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# INTERNATIONAL AS FURTHER MATHEMATICS FM02

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

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Mark scheme

January 2022

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Version: 1.1 Final



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**Key to mark scheme abbreviations**

<b>M</b>	Mark is for method
<b>m</b>	Mark is dependent on one or more M marks and is for method
<b>A</b>	Mark is dependent on M or m marks and is for accuracy
<b>B</b>	Mark is independent of M or m marks and is for method and accuracy
<b>E</b>	Mark is for explanation
✓ <b>or ft</b>	Follow through from previous incorrect result
<b>CAO</b>	Correct answer only
<b>CSO</b>	Correct solution only
<b>AWFW</b>	Anything which falls within
<b>AWRT</b>	Anything which rounds to
<b>ACF</b>	Any correct form
<b>AG</b>	Answer given
<b>SC</b>	Special case
<b>oe</b>	Or equivalent
<b>A2, 1</b>	2 or 1 (or 0) accuracy marks
<b>-x EE</b>	Deduct x marks for each error
<b>NMS</b>	No method shown
<b>PI</b>	Possibly implied
<b>SCA</b>	Substantially correct approach
<b>sf</b>	Significant figure(s)
<b>dp</b>	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$\mathbf{C}^T = \begin{bmatrix} 4 & 3 & 0 \\ -3 & 0 & k \\ 0 & -k & -3 \end{bmatrix}$	B1	
		1	

Q	Answer	Marks	Comments
1(b)(i)	$\mathbf{CC}^T = \begin{bmatrix} 4 & -3 & 0 \\ 3 & 0 & -k \\ 0 & k & -3 \end{bmatrix} \begin{bmatrix} 4 & 3 & 0 \\ -3 & 0 & k \\ 0 & -k & -3 \end{bmatrix}$  $= \begin{bmatrix} 25 & 12 & -3k \\ 12 & 9+k^2 & 3k \\ -3k & 3k & k^2+9 \end{bmatrix}$	M1  A1	Attempt to multiply matrices with at least three elements correct
		2	

Q	Answer	Marks	Comments
1(b)(ii)	$k^2 + 9 = 25$  $k = 4$ <b>and</b> $k = -4$	M1  A1	Sets their $k^2 + 9$ equal to 25 <b>PI</b>
		2	

Q	Answer	Marks	Comments
1(c)(i)	The number of columns in the matrix <b>C</b> must be the same as the number of rows in the matrix <b>D</b>	E1	oe
		1	

Q	Answer	Marks	Comments
1(c)(ii)	$\mathbf{DC} = \begin{bmatrix} 1 & 1 & 1 \\ -2 & -2 & -2 \end{bmatrix}$	B1 B1	For a 2 by 3 matrix For a fully correct answer
		2	

	<b>Question 1 Total</b>	<b>8</b>	
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Q	Answer	Marks	Comments
2(a)	Lets $f(x) = 2.7^x - 2x - 5$ $f(2) = -1.71$ $f(3) = 8.683$	M1	Correct evaluation of a suitable interval
	Since change of sign between $x = 2$ and $x = 3$ and as the curve is continuous [on this interval] then there is a root in the interval $2 < x < 3$	A1	Must mention change of sign and continuous curve in conclusion
		2	

Q	Answer	Marks	Comments
2(b)	$f(2) = -1.71$ $f(3) = 8.683$	M1	Attempting to calculate $f$ (mid-point) of at least two sets of values
	$f(2.5) = 1.9786...$ $2 < x < 2.5$ $f(2.25) = -0.1552...$		
	$2.25 < x < 2.5$ $f(2.375) = 0.8300...$ $2.25 < x < 2.375$	A1	Interval stated. PI
	$f(2.3125) = 0.3182...$ $2.25 < x < 2.3125$	A1	Interval stated. PI
	$[a] = 2.3$	A1	CSO Must see $2.25 < x < 2.3125$ or better
		4	

