OXFORD

INTERNATIONAL AQA EXAMINATIONS

INTERNATIONAL A-LEVEL

FURTHER MATHEMATICS

FM04

(9665/FM04) Unit FS2 Statistics

Mark scheme

January 2020

Version: 1.0 Final

201xFM04/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from oxfordaqaexams.org.uk

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1	TT median 0 TH or HT median 5 TT median 10	M1 m1	Identify at least one possible combination and median Identify all possible combinations and medians
	<table><tr><td>m</td><td>0</td><td>5</td><td>10</td></tr><tr><td>$\mathrm{P}(M = m)$</td><td>$\frac{1}{4}$</td><td>$\frac{1}{2}$</td><td>$\frac{1}{4}$</td></tr></table>	A1	Accept in function form
	Total	**3**	

Q	Answer	Marks	Comments
2(a)	Relative Efficiency = $\dfrac{\frac{100}{\sigma^2}}{\frac{10}{\sigma^2}}$	M1	Uses relative efficiency formula oe
	= 10	A1	
	Linda's estimator is more efficient than Andrew's estimator	E1	oe
2(b)(i)	$\mathrm{E}(P) = \dfrac{E\left(\overline{X_1}\right)+E\left(\overline{X_2}\right)}{2} = \dfrac{\mu+\mu}{2}$	M1	
	= μ therefore unbiased	A1	
2(b)(ii)	$\mathrm{Var}(P) = \dfrac{Var\left(\overline{X_1}\right)+Var\left(\overline{X_2}\right)}{2^2} = \dfrac{\frac{\sigma^2}{10}+\frac{\sigma^2}{100}}{2^2}$	M1	
	= $\dfrac{11\sigma^2}{400}$	A1	oe
	Total	**7**	

Q	Answer	Marks	Comments
3(a)	$\bar{x} = 66.404$	B1	
	$s^2 = \frac{1}{500-1}\left(2210000 - \frac{33202^2}{500}\right)$	M1	Attempt at variance formula Allow one slip Implied by correct answer
	= 10.5(2984369)	A1	AWRT 10.5
	z = 2.0537	B1	AWRT 2.054
	$66.404 \pm 2.0537\sqrt{\frac{10.52...}{500}}$	M1	Applies confidence interval formula with their values
	(66.106, 66.702)	A1	AWRT
3(b)	Rejects as 67 is outside the confidence interval	E1ft	Follow through their confidence interval
3(c)	Still valid as sample size is large enough	E1	oe
	for central limit theorem to apply	E1	oe
	Total	**9**	

Q	Answer	Marks	Comments
4(a)(i)	P($X \leq$ 25) = 0.9427 P($X \leq$ 26) = 0.9686	M1	Both seen
	Accept if $X \leq$ 26	M1	oe
	B(50, 0.5)	M1	Use of PI
	Type II probability = 0.664	A1	AWRT
4(a)(ii)	Power = 1 – 0.664	M1	Applies power formula with their Type II probability
	= 0.336	A1F	AWRT
4(b)	The power of the test would increase	E1	
	Total	**7**	

Q	Answer	Marks	Comments
5	H_0: $\sigma_A = \sigma_B$ H_1: $\sigma_A \neq \sigma_B$	**B1**	Both hypotheses
	dof 20, 25	**B1**	
	F = $\dfrac{s_A^2}{s_B^2} = \dfrac{0.3^2}{0.2^2}$	**M1**	
	= 2.25	**A1**	
	$F_{20,25}$ = 2.70	**B1**	AWRT
	Accept H_0	**A1ft**	Follow through their F and $F_{20,25}$ Implied by correct conclusion in context
	Evidence to suggest/support that the population variances of machines *A* and *B* are equal	**E1**	Must be consistent with their conclusion on whether to accept H_0 or not or their F and $F_{20,25}$ if not explicitly stated
	Total	**7**	

Q	Answer	Marks	Comments
6	H_0: There is no association between the office an employee works in and the transport they use to get to work H_1: There is an association between the office an employee works in and the transport they use to get to work	B1	Both hypotheses, variables must be referenced in at least the null hypothesis oe
	(see table A below)	M1	At least three values AWRT PI
		A1	All values PI
	(see tables B below)	M1	Combine two columns for expected and actual values
	dof $v = 2$	B1	
	$\sum\frac{(O-E)^2}{E}=\frac{(4-13.6)^2}{13.6}+\frac{(22-12.4)^2}{12.4}+\frac{(32-24.5...)^2}{24.5...}+\frac{(15-22.4...)^2}{22.4...}+\frac{(32-29.8...)^2}{29.8...}+\frac{(25-27.1...)^2}{27.1...}$	M1	Attempts $\sum\frac{(O-E)^2}{E}$ using their table even if columns not combined
	= 19.2 (B+T combined) or = 17.9 (C+T combined) or = 21.3 (W+T combined)	A1	Accept 19.1 AWRT

Table A:

	W	C	B	T
N	13.6	24.6	26.2	3.66
S	12.4	22.4	23.8	3.34

Tables B:

	B+T Expected	B+T Actual
N	29.8	32
S	27.2	25

or

	C+T Expected	C+T Actual
N	28.2	34
S	25.7	20

or

	W+T Expected	W+T Actual
N	17.3	6
S	15.7	27

	Test statistic = 9.2	**B1**	AWRT
	Reject H_0	**A1ft**	Follow through their $\sum\frac{(O-E)^2}{E}$ and test statistic Implied by correct conclusion in context
	Evidence to suggest/support that there is an association between the office an employee works in and the transport they use to get to work	**E1**	Must be consistent with their conclusion on whether to accept H_0 or not or their $\sum\frac{(O-E)^2}{E}$ and test statistic if not explicitly stated
	Total	**10**	

Q	Answer	Marks	Comments
7(a)	H_0: $\mu_A = \mu_B$ H_1: $\mu_A \neq \mu_B$	B1	Both hypotheses
	$z = \dfrac{\lvert 504 - 502 \rvert}{\sqrt{\dfrac{2^2}{10} + \dfrac{3^2}{12}}}$	M1 M1	Numerator Denominator
	= 1.87	A1	AWRT
	z_{crit} = 1.96 or -1.96	B1	Seen anywhere
	Accept H_0	A1ft	Follow through their z and z_{crit} Implