OXFORD

INTERNATIONAL AQA EXAMINATIONS

# INTERNATIONAL AS
# FURTHER MATHEMATICS
# FM01

(9665/FM01) Unit FP1 Pure Mathematics

Mark scheme

January 2022

Version: 1.0 Final

2 2 1 X F M 0 1 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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**Key to mark scheme abbreviations**

| | |
|---|---|
| **M** | Mark is for method |
| **m** | Mark is dependent on one or more M marks and is for method |
| **A** | Mark is dependent on M or m marks and is for accuracy |
| **B** | Mark is independent of M or m marks and is for method and accuracy |
| **E** | Mark is for explanation |
| √ **or ft** | Follow through from previous incorrect result |
| **CAO** | Correct answer only |
| **CSO** | Correct solution only |
| **AWFW** | Anything which falls within |
| **AWRT** | Anything which rounds to |
| **ACF** | Any correct form |
| **AG** | Answer given |
| **SC** | Special case |
| **oe** | Or equivalent |
| **A2, 1** | 2 or 1 (or 0) accuracy marks |
| **–*x* EE** | Deduct *x* marks for each error |
| **NMS** | No method shown |
| **PI** | Possibly implied |
| **SCA** | Substantially correct approach |
| **sf** | Significant figure(s) |
| **dp** | Decimal place(s) |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 1 | $\left[\sum_{r=n+1}^{2n} r^3 =\right] \sum_{r=1}^{2n} r^3 - \sum_{r=1}^{n} r^3$ | M1 | If **M0** awarded, allow **SC1** for sight of $\frac{1}{4}(2n)^2(2n+1)^2$ and $\frac{1}{4}n^2(n+1)^2$ |
| | $= \frac{1}{4}(2n)^2(2n+1)^2 - \frac{1}{4}n^2(n+1)^2$ | A1 | |
| | $= \frac{1}{4}n^2\left\{4(2n+1)^2 - (n+1)^2\right\}$ | M1 | Factorising at least $n^2$ using consistent working |
| | $= \frac{1}{4}n^2\left\{16n^2 + 16n + 4 - \left(n^2 + 2n + 1\right)\right\}$ | M1 | Expands the two squared brackets<br>**or**<br>uses difference of two squares<br>Allow one slip |
| | $= \frac{1}{4}n^2\left\{15n^2 + 14n + 3\right\}$ | | |
| | $= \frac{1}{4}n^2(5n+3)(3n+1)$ | A1 | |

| | | | |
|---|---|---|---|
| | **Question 1 Total** | **5** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 2(a) | $\frac{7-3\text{i}}{k-5\text{i}} \times \frac{k+5\text{i}}{k+5\text{i}}$ | M1 | **or**<br>$z = x + \text{i}y$<br>$(x+\text{i}y)(k-5\text{i}) = 7-3\text{i}$<br>Then multiplies out and equates real and imaginary parts |
| | Real part $= \frac{7k+15}{k^2+25}$ | A1 | Seen anywhere |
| | Imaginary part $= \frac{35-3k}{k^2+25}$ | A1 | Condone $\left(\frac{35-3k}{k^2+25}\right)\text{i}$ |
| | | 3 | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 2(b) | substituting $k = 2$<br>or<br>$\frac{35-3k}{7k+15}$ seen | M1 | |
| | $\left[\frac{7-3\text{i}}{2-5\text{i}} = \frac{29}{29} + \text{i}\left(\frac{29}{29}\right) = \right]$ $1+\text{i}$<br>or<br>$\frac{35-3\times 2}{7\times 2+15}$ | A1 | |
| | $\arg\left(\frac{7-3\text{i}}{2-5\text{i}}\right) = \arg(1+\text{i}) = \left[\tan^{-1}\left(\frac{1}{1}\right) = \right]$ $\frac{\pi}{4}$ | A1 | **AG**<br>Condone $\tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$<br>where $\theta = \arg\left(\frac{7-3\text{i}}{2-5\text{i}}\right)$ |
| | | 3 | |