Q	Answer	Marks	Comments
2(c)	$f(-3) = 1.0508...$ $f(-2) = -0.8628...$	M1	PI by correct answer or by $\beta = -2.4564...$
	$n = -3$	A1	Condone $-3 < \beta < -2$ with no incorrect work
		2	

	<b>Question 2 Total</b>	<b>8</b>	
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Q	Answer	Marks	Comments
3(a)(i)	$\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 & 3 \\ 0 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 5 & 7 & 3 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ <p>(1,0), (5,1), (7,1), (3,0)</p>	<p><b>M1</b></p> <p><b>A1</b></p>	<p>Attempts to multiply matrices or multiply <math>\begin{bmatrix} 1 \\ 0 \end{bmatrix}</math>, <math>\begin{bmatrix} 1 \\ 1 \end{bmatrix}</math>, <math>\begin{bmatrix} 3 \\ 1 \end{bmatrix}</math> or <math>\begin{bmatrix} 3 \\ 0 \end{bmatrix}</math> by M</p> <p><b>CAO</b></p>
		<b>2</b>	

Q	Answer	Marks	Comments
3a(ii)	<p>Shear Parallel to the <math>x</math>-axis</p>	<p><b>B1</b></p> <p><b>B1</b></p>	<p><b>oe</b> for example 'leaving all the points on the <math>x</math>-axis as invariant'</p> <p>Condone 'x-axis is invariant'</p>
		<b>2</b>	

Q	Answer	Marks	Comments
3(b)	$\begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ a \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ <p><math>a = 2</math></p>	<p><b>M1</b></p> <p><b>A1</b></p>	<p>Multiplying matrices or using coordinates from the diagram to find <math>a</math></p> <p><b>PI</b> by correct answer</p>
		<b>2</b>	

Q	Answer	Marks	Comments
3(c)	$\mathbf{NM} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 2 & 9 \end{bmatrix}$ <p><math>\det(\mathbf{NM}) = 9 - 8 = 1</math></p> <p>Area of the shape remains the same</p>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>E1ft</b></p>	<p>Calculating <b>NM</b> using their <math>a</math></p> <p><b>PI</b> by correct value for <math>\det(\mathbf{NM})</math></p> <p>Concluding statement: <b>ft</b> if <math>\det(\mathbf{NM}) = 1</math></p>
		<b>3</b>	
3(c) ALT	<p><math>\det \mathbf{N} = 1</math> <math>\det \mathbf{M} = 1</math></p> <p><math>\det \mathbf{NM} = \det \mathbf{N} \times \det \mathbf{M} = 1</math></p> <p>Area of the shape remains the same</p>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>E1</b></p>	<p>Calculation of <math>\det \mathbf{N}</math> and <math>\det \mathbf{M}</math> using their <math>a</math></p> <p><b>PI</b> by correct value for <math>\det(\mathbf{NM})</math></p> <p>Concluding statement</p>
		<b>3</b>	

