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FM05

(9665/FM05) Unit FM2 Mechanics

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	$2\begin{bmatrix}4\\1\end{bmatrix}+m\begin{bmatrix}2\\U\end{bmatrix}=(m+2)\begin{bmatrix}2.8\\-1\end{bmatrix}$ $8+2m=2.8m+5.6$ $2.4=0.8m$ $m=3$	**M1** **A1**	Forms equation based on conservation of momentum in one or two dimensions Correct value for m
		2	

Q	Answer	Marks	Comments
1(b)	$2+3U=-5$ $U=-\frac{7}{3}$	**M1** **A1ft** **A1**	Forms equation for second component based on conservation of momentum, with at least one side of the equation correct. Correct equation, for their m Correct value for U **AWRT** -2.3
		3	

Q	Answer	Marks	Comments
1(c)	$\mathbf{I}=2\begin{bmatrix}2.8\\-1\end{bmatrix}-2\begin{bmatrix}4\\1\end{bmatrix}$ $=\begin{bmatrix}-2.4\\-4\end{bmatrix}$ $I=\sqrt{2.4^2+4^2}=4.7\ [\text{Ns}]$	**M1** **A1** **A1**	Uses impulse formula in vector form Obtains correct impulse expression Obtains correct magnitude **AWRT** 4.7, such as 4.66476
		3	

	Question 1 Total	**8**	

Q	Answer	Marks	Comments
2(a)	$3\cos 60° = v\cos 30°$ $v = \dfrac{3\cos 60°}{\cos 30°}\left[= \sqrt{3}\right]$ $v\sin 30° = e \times 3\sin 60°$ $e = \dfrac{3\cos 60° \times \sin 30°}{\cos 30° \times 3\sin 60°}$ $= \dfrac{1}{3}$	**M1** **A1** **M1** **M1** **A1**	Forms equation for motion parallel to the wall Correct v Forms equation for motion perpendicular to the wall. Eliminates v Correct value for e AWRT 0.33
		5	

Q	Answer	Marks	Comments
2(b)	$I = 0.08 \times \dfrac{3\cos 60°}{\cos 30°} 3\sin 30° - 0.08 \times \left(-3\sin 60°\right)$ $= \dfrac{3\sqrt{3}}{25} + \dfrac{\sqrt{3}}{25}$ $= \dfrac{4\sqrt{3}}{25}$ [N s]	**M1** **A1** **A1**	Forms equation to find impulse. Allow sign errors and their v Correct equation Correct impulse in exact form. Accept $0.16\sqrt{3}$
		3	

	Question 2 Total	**8**	

Q	Answer	Marks	Comments
3(a)	$4 = \frac{2\pi}{\omega}$ $\omega = \frac{\pi}{2}$ $6 = a \times \frac{\pi}{2}$ $a = \frac{12}{\pi}$ [m]	 B1 M1 A1	 Correct ω Forms equation to find the amplitude using their ω Correct amplitude
		3	

Q	Answer	Marks	Comments
3(b)	$5^2 = \left(\frac{\pi}{2}\right)^2\left(\left(\frac{12}{\pi}\right)^2 - x^2\right)$ $x = \pm\frac{\sqrt{44}}{\pi} = \pm\frac{2\sqrt{11}}{\pi}$ [m]	M1 A1 A1	Forms equation to find displacement At least one correct displacement Both displacements correct and no others. Accept $\pm\frac{6.6}{\pi}$ and $\pm\frac{\sqrt{44}}{\pi}$
		3	

	Question 3 Total	**6**	

Q	Answer	Marks	Comments
4	$2\dfrac{\mathrm{d}v}{\mathrm{d}t} = -0.1\times 2\times 9.8 - 0.49v^2$	**M1**	Forms a three term differential equation using $F = ma$
		A1	Correct differential equation
	$\dfrac{\mathrm{d}v}{\mathrm{d}t} = -0.245(4+v^2)$		
	$\int \dfrac{1}{4+v^2}\mathrm{d}v = \int -0.245\mathrm{d}t$	**M1**	Separates the variables
		M1	Integrates to obtain a $\tan^{-1}$ term
	$\dfrac{1}{2}\tan^{-1}\left(\dfrac{v}{2}\right) = -0.245t + c$	**A1**	Correct integration. Condone missing constant of integration.
	$t = 0,\ v = 20 \Rightarrow c = \dfrac{1}{2}\tan^{-1}(10)$	**M1** **A1**	Finds c Correct c
	$v = 0$	**M1**	Substitutes $v = 0$
	$0 = -0.245t + \dfrac{1}{2}\tan^{-1}(10)$		
	$t = \dfrac{1}{0.49}\tan^{-1}(10) = 3.0\,[\mathrm{s}]$	**A1**	Correct time. **AWRT** 3.0, such as 3.002
		9	

	Question 4 Total	**9**	

Q	Answer	Marks	Comments
5(a)	$2.5\times 9.8 = k\times 0.05$ $k = 490$	**M1** **A1**	Equation to find stiffness Allow $5k$ instead of $0.05k$ Correct stiffness Allow 490.5 from g = 9.81
		2	