| | | | |
|---|---|---|---|
| | **Question 2 Total** | **6** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 3 | $A = \pi r^2$<br>$\frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r$ | **B1** | |
| | $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{d}r}{\mathrm{d}A} \times \frac{\mathrm{d}A}{\mathrm{d}t}$ | **M1** | Seen or used |
| | $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{2\pi r} \times 3$ | **A1ft** | their value for $r$ may be substituted in<br>**ft** their $\frac{\mathrm{d}A}{\mathrm{d}r}$ |
| | When $A = 36\pi$, $r = 6$ | **B1** | |
| | $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{2\pi(6)} \times 3$ | **M1** | **ft** their $\frac{\mathrm{d}r}{\mathrm{d}t}$ and their value of $r$ |
| | $= \frac{1}{4\pi}$ [metres/day] | **A1** | **CAO** |

| | | | |
|---|---|---|---|
| | **Question 3 Total** | **6** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 4(a) | $2x-\frac{\pi}{2}=2n\pi\pm\frac{2\pi}{3}$ | B1 | **oe** |
| | $x=\frac{1}{2}\left(2n\pi\pm\frac{2\pi}{3}+\frac{\pi}{2}\right)$ | M1 | Rearranging to make $x$ the subject going from $\left(2x-\frac{\pi}{2}\right)$ to $x$ Allow one slip |
| | $x=n\pi+\frac{\pi}{4}\pm\frac{\pi}{3}$ | A1 A1 | **oe**, eg $x=n\pi+\frac{7\pi}{12}$ or $x=n\pi-\frac{\pi}{12}$ |
| | | 4 | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 4(b) | $k=1$ : $\frac{7\pi}{12},\frac{11\pi}{12}$ <br> $k=2$ : also $\frac{19\pi}{12},\frac{23\pi}{12},\frac{31\pi}{12},\frac{35\pi}{12}$ | M1 | For investigating at least one positive value of $k$ with their general solution from **part (a)** |
| | $k=1$: 2 solutions <br> $k=2$: 6 solutions [etc] | M1 | For finding the number of solutions for at least two values of $k$ using their general solution from **part (a)** |
| | $4k-2$ | A1 | **CAO** |
| | | 3 | |

| | | | |
|---|---|---|---|
| | **Question 4 Total** | **7** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 5(a) | $\alpha + \beta = -5$ | B1 | |
| | $\alpha\beta = 9$ | B1 | |
| | | 2 | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 5(b) | $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 25 - 18$ | M1 | or other valid method |
| | $\alpha^2 + \beta^2 = 7$ | A1 | |
| | | 2 | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 5(c) | $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ | M1 | or other valid method |
| | $= -125 - 3(9)(-5) = 10$ | A1 | |
| | | 2 | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 5(d) | Sum of roots $= \alpha + \dfrac{\beta}{\alpha} + \beta + \dfrac{\alpha}{\beta}$ <br> $= \alpha + \beta + \dfrac{\beta^2 + \alpha^2}{\alpha\beta}$ | M1 | |
| | $= -5 + \dfrac{7}{9} = -\dfrac{38}{9}$ | A1ft | **ft** their $\alpha^2 + \beta^2$ from **part (b)** |
| | Product of roots $= \left(\alpha + \dfrac{\beta}{\alpha}\right)\left(\beta + \dfrac{\alpha}{\beta}\right)$ <br> $= \alpha\beta + \dfrac{\alpha^2}{\beta} + \dfrac{\beta^2}{\alpha} + 1$ | M1 | |
| | $= \alpha\beta + \dfrac{\alpha^3 + \beta^3}{\alpha\beta} + 1$ | m1 | Converts $\dfrac{\alpha^2}{\beta} + \dfrac{\beta^2}{\alpha}$ into $\dfrac{\alpha^3 + \beta^3}{\alpha\beta}$ |
| | $= 9 + \dfrac{10}{9} + 1 = \dfrac{100}{9}$ | A1 | |
| | $9x^2 + 38x + 100 = 0$ | A1 | Correct quadratic equation with integer coefficients |
| | | 6 | |