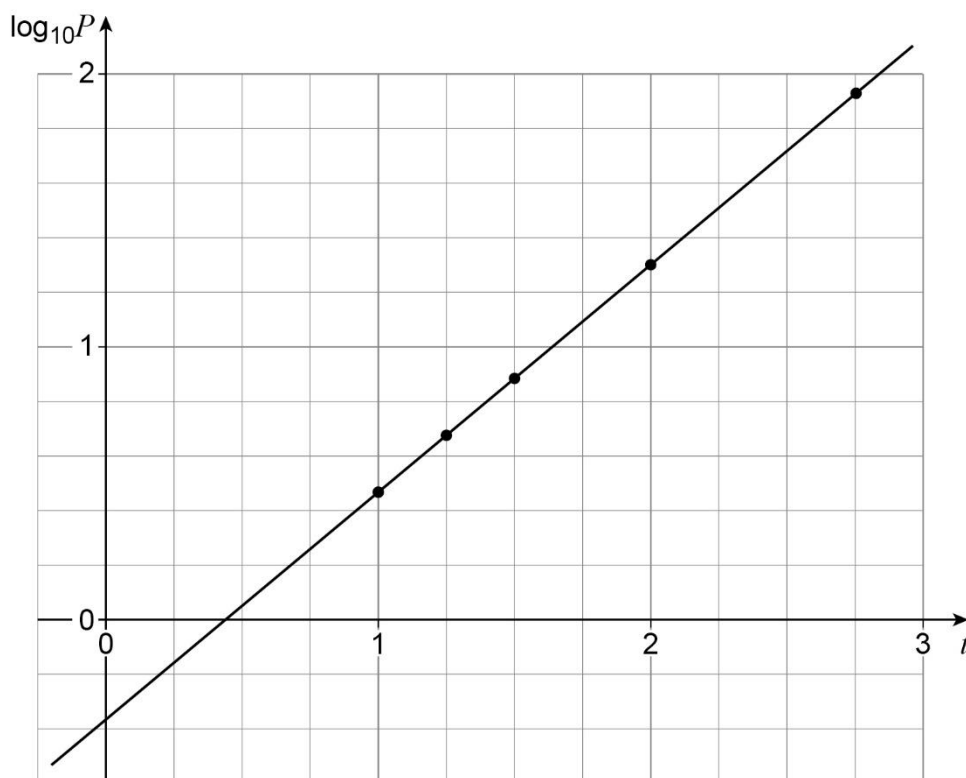
	<b>Question 3 Total</b>	<b>9</b>	
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Q	Answer	Marks	Comments
4	$hf(x, y) = 0.1 \left( 2 - \frac{1.5^3}{2} \right)$ $y_1 = 1.5 + 0.1 \times 0.3125 = \frac{49}{32} = 1.53125$ $y_2 = 1.53125 + 0.1 \left( 2.1 - \frac{1.53125^3}{2.1} \right)$ $y = 1.5703 \quad (\text{to 4 dp})$	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>Condone slip on substitution <b>PI</b></p> <p><b>AWRT 1.53</b> <b>PI</b></p> <p>Correct use of formula. <b>PI</b></p> <p><b>CAO</b></p>
		4	
	<b>Question 4 Total</b>	<b>4</b>	

Q	Answer	Marks	Comments
5(a)	$P = a \times 10^{kt}$ $\log_{10} P = \log_{10} (a \times 10^{kt})$ $= \log_{10} a + \log_{10} (10^{kt})$ $= \log_{10} a + kt$	B1	Using the rules for logs correctly <b>AG</b> Full steps must be shown
		1	

Q	Answer	Marks	Comments						
5(b)	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td><math>\log_{10} P</math></td> <td>0.47</td> <td>0.68</td> <td>0.89</td> <td>1.30</td> <td>1.93</td> </tr> </table>	$\log_{10} P$	0.47	0.68	0.89	1.30	1.93	B1	CAO
$\log_{10} P$	0.47	0.68	0.89	1.30	1.93				
		1							

Q	Answer	Marks	Comments
5(c)	'Their' points correctly plotted Line of best fit drawn	B1 ft B1 ft	All points plotted $\pm 0.25$ square See image below



		2	
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Q	Answer	Marks	Comments
5(d)(i)	Their intercept = $\log_{10}a$	<b>M1</b>	Allow answers $-0.4 \leq \log_{10}a \leq -0.3$
	$\log_{10}a = -0.35$ $a = 10^{-0.35} = 0.45$	<b>A1</b>	Allow answers in the range $0.40 \leq a \leq 0.50$
	Their gradient = $k$ $k = 0.83$	<b>B1</b>	Allow answers in the range $0.70 \leq k \leq 0.90$
		<b>3</b>	

Q	Answer	Marks	Comments
5(d)(ii)	$P = 0.45 \times 10^{0.83t}$	<b>B1ft</b>	ft their values of $a$ and $k$
		<b>1</b>	

Q	Answer	Marks	Comments
5(d)(iii)	$P = 0.45 \times 10^{0.83 \times 4}$ $= 940$	<b>M1</b>	Substitute $t = 4$ into their formula
	Total profit is 940 million dollars	<b>A1</b>	Must include units Allow answers in the range $250 \text{ million} < P < 2000 \text{ million}$
		<b>2</b>	

