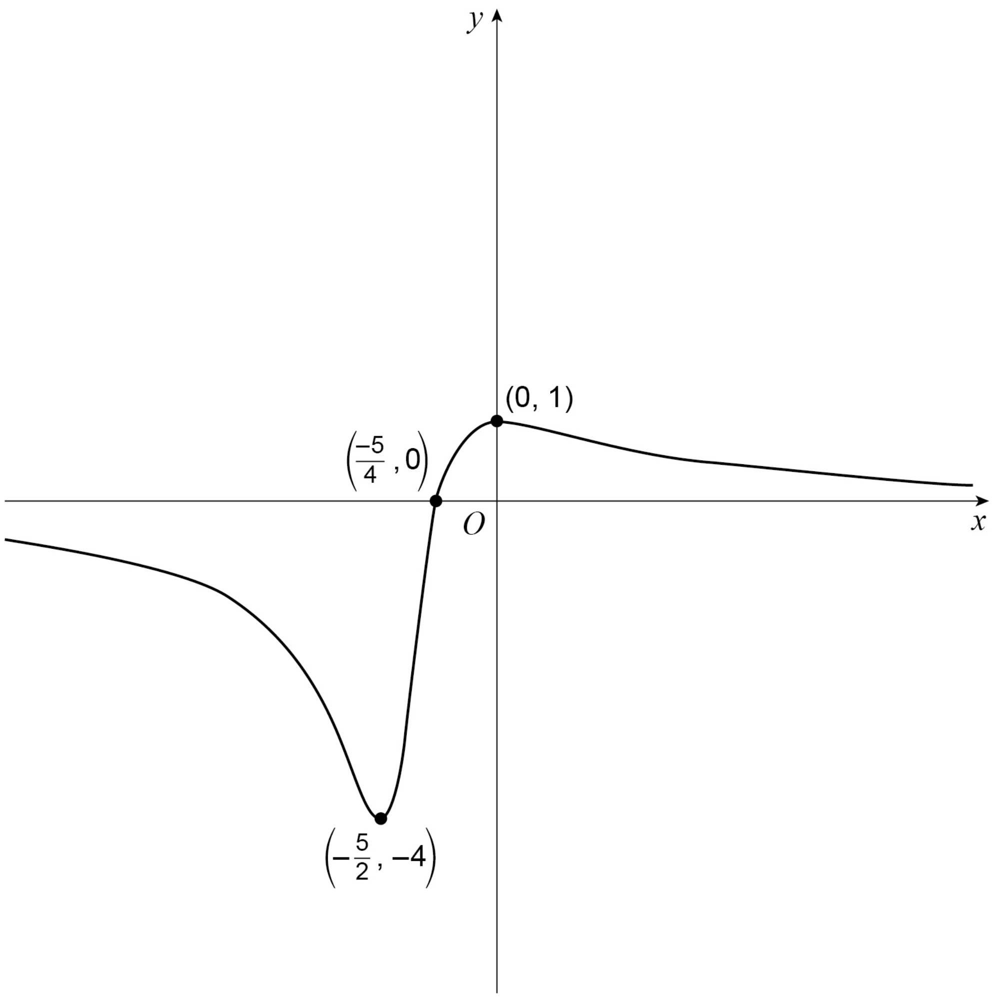
	Question 8 Total	9	

Q	Answer	Marks	Comments
9(a)(i)	denominator $= x^2+4x+5$ $= (x+2)^2+1$ $\neq 0$ [for all real x]	E1	Considers the denominator equal to zero or Considers the discriminant of the denominator
	[As] the denominator cannot equal zero [there are no vertical asymptotes]	E1	Shows that the denominator is non-zero for all [real] x This **E1** mark is dependent on the first **E1** mark
		2	

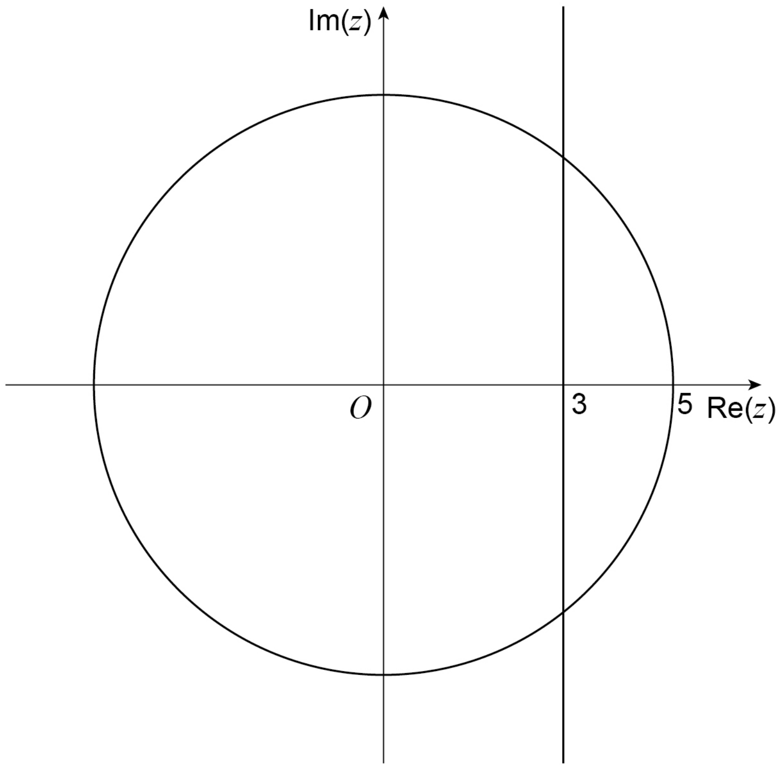
Q	Answer	Marks	Comments
9(a)(ii)	$y=0$	B1	
		1	