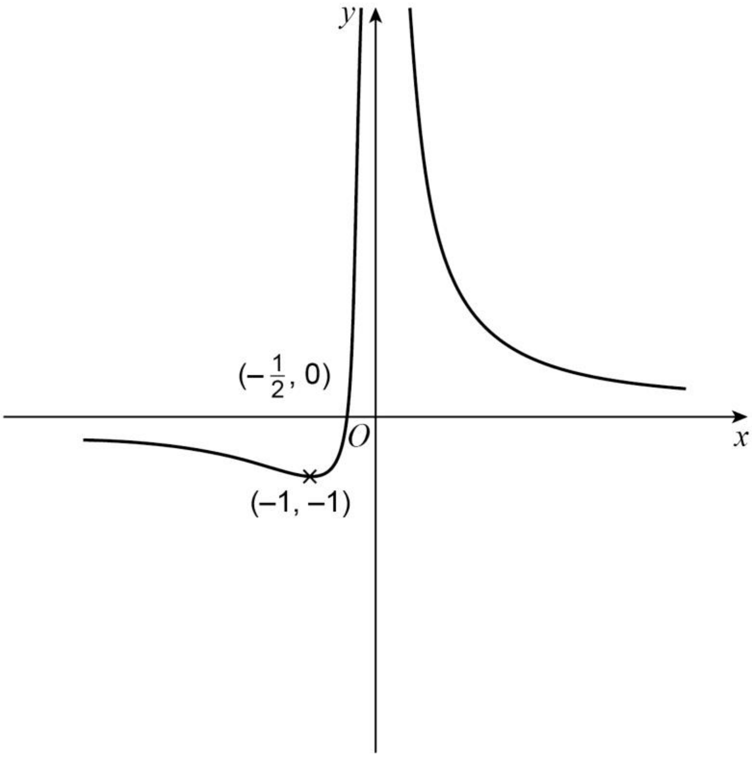
| | | | |
|---|---|---|---|
| | **Question 5 Total** | **12** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 6(a) | $x = 0$ | B1 | |
| | $y = 0$ | B1 | |
| | | 2 | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 6(b) | $k = \frac{2x+1}{x^2}$<br>$kx^2 - 2x - 1 = 0$ | M1 | |
| | $\Delta \geq 0$ so $(-2)^2 - 4k(-1) \geq 0$ | B1 | Explicit use of the discriminant |
| | $4 + 4k \geq 0$<br>so $k \geq -1$ | A1 | AG |
| | | 3 | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 6(c) | When $k = -1$, $-x^2 - 2x - 1 = 0$<br>$(x+1)^2 = 0 \Rightarrow x = -1$ | M1 | |
| | Stationary point is $(-1, -1)$ | A1 | |
| | | 2 | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **6(d)** | See below | **B1** | Correct general shape |
| | See below | **B1** | Axis intercept correctly labelled (condone $x$-coordinate only) and stationary point correctly marked and labelled |
| | See below | **B1** | Graph approaches all asymptotes |
| | | **3** | |




| Q | Answer | Marks | Comments |
|---|---|---|---|
| **6(e)** | $\frac{2x+1}{x^2} > 3 \therefore 2x+1 > 3x^2$ | **M1** | Allow equation if followed by attempt to solve inequality |
| | $3x^2 - 2x - 1 < 0$<br>$(3x+1)(x-1) < 0$ | **M1** | Or for solving the corresponding equation |
| | $-\frac{1}{3} < x < 1$ | **A1** | **PI** |
| | $-\frac{1}{3} < x < 0,\ 0 < x < 1$ | **A1** | **ACF,** e.g. $-\frac{1}{3} < x < 1$ and $x \neq 0$ |
| | | **4** | |

| | | | |
|---|---|---|---|
| | **Question 6 Total** | **14** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 7(a) | Because one of the limits is infinite | E1 | Or 'the range of integration is infinite' |
| | | 1 | |