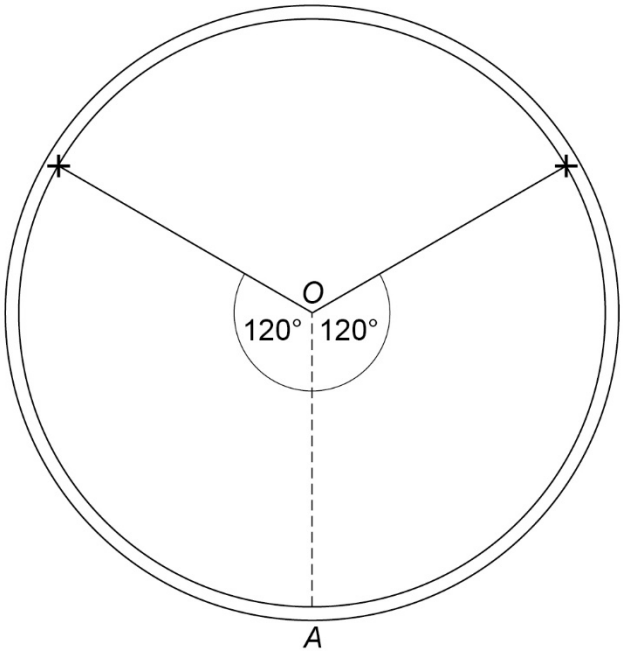
Q	Answer	Marks	Comments
5(b)(i)	$x =$ Displacement below equilibrium position $2.5\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = 2.5\times 9.8 - 490(0.05 + x)$ $2.5\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = 24.5 - 24.5 - 490x$ $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = -196x$ $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} \propto -x$ $\therefore$ Simple Harmonic Motion	**M1** **A1** **A1** **A1** **A1**	Forms three term differential equation At least two correct terms Correct differential equation Correct simplified differential equation Correct conclusion Allow 196.2 from g = 9.81
		5	

Q	Answer	Marks	Comments
5(b)(ii)	$2 = a\times 14$ $a = \dfrac{2}{14} = \dfrac{1}{7}\,[\mathrm{m}]$	**M1** **A1**	Equation to find amplitude based on $v_{\max} = a\times\omega$ with their ω Correct amplitude Allow $\dfrac{2}{\sqrt{196.2}}$ from g = 9.81
		2	

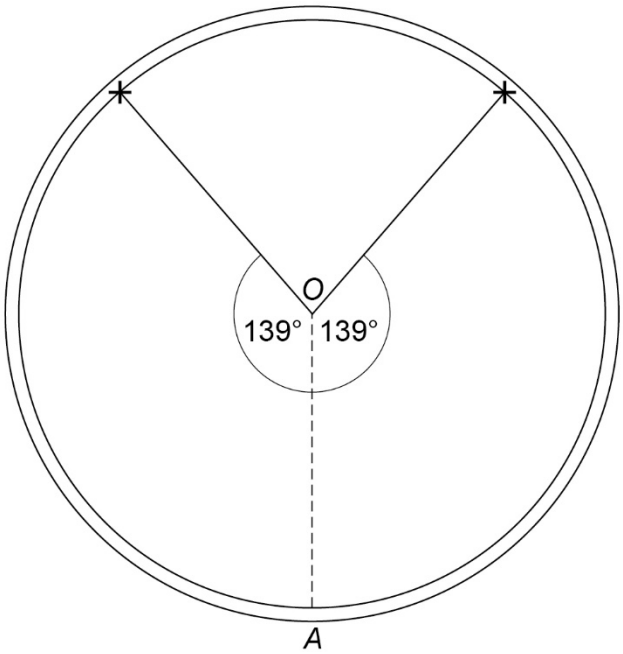
Q	Answer	Marks	Comments
5(b)(iii)	$x = \frac{1}{7}\sin(14t)$	**B1**	Expression for the displacement Accept $x = A\sin(\omega t)$ or $x = A\cos\left(\omega t - \frac{\pi}{2}\right)$ with their A and ω
	$0.1 = \frac{1}{7}\sin(14t)$	**M1**	Equation to find time for displacement of 10 cm with their A and ω Allow 10 instead of 0.1
	$t = 0.0554$	**A1**	Correct time Note g = 9.81 gives 0.0554 to 3 sf
	Period $= \frac{2\pi}{14} = \frac{\pi}{7} = 0.4488$	**B1**	Correct period, such as 0.4488 s, for their ω Note 0.4486 from g = 9.81
	$\frac{4 \times 0.0554}{0.4488} \times 100 = 49\%$	**M1**	Calculation to find percentage using their time
		A1	Correct percentage Note g = 9.81 gives 49% to 2 sf
		6	