Q	Answer	Marks	Comments
9(b)	$k\left(x^2+4x+5\right)=4x+5$	M1	Forms a quadratic equation in x in terms of k
	$kx^2+(4k-4)x+(5k-5)=0$	A1	Correct quadratic equation in x in terms of k equal to zero [= 0 can be implied]
	For real roots $\Delta \geq 0$ $(4k-4)^2-4k(5k-5)\geq 0$	m1	**oe**
	$16k^2-32k+16-20k^2+20k\geq 0$		
	$-4k^2-12k+16\geq 0$		
	$k^2+3k-4\leq 0$	A1	**AG** Must be convincingly shown
		4	

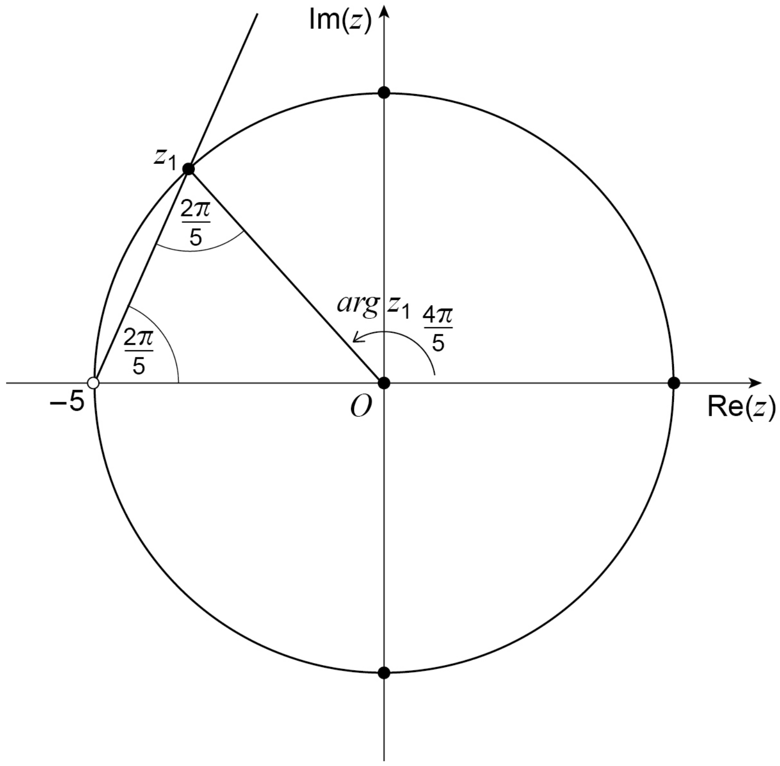
Q	Answer	Marks	Comments
9(c)	At stationary points, $y^2+3y-4=0$	M1	**PI**
	$y=-4$ and $y=1$	m1	Solves the quadratic equation
	When $y=-4$, $-4x^2-20x-25=0$ When $y=1$, $x^2=0$	A1	**PI** Substitutes one of their y values to find a correct quadratic equation in x
	$x=-\frac{5}{2}$ or $x=0$	B1	Finds at least one correct value for x
	$\left(-\frac{5}{2},-4\right)$ and $(0,1)$	B1	Both coordinates correct and no others Must be written as coordinates
		5	

Q	Answer	Marks	Comments
9(d)	Graph of $y = \mathrm{f}(x)$, correct shape, tending to x-axis at both extremes	B1	
	Stationary points shown	B1ft	**ft** their **part (c)**
	Values at axis intercepts shown	B1	
			
		3	

	Question 9 Total	15	

Q	Answer	Marks	Comments
10(a)	Circle, centre 0, radius 5	**B1**	
	Vertical line through the point $z = 3$	**B1**	Ignore extras
			
		2	

Q	Answer	Marks	Comments
10(b)	$\lvert z \rvert = 6$ or $\lvert z \rvert = 4$	**B1**	For either (or equivalent Cartesian form) **PI**
	$\text{Re}(z) = 2$ or $\text{Re}(z) = 4$	**B1**	Must have both (or equivalent Cartesian form) **PI**
	$[z =]4$	**B1**	
	$[z =]4 \pm 2\sqrt{5}\,\text{i}$	**B1**	
	$[z =]2 \pm 2\sqrt{3}\,\text{i}$	**B1**	
	$[z =]2 \pm 4\sqrt{2}\,\text{i}$	**B1**	Condone solutions given as coordinates Award a maximum of 5 marks if any extra incorrect answers are given
		6	

Q	Answer	Marks	Comments
10(c)	Triangle is isosceles so angle at $z_1 = \frac{2\pi}{5}$	M1	
	Exterior angle of triangle: $\arg z_1 = \frac{2\pi}{5} + \frac{2\pi}{5} = \frac{4\pi}{5}$	M1	
	$z_1 = 5\left(\cos\frac{4\pi}{5} + \mathrm{i}\sin\frac{4\pi}{5}\right)$	A1	condone extra inclusion of $z_1 = 5(\cos\pi + \mathrm{i}\sin\pi)$
			
		3	

	Question 10 Total	**11**	