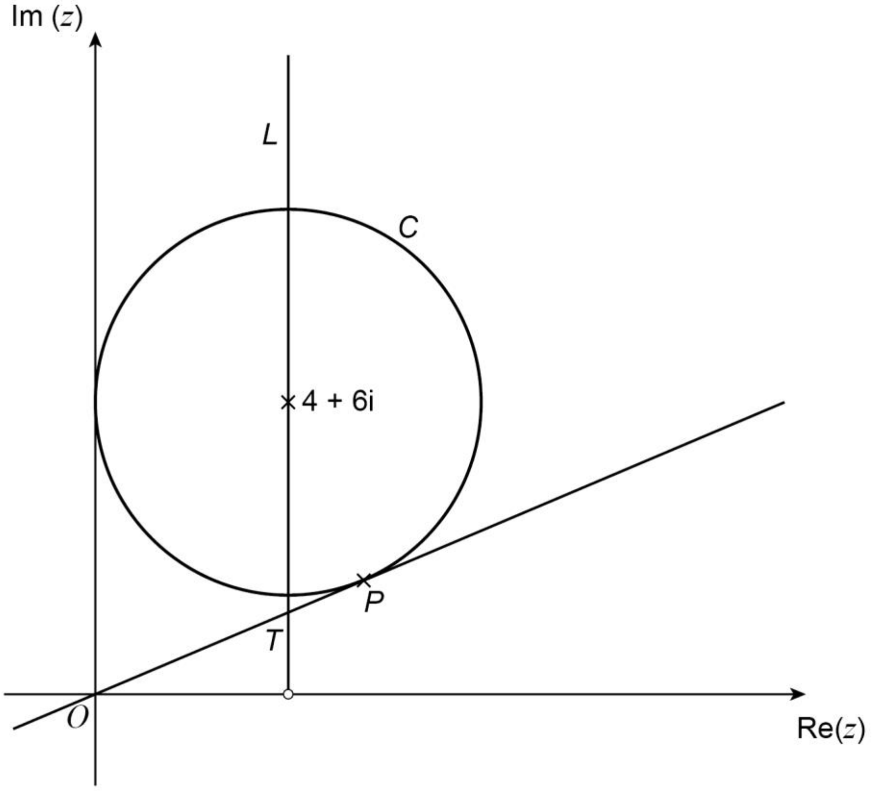
| Q | Answer | Marks | Comments |
|---|---|---|---|
| 7(b) | Because the integrand is not defined at one of the limits of integration<br>**or**<br>Because the integrand is not defined when $x = 0$ | E1 | |
| | | 1 | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 7(c) | $I_2 = \lim_{h \to 0} \int_h^{64} \frac{1}{\left(\sqrt[3]{x}\right)^2} \mathrm{d}x$ | M1 | Limiting process seen in the solution |
| | $\left[ = \lim_{h \to 0} \int_h^{64} x^{-\frac{2}{3}} \mathrm{d}x \right]$ | | |
| | $= \lim_{h \to 0} \left[ 3x^{\frac{1}{3}} \right]_h^{64}$ | m1 | Condone 0 as lower limit if 1st **M1** was awarded |
| | $= 3(4) - 3(0)$<br>$= 12$ | A1 | Correct answer with no limiting process shown is **SC1** |
| | | 3 | |

| | | | |
|---|---|---|---|
| | **Question 7 Total** | **5** | |

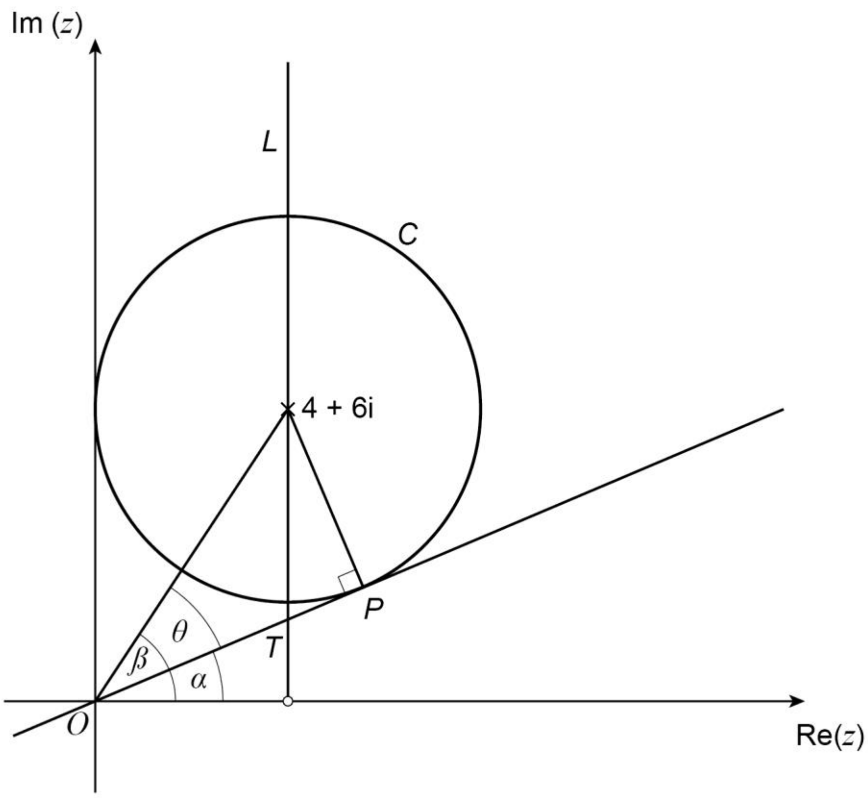
| Q | Answer | Marks | Comments |
|---|---|---|---|
| **8(a)** | $4+6\text{i}$ | **B1** | |
| | | **1** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **8(b)** | *L* drawn correctly | **B1** | Condone no indication that end point is not included, but must not be below the real axis |
| | *OP* drawn correctly | **B1** | No need to extend beyond *O* or *P* |
| | *T* marked correctly | **B1ft** | The intersection of their line *OP* with the correct half-line *L* **See artwork below** |




