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FURTHER MATHEMATICS

FM04

(9665/FM04) Unit FS2 Statistics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1(a)	H_0: There is **not** an association between use of app and passing maths examination H_1: There **is** an association between use of app and passing maths examination	B1	Must have both H_0 and H_1 Condone 'change the chance' in place of 'association'
		1	

Q	Answer	Marks	Comments
1(b)	$\sum \frac{\left(\lvert O-E\rvert-0.5\right)^2}{E}$ $=\frac{\left(\lvert 72-66\rvert-0.5\right)^2}{66}+\frac{\left(\lvert 28-34\rvert-0.5\right)^2}{66}+\frac{\left(\lvert 60-66\rvert-0.5\right)^2}{34}+\frac{\left(\lvert 40-34\rvert-0.5\right)^2}{34}$	**M1**	Allow **SC1** for using $\sum \frac{(O-E)^2}{E}$ (i.e. not using Yates correction) which is possibly implied by 3.2085…
	$\left[=\frac{5.5^2}{66}+\frac{5.5^2}{66}+\frac{5.5^2}{34}+\frac{5.5^2}{34}\right]$		
	$=2.6960\left[78\ldots\right]$	**A1**	**AG oe** $\frac{275}{102}$ each term shown
		2	

Q	Answer	Marks	Comments
1(c)	Degrees of freedom [dof], $\nu = 1$ cv = $\chi_1^2(0.9) = 2.706$ $2.696 < \chi_1^2(0.9) = 2.706$, do not reject H_0 Evidence to suggest there is **not** an association between use of app and passing maths examination	**B1** **B1** **M1** **A1**	**PI** Critical value for $\nu = 1$, **AWRT** 2.71 **PI**, allow 'accept H_0' **ft** their critical value. If their cv < 2.696, must see reject H_0 Must be correct contextual statement and consistent with their cv Allow 'support company's belief'
		4	
	Question 1 Total	**7**	

Q	Answer	Marks	Comments
2	$\text{P}\left(\bar{X} < 0.16 \mid \mu = 0.1468\right)$ $= \text{P}\left(z < \dfrac{0.16 - 0.1468}{0.04}\right)$ P($z <$ 0.33) = 0.6293 Power = 1 – 0.6293 = 0.371 [to 3 sf]	**M1** **A1** **A1**	Correct probability statement or standardises correct probability (**PI** by 0.33 seen)
	Question 2 Total	**3**	

Q	Answer	Marks	Comments
3(a)	H_0: Distribution is uniform H_1: Distribution is not uniform	B1	
	dof $v = 6 - 1 = 5$	B1	**PI** by correct critical value
	expected values = 50	B1	Seen or used
	$\sum\frac{(O-E)^2}{E} = \frac{(50-50)^2}{50} + \frac{(43-50)^2}{50} + \frac{(38-50)^2}{50} + \frac{(63-50)^2}{50} + \frac{(61-50)^2}{50} + \frac{(45-50)^2}{50}$	M1	**PI**
	$= 10.16$	A1	**oe** $\frac{254}{25}$
	$\chi_5^2(0.99) = 15.086$	B1	Finds critical value
	$10.16 < 15.086$, do not reject H_0	A1ft	Allow 'accept H_0' **ft** their test statistic and critical value Implied by correct conclusion in context
	Evidence to suggest that the die is **fair**	E1	Must be consistent with their conclusion on whether to accept H_0 **or** not **or** their test statistic and critical value if not explicitly stated Must not be definite
		8	

Q	Answer	Marks	Comments
3(b)	$\chi_5^2(0.90) = 9.236$ **and** rejection of H_0 with their $10.16 > 9.236$	B1ft	**ft** their degrees of freedom from **(a)** [Note $\chi_4^2(0.90) = 7.779$]
	For the **higher** significance level, there is a **lower** χ^2 value rejection of H_0 **or** the critical region (tail) is **increased**	E1ft	**oe**
		2	
	Question 3 Total	10	

Q	Answer	Marks	Comments
4(a)	Both T and V are: Functions of the **random variables** of a **sample and** not dependent on **population parameters**	E2	Must contain emboldened key words **E1** for one of the three statements: • Uses **Random Variables** • Calculated from a **sample** (Allow **observations**) • Not dependent on any **population parameters** (Allow **unknown parameters**)
		2	