Q	Answer	Marks	Comments
5d(iv)	This total profit is extrapolated [so therefore may be unreliable]	<b>E1</b>	Reference to extrapolation
		<b>1</b>	

	<b>Question 5 Total</b>	<b>11</b>	
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Q	Answer	Marks	Comments
6(a)	Discrete Uniform (Distribution)	B1	Condone uniform distribution
		1	

Q	Answer	Marks	Comments
6(b)	0.75	B1	oe
		1	

Q	Answer	Marks	Comments
6(c)(i)	$p = 0.2$	B1	oe
		1	

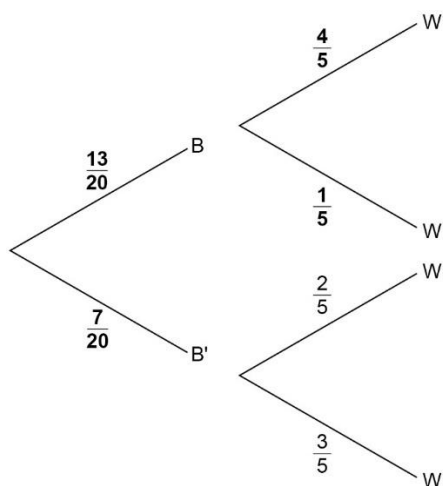
Q	Answer	Marks	Comments
6(c)(ii)	$E(X) = 2.5$	B1	oe may not be evaluated
	$E(2X - 5Y) = 2E(X) - 5E(Y)$	M1	Applies formula Implied by sight of $2 \times 2.5 - 5 \times 5$ for their 2.5
	$= -20$	A1	PI
		3	

	<b>Question 6 Total</b>	<b>6</b>	
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Q	Answer	Marks	Comments
7(a)	$P(\text{not win}) = \frac{17}{50}$	<b>B1</b>	oe, seen or used
	$P(\text{Basia plays}) = \frac{33}{50} \times \frac{26}{33} + \frac{17}{50} \times \frac{13}{34}$	<b>M1</b>	
	$= \frac{13}{20}$	<b>A1</b>	oe
		<b>3</b>	

Q	Answer	Marks	Comments
7(b)	$P(\text{win} \mid \text{Basia plays}) = \frac{\frac{33}{50} \times \frac{26}{33}}{\frac{13}{20}}$	<b>M1</b>	ft their P(Basia plays) provided between 0 and 1
	$= \frac{4}{5}$	<b>A1</b>	<b>AG</b> , be convinced
		<b>2</b>	

Q	Answer	Marks	Comments
7(c)	See image below	<b>B1ft</b>	ft their P(B) provided between 0 and 1 oe



		<b>1</b>	
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	<b>Question 7 Total</b>	<b>6</b>	
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Q	Answer	Marks	Comments
8(a)	$G_X(t) = 0.4 + 0.25t + 0.35t^2$	<b>B1</b>	oe eg $\frac{2}{5} + \frac{1}{4}t + \frac{7}{20}t^2$
		<b>1</b>	

Q	Answer	Marks	Comments
8(b)(i)	$G_{X+Y}(t) = G_X(t)G_Y(t)$	<b>M1</b>	Applies formula
	$0.19t + 0.32875t^2 + 0.2975t^3 + 0.18375t^4$	<b>A1</b>	oe eg $\frac{19}{100}t + \frac{263}{800}t^2 + \frac{119}{400}t^3 + \frac{147}{800}t^4$
		<b>2</b>	