	Question 5 Total	**15**	

Q	Answer	Marks	Comments
6(a)	$\frac{1}{2}mU^2 = \frac{1}{2}mv^2 + mga(1-\cos\theta)$	M1	Uses conservation of energy.
	$v^2 = U^2 - 2ga(1-\cos\theta)$	A1	Correct energy equation.
	$R - mg\cos\theta = \frac{mv^2}{a}$	M1 A1	Resolves along a radius. Correct equation.
	$R = m\left(\frac{U^2}{a} - 2g + 3g\cos\theta\right)$	A1	Obtains given result from correct working **AG**
		5	

Q	Answer	Marks	Comments
6(b)(i)	$R = m\left(\frac{7ag}{2a} - 2g + 3g\cos\theta\right)$ $= m\left(\frac{7g}{2} - 2g + 3g\cos\theta\right)$ $0 = \frac{7g}{2} - 2g + 3g\cos\theta$	M1	Substitutes for U and sets $R = 0$
	$\cos\theta = -\frac{1}{2}$	A1	Obtains correct value for $\cos\theta$
	$\theta = 120°$ **or** $-120°$	A1	Obtains at least one correct value for θ
		A1	Shows both positions correctly on the diagram. Accept 120° or 240°, $\pm\frac{2\pi}{3}$, $\frac{2\pi}{3}$ or $\frac{4\pi}{3}$ Accept answers in radians (±2.09 or 4.19).

		4	

Q	Answer	Marks	Comments
6(b)(ii)	$v^2 = ga\left(\frac{7}{2} - 2(1-\cos\theta)\right)$ $v^2 = ga\left(\frac{3}{2} + 2\cos\theta\right)$ $0 = \frac{3}{2} + 2\cos\theta$ $\cos\theta = -\frac{3}{4}$ $\theta = 139°$ **or** $-139°$  	**M1** **A1** **A1** **A1**	Substitutes for U and sets $v = 0$ Obtains correct value for $\cos\theta$ Obtains at least one correct value for θ Shows both positions correctly on the diagram Accept 139° or 221° Accept answers in radians (±2.42 or 3.86)
		4	

	Question 6 Total	**13**	

Q	Answer	Marks	Comments
7	Max KE when rod at equilibrium position.		
	$T_1 = \frac{4mg}{d}e$	**M1**	Finds tensions in both strings at equilibrium
	$T_2 = \frac{3mg}{d}(2d-e)$	**A1**	Correct tensions
	$mg + 2\times\frac{3mg}{d}(2d-e) = \frac{4mg}{d}e$	**M1**	Equation to find extension at equilibrium with three terms
	$d + 12d - 6e = 4e$		
	$e = \frac{13d}{10}$	**A1**	Correct equation
		A1	Correct extension
	$\text{Initial EPE} = \frac{1}{2}\times\frac{4mg}{d}\times(2d)^2 = 8mgd$	**B1**	Correct initial EPE
	$8mgd = mg\times\frac{7d}{10} + \frac{1}{2}\times\frac{4mg}{d}\times\left(\frac{13d}{10}\right)^2 + 2\times\frac{1}{2}\times\frac{3mg}{d}\times\left(\frac{7d}{10}\right)^2 + \text{KE}$	**M1**	Five term energy equation
	$8mgd = \frac{7mgd}{10} + \frac{169mgd}{50} + \frac{147mgd}{100} + \text{KE}$	**A1**	At least three terms correct
	$\text{KE} = \frac{49mgd}{20} = 2.45mgd$	**A1**	All terms correct
		A1	Correct KE
		10	

	Question 7 Total	**10**	

Q	Answer	Marks	Comments
8(a)	$25\sin 30°t - \frac{1}{2}g\cos 20°t^2 = U\sin 60°t - \frac{1}{2}g\cos 20°t^2$	M1	Equation for motion perpendicular to the plane
		A1	Correct equation
	$25\sin 30°t = U\sin 60°t$	M1	Solves for U
	$U = \frac{25\sin 30°}{\sin 60°} = \frac{25\sqrt{3}}{3}$	A1	Correct U
	$25\cos 30°t + \frac{1}{2}g\sin 20°t^2 = 10 + U\cos 60°t + \frac{1}{2}g\sin 20°t^2$	M1	Equation for motion parallel to the plane
		A1	Correct equation
	$25\cos 30°t = 10 + \frac{25\sqrt{3}}{3}\cos 60°t$	M1	Solves for t
	$\frac{25\sqrt{3}}{2}t = 10 + \frac{25\sqrt{3}}{6}t$	A1	Any correct version of t AWRT 0.69
	$t = \frac{2\sqrt{3}}{5}$ s	A1	Correct t in exact form
		9	

Q	Answer	Marks	Comments
8(b)	$y_{\max} = 25\sin 30° \times \frac{2\sqrt{3}}{5} - \frac{1}{2}g\cos 20°\left(\frac{2\sqrt{3}}{5}\right)^2$	M1	Substitutes their time into correct equation
	$= 6.5$ m	A1	Obtains correct height. **AWFW** 6.4 to 6.5, such as 6.450097
		2	

	Question 8 Total	**11**	