Q	Answer	Marks	Comments
4(b)	$E(T) = \sum_{k=1}^{n} E(X_k) = \sum_{k=1}^{n} \mu$	M1	Allow X for X_k
	$= n\mu \neq \mu$ [therefore not unbiased]	A1	Must see $n\mu \neq \mu$
		2	

Q	Answer	Marks	Comments
4(c)(i)	$\text{Var}(X_k) = \text{E}(X_k^2) - \text{E}(X_k)^2$ $\text{E}(X_k^2) = \text{Var}(X_k) + \text{E}(X_k)^2$ $\left[\text{Var}(X_k) = \sigma^2,\ \text{E}(X_k) = \mu\right]$		Allow X for X_k
	$\text{E}(X_k^2) = \sigma^2 + \mu^2$	B1	**AG** Be convinced
		1	

Q	Answer	Marks	Comments
4(c)(ii)	$\text{Var}(T)=\text{E}(T^2)-\text{E}(T)^2$	M1	Either for rearranging **or** $\text{Var}(T)=n\sigma^2$
	$\text{Var}(T)=n\text{Var}(X)=n\sigma^2$		
	$\text{E}(T)^2=(n\mu)^2=n^2\mu^2$		
	$\text{E}(T^2)=n\sigma^2+n^2\mu^2$	A1	**AG**, must see evidence of rearranging and $\text{Var}(T)=n\sigma^2$
		2	

Q	Answer	Marks	Comments
4(d)	$\sum_{k=1}^{n}E(X_k^2)=n(\sigma^2+\mu^2)$	B1	Seen or used
	$E\left(\frac{nV}{n-1}\right)=\frac{n}{n-1}\left(\frac{1}{n}n(\sigma^2+\mu^2)-\frac{(n\sigma^2+n^2\mu^2)}{n^2}\right)$	M1	Requires substitution of their $\sum_{k=1}^{n}E(X_k^2)$
	$=\sigma^2$, therefore unbiased	A1	Must see σ^2 and conclusion
		3	
	Question 4 Total	**10**	

Q	Answer	Marks	Comments
5(a)	$\bar{x} = 32.82$	**B1**	
	$s^2 = \frac{1}{10-1}\left(10843.9 - \frac{328.2^2}{10}\right)$	**M1**	
	$s^2 = 8.0417$ or $s = 2.8358(02845)$	**A1**	**AWRT** 8.04 (s^2) or 2.84 (s) **oe** $s^2 = \frac{9047}{1125}$
	$t_9(0.99) = 2.821$	**B1**	
	$32.82 \pm 2.821\sqrt{\frac{8.0417..}{10}}$	**M1**	Calculates confidence interval limits with their mean, their sample variance And their t-value. **PI** by correct answer
	(30.29, 35.35)	**A1**	**CAO**
		6	

Q	Answer	Marks	Comments
5(b)	$z = [+]\ 2.3263$	**B1**	Seen or used, **AWRT** 2.326
	$\sigma = 3$	**B1**	**PI**
	$0.5 > 2.3263\sqrt{\frac{3^2}{n}}$	**M1**	**ft** their z Allow = sign
	n = 195 [from 194.828….]	**A1**	Must be 195 **and not** 194 if z used If t used accept $n > 198$ from t = 2.345 to 2.351
		4	
	Question 5 Total	**10**	

Q	Answer	Marks	Comments
6(a)(i)	$\mathrm{E}\left(\bar{X}\right) = \mu$	**B1**	
	$\mathrm{Var}\left(\bar{X}\right) = \dfrac{\sigma^2}{n}$	**B1**	
		2	

Q	Answer	Marks	Comments
6(a)(ii)	$\mathrm{E}\left(\bar{X}\right) = \mu$, so estimator is unbiased	**B1**	Conclusion required
	$\mathrm{Var}\left(\bar{X}\right) \to 0$ as $n \to \infty$, so estimator is consistent	**B1**	Conclusion required
		2	