Q	Answer	Marks	Comments
8(b)(ii)	$G'_{X+Y}(t) = 0.19 + 0.6575t + 0.8925t^2 + 0.735t^3$	<b>M1</b>	Differentiates once oe eg $\frac{19}{100} + \frac{263}{400}t + \frac{357}{400}t^2 + \frac{147}{200}t^3$ Condone one slip
	$G''_{X+Y}(t) = 0.6575 + 1.785t + 2.205t^2$	<b>M1</b>	Differentiates twice oe eg $\frac{263}{400} + \frac{357}{200}t + \frac{441}{200}t^2$ Condone one slip ft their $G'_{X+Y}(t)$
	$G'_{X+Y}(1) = 2.475$ or $G''_{X+Y}(1) = 4.6475$	<b>A1</b>	Finds one of $G'_{X+Y}(1)$ or $G''_{X+Y}(1)$ oe eg $\frac{99}{40}$ or $\frac{1859}{400}$ <b>PI</b>
	$\sigma^2 = G''_{X+Y}(1) + G'_{X+Y}(1) - (G'_{X+Y}(1))^2$	<b>M1</b>	Applies variance formula to their $G_{X+Y}(t)$
	$= 0.996875$	<b>A1</b>	oe, eg $\frac{319}{320}$
		<b>5</b>	<b>SC</b> a fully correct solution that does not use the result obtained in part (b)(i) scores 3/5

	<b>Question 8 Total</b>	<b>8</b>	
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Q	Answer	Marks	Comments
9	$[s] = L$ $[vt] = LT^{-1} \times T = L$ $\left[\frac{1}{2}at^2\right] = LT^{-2} \times T^2 = L$ $\therefore$ Dimensionally consistent	B1 B1 B1	For correct dimensions of at least two terms (condone use of units) All three dimensions correct Obtains same dimensions and states conclusion
		3	
	<b>Question 9 Total</b>	<b>3</b>	

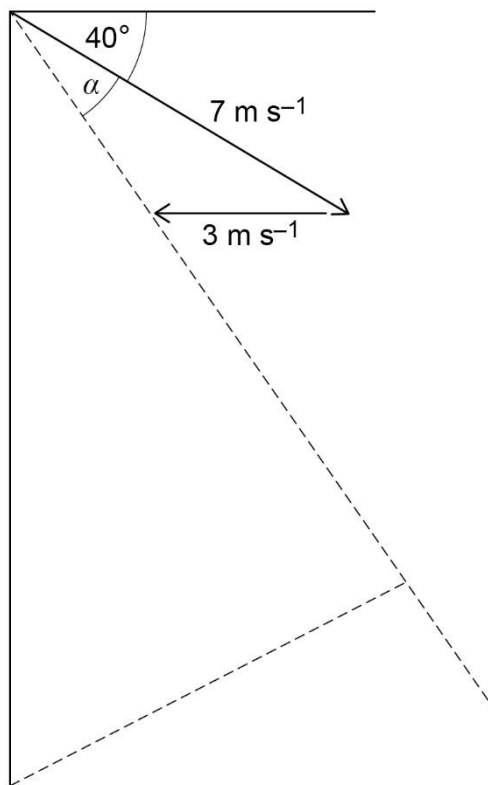
Q	Answer	Marks	Comments
<b>10(a)</b>	$7 \times 4 + 3 \times (-5) = 7v_A + 3v_B$	<b>M1</b>	Conservation of momentum equation Condone sign errors
	$13 = 7v_A + 3v_B$		
	$v_A - v_B = -0.9(4 - (-5))$	<b>M1</b>	Coefficient of restitution equation Condone sign errors
	$v_A - v_B = -8.1$	<b>A1</b>	Both equations correct
	$v_A = -1.13$	<b>A1</b>	Correct speed for A
Speed of A = $1.13 \text{ m s}^{-1}$			
$v_B = 6.97$	<b>A1</b>	Correct speed for B	
Speed of B = $6.97 \text{ m s}^{-1}$			
		<b>5</b>	

Q	Answer	Marks	Comments
<b>10(b)</b>	Sphere A changes direction	<b>B1</b>	Both statements correct
	Sphere B changes direction		
		<b>1</b>	