| | | | |
|---|---|---|---|
| | | **3** | |

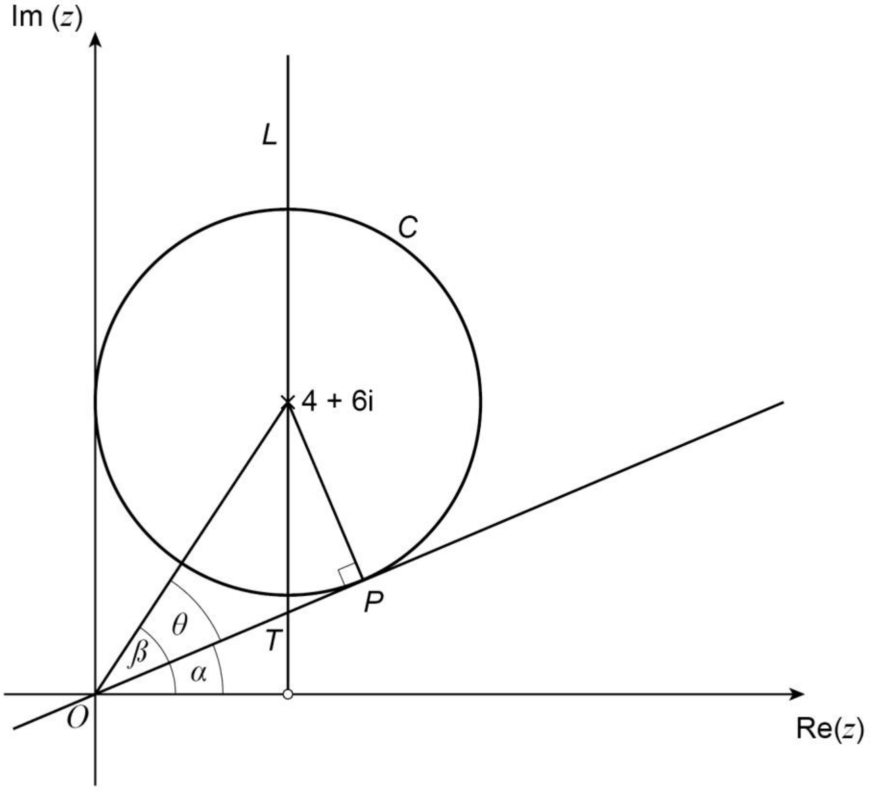
| Q | Answer | Marks | Comments |
|---|---|---|---|
| 8(c) | $\tan\beta = \frac{3}{2}$ | B1 | See diagram below |
| | $\left[\sin\theta = \frac{2}{\sqrt{13}} \text{ so}\right] \tan\theta = \frac{2}{3}$ | B1 | |
| | $\arg z = \alpha = \beta - \theta$ | M1 | |
| | $\tan\alpha = \frac{5}{12}$ [or $\alpha = 0.39479...$] | A1 | |
| | $z = 4 + (4\tan\alpha)\mathrm{i}$ | M1 | |
| | $z = 4 + \frac{5}{3}\mathrm{i}$ | A1 | Exact value for real and imaginary parts |