Q	Answer	Marks	Comments
6(b)	Efficiency $\dfrac{1}{\mathrm{Var}\left(\bar{X}_A\right)} = \dfrac{40}{\sigma^2}$ **or** $\dfrac{1}{\mathrm{Var}\left(\bar{X}_B\right)} = \dfrac{60}{\sigma^2}$	**M1**	Either expression or maybe seen in Relative Efficiency
	Relative Efficiency $= \dfrac{\left(\dfrac{1}{\mathrm{Var}\left(\bar{X}_B\right)}\right)}{\left(\dfrac{1}{\mathrm{Var}\left(\bar{X}_A\right)}\right)} = \dfrac{\left(\dfrac{60}{\sigma^2}\right)}{\left(\dfrac{40}{\sigma^2}\right)}$		
	Relative Efficiency = 1.5	**A1**	**AG**
		2	

Q	Answer	Marks	Comments
6(c)	$\text{Var}(T) = p^2\ \text{Var}(\bar{X}_A) + (1-p)^2\ \text{Var}(\bar{X}_B)$ $\dfrac{\mathrm{d}(\text{Var}(T))}{\mathrm{d}p} = 2p\dfrac{\sigma^2}{40} + 2(p-1)\dfrac{\sigma^2}{60}$	B1	**PI** by correct derivative
	and $\dfrac{\mathrm{d}(\text{Var}(T))}{\mathrm{d}p} = 0$	M1	May be seen as derivative of Efficiency
	$2p\dfrac{\sigma^2}{40} + 2(p-1)\dfrac{\sigma^2}{60} = 0$, leading to $p = 0.4$	A1	**oe** by completing the square
	$\dfrac{\mathrm{d}^2(\text{Var}(T))}{\mathrm{d}p^2} = \dfrac{\sigma^2}{12} > 0$, so minimum variance **or** maximum efficiency	B1	Conclusion required with check
		4	
	Question 6 Total	**10**	

Q	Answer	Marks	Comments
7(a)	$S_m^2 = 2140.8$ or $S_f^2 = 2094.36...$ $[S_m = 46.2687....$ or $S_f = 45.7641....]$	B1	**PI** allow any choice of subscripts or labelling, $S_m^2 = \frac{10704}{5}$, $S_f^2 = \frac{75397}{36}$ **AWRT** 2141 or 2094 or **AWRT** 46.27 or 45.76
	$S_p^2 = \frac{(n_m - 1)S_m^2 + (n_f - 1)S_f^2}{n_m + n_f - 2}$ $n_m = 11$, $n_f = 9$	M1	**oe**
	$S_p^2 = 2120$	A1	**AWRT** 2120
		3	

Q	Answer	Marks	Comments
7(b)	H_0: $\mu_m = \mu_f$ H_1: $\mu_m > \mu_f$	B1	Both hypotheses
	$\overline{m} - \overline{f} = 274 - 232.89 = 41.11$	M1	**oe** $274 - \frac{2096}{9} = \frac{370}{9}\left[= 41\frac{1}{9}\right]$
	$t_{\text{calc}} = \frac{41.11 - 0}{\sqrt{2120\left(\frac{1}{11} + \frac{1}{9}\right)}}$	M1	**ft** their $\overline{m} - \overline{f}$ and their answer to **(a)**
	$= 1.986[47]$	A1	**AWRT** 1.986 **or** 1.987 $p = 0.0312(12)$
	dof $v = [11 + 9 - 1 - 1 =]$ 18	B1	**PI** by correct critical value
	$t_{\text{crit}} = t_{18}(0.95) = 1.734$	B1	
	$1.986 > 1.734$, reject H_0		
	Evidence to support that the mean number of platelets is greater for males than females	E1ft	Must be in context Statement should not be definitive and not contradict any **ft** accepting H_0
		7	

Q	Answer	Marks	Comments
7(c)	H0: $\sigma_m^2 = \sigma_f^2$ H1: $\sigma_m^2 \neq \sigma_f^2$	B1	Both hypotheses
	$F_{calc} = \frac{S_m^2}{S_f^2} = \frac{2140.8}{2094.36...}$	M1	**ft** on variances
	$= 1.0223\ldots$	A1	**AWRT** 1.022
	$v_1 = 10,\ v_2 = 8$	B1	Both dof correct, **PI** from correct F_{crit}
	$F_{10,8}(0.95) = 4.295$	B1	**PI** , $p = 0.9947$
	$1.0223 < 4.295$, insufficient evidence for the variance in number of platelets between males and females to be different from zero. The assumption is supported.	E1	No need to compare with other lower critical value [0.2594] as $F_{calc} > 1$ Condone omission of statement context. Statement should not be definite.
		6	
	Question 7 Total	**16**	