Q	Answer	Marks	Comments
<b>10(c)</b>	$\int_0^{0.02} kt(0.02 - t)dt$	<b>M1</b>	Forms integral
	$= \frac{k}{750000}$	<b>A1</b>	Evaluates integral correctly
	$I = 7 \times (-1.13) - 7 \times 4$ or $I = 3 \times 6.97 - 3 \times (-5)$	<b>M1</b>	Uses impulse equation Condone sign errors
	$= -35.91$ or $35.91$	<b>A1</b>	Correct impulse
	$k = 750000 \times 35.91$ $= 26900000$	<b>A1</b>	<b>CAO</b>
		<b>5</b>	

	<b>Question 10 Total</b>	<b>11</b>	
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Q	Answer	Marks	Comments
11	$v_{SF}^2 = 7^2 + 3^2 - 2 \times 7 \times 3 \cos 40^\circ$ $v_{SF} = \sqrt{58 - 42 \cos 40^\circ} = 5.082$ $\frac{\sin \alpha}{3} = \frac{\sin 40^\circ}{\sqrt{58 - 42 \cos 40^\circ}}$ $\alpha = 22.3^\circ$ Minimum Distance = $1500 \sin(90 - 50 - 22.3)$ = 697 metres	<b>M1</b> <b>A1</b>  <b>M1</b> <b>A1</b>  <b>M1</b> <b>A1</b>	Correct method to find relative speed Correct relative speed  Correct method to find unknown angle Correct angle. <b>PI</b> by sight of $27.7^\circ$  Correct method to find minimum distance, eg $1500 \sin(27.7)$ Correct minimum distance



		<b>6</b>	
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Q	Answer	Marks	Comments
<b>11</b> <b>ALT 1</b>	$v_{SF} = \begin{bmatrix} 7 \cos 40^\circ \\ 7 \sin 40^\circ \end{bmatrix} - \begin{bmatrix} 3 \\ 0 \end{bmatrix}$ $v_{SF} = \begin{bmatrix} 7 \cos 40^\circ - 3 \\ 7 \sin 40^\circ \end{bmatrix}$ $\tan \theta = \frac{7 \sin 40^\circ}{7 \cos 40^\circ - 3}$ $\theta = 62.3^\circ$ <p>Minimum Distance = <math>1500 \sin(90 - 62.3)</math>                      = 697 metres</p>	<b>M1</b>  <b>A1</b>  <b>M1</b>  <b>A1</b>  <b>M1</b>  <b>A1</b>	Correct method to find relative velocity  Correct relative velocity  Correct method to find unknown angle  Correct angle. <b>PI</b> by sight of $27.7^\circ$  Correct method to find minimum distance Correct minimum distance
<b>11</b> <b>ALT 2</b>	$r_{SF} = \begin{bmatrix} 7t \cos 40^\circ \\ 1500 - 7t \sin 40^\circ \end{bmatrix} - \begin{bmatrix} 3t \\ 0 \end{bmatrix}$ $r_{SF} = \begin{bmatrix} t(7 \cos 40^\circ - 3) \\ 1500 - 7t \sin 40^\circ \end{bmatrix}$ $t^2(7 \cos 40^\circ - 3)^2 + (1500 - 7t \sin 40^\circ)^2$ $= t^2((7 \cos 40^\circ - 3)^2 + 49 \sin^2 40^\circ) - 21000t \sin 40^\circ + 1500^2$ $t_{\min} = \frac{21000 \sin 40^\circ}{2((7 \cos 40^\circ - 3)^2 + 49 \sin^2 40^\circ)} = 261.3\dots$ <p>Minimum Distance</p> $= \sqrt{261.3\dots^2(7 \cos 40^\circ - 3)^2 + (1500 - 7 \times 261.3\dots \sin 40^\circ)^2}$ = 697 metres	<b>M1</b>  <b>A1</b>  <b>M1</b>  <b>A1</b>  <b>M1</b>  <b>A1</b>	Correct method to find relative position vector  Correct relative position vector  Correct method to find unknown time  Correct time. <b>AWRT</b> 261  Correct method to find minimum distance Correct minimum distance

	<b>Question 11 Total</b>	<b>6</b>	
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