| | | | |
|---|---|---|---|
| | | 6 | |

| | | | |
|---|---|---|---|
| | Question 8 Total | 10 | |

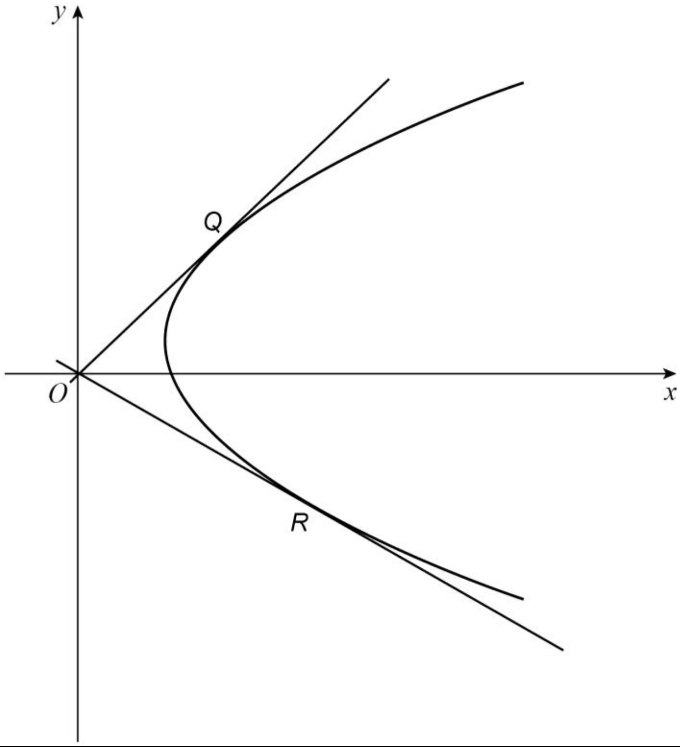
| Q | Answer | Marks | Comments |
|---|---|---|---|
| **8(c) ALT** | Equation of line *OP* is $y = mx$<br>$(x-4)^2 + (mx-6)^2 = 16$ | **M1** | |
| | $(m^2+1)x^2 - (8+12m)x + 36 = 0$ | **A1** | |
| | $\Delta = 0 \Rightarrow (8+12m)^2 - 4\times(m^2+1)\times 36 = 0$ | **M1** | |
| | $m = \dfrac{5}{12}$ | **A1** | |
| | $y$-coordinate of *T*<br>$y = \dfrac{5}{12}\times 4$ | **M1** | **ft** their $m$ |
| | $z = 4 + \dfrac{5}{3}\text{i}$ | **A1** | Exact value for real and imaginary parts |



| Q | Answer | Marks | Comments |
|---|---|---|---|
| **9(a)** | $(x-12)^2 + y^2 = (x+12)^2$ | **M1** | |
| | $x^2 - 24x + 144 + y^2 = x^2 + 24x + 144$<br>$y^2 = 48x$ | **A1** | |
| | | **2** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **9(b)** | $(y-4)^2 = 48(x-5)$ | **B1 B1** | **B1** for LHS, **B1** for RHS |
| | | **2** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **9(c)(i)** | Parabola with vertex facing left | **B1** | |
| | Parabola with vertex in 1st quadrant | **B1** | **ft** their answer to part **(b)** if translation $\begin{bmatrix} 5 \\ -4 \end{bmatrix}$ is used, which leads to a parabola with a vertex in the 4th quadrant |
| | Lines *OQ* and *OR*, *Q* and *R* marked correctly, *R* to the right of *Q* | **B1** | **See artwork below** |
| | | **3** | |



| Q | Answer | Marks | Comments |
|---|---|---|---|
| **9(c)(ii)** | $y = mx$ is a tangent<br>so $(mx-4)^2 = 48(x-5)$ | **M1** | |
| | $m^2x^2 - 8mx + 16 - 48x + 240 = 0$<br>$m^2x^2 - (8m+48)x + 256 = 0$ | **A1ft** | **ft** their **part (b)** |
| | $\Delta = 0 \Rightarrow (8m+48)^2 - 4m^2(256) = 0$ | **M1** | |
| | $960m^2 - 768m - 2304 = 0$<br>$5m^2 - 4m - 12 = 0$ | **A1ft** | for either equation (**oe**)<br>**ft** their **part (b)** |
| | $m = 2$ or $m = -\frac{6}{5}$ | **A1ft** | **ft** their **part (b)** |
| | $m = 2$: $4x^2 - 64x + 256 = 0$ and $x = 8$<br>or<br>$m = -\frac{6}{5}$: $\frac{36}{25}x^2 - \frac{192}{5}x + 256 = 0$ and $x = \frac{40}{3}$ | **M1** | |
| | $Q(8, 16)$ | **A1** | |
| | $R\left(\frac{40}{3}, -16\right)$ | **A1** | |
| | | **8** | |

| | | | |
|---|---|---|---|
| | **Question 9 Total** | **15** | |