Q	Answer	Marks	Comments
8(a)	$M_{X_k}(t) = E\left(e^{tX_k}\right) = \sum_{x=1}^{\infty} e^{tx} p(1-p)^{x-1}$	M1	Applies mgf formula
	$= pe^t \sum_{x=1}^{\infty}\left((1-p)e^t\right)^{x-1}$	A1	Correct formula Must start from $x = 1$ and sum to infinity or imply infinite series
	$[S_\infty =] \quad \dfrac{pe^t}{1-(1-p)e^t} = \dfrac{p}{e^{-t}-(1-p)}$	M1 A1	**M1:** Identify as geometric progression, $a = pe^t, \quad r = (1-p)e^t$ **oe** [This may be seen in S_∞] **A1: AG** Be convinced
		4	

Q	Answer	Marks	Comments
8(b)	$M'_{X_k}(t) = \dfrac{pe^{-t}}{\left(e^{-t}-(1-p)\right)^2}$	M1	Attempt to differentiate
	$\mu = M'_{X_k}(0) = \dfrac{p}{p^2} = \dfrac{1}{p}$	A1	Substitutes $t = 0$ into correct expression and gives correct answer
		2	

Q	Answer	Marks	Comments
8(c)(i)	$\left[\left(M_{X_k}(t)\right)^2 =\right] \dfrac{\left(\frac{1}{6}\right)^2}{\left(\mathrm{e}^{-t}-\left(1-\frac{1}{6}\right)\right)^2}$	M1	$\dfrac{p^2}{\left(\mathrm{e}^{-t}-(1-p)\right)^2}$
	$=\dfrac{1}{\left(6\mathrm{e}^{-t}-5\right)^2}$	A1	**AG**
		2	

Q	Answer	Marks	Comments
8(c)(ii)	$\left[\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{p^2}{\left(\mathrm{e}^{-t}-(1-p)\right)^2}\right)=\right] \dfrac{12\mathrm{e}^{-t}}{\left(6\mathrm{e}^{-t}-5\right)^3}$	M1	Attempt at differentiation
	When $t=0$, $\mu=\dfrac{12\mathrm{e}^0}{\left(6\mathrm{e}^0-5\right)^3}$		
	$\mu=12$	A1	**AG** Be convinced Must see clear evidence of use of $t=0$
		2	

Q	Answer	Marks	Comments
8(d)	$\left[M_Y(t)=\right]\ \dfrac{p^n}{\left(\mathrm{e}^{-t}-(1-p)\right)^n}=\dfrac{1}{\left(6\mathrm{e}^{-t}-5\right)^n}$	B1	Identifies correct moment generating function
	$\left[M_Y'(t)=\right]\ \dfrac{np^n\mathrm{e}^{-t}}{\left(\mathrm{e}^{-t}-(1-p)\right)^{n+1}}=\dfrac{6n\mathrm{e}^{-t}}{\left(6\mathrm{e}^{-t}-5\right)^{n+1}}$		
	$\left[M_Y''(t)=\right]\ \dfrac{6n\mathrm{e}^{-t}\left(6n\mathrm{e}^{-t}+5\right)}{\left(6\mathrm{e}^{-t}-5\right)^{n+2}}$	M1	Attempt to find first derivative **and** second derivative
	Use of $\sigma^2=M_Y''(0)-M_Y'(0)^2$	M1	Must substitute $t=0$ correctly
	$M_Y'(0)=\dfrac{6n}{(6-5)^{n+1}}=6n$		
	$M_Y''(0)=\dfrac{6n(6n+5)}{(6-5)^{n+2}}=36n^2+30n$		
	$\sigma^2=36n^2+30n-(6n)^2$		
	$\sigma^2=30n$	A1	
		4	
	Question 8 Total	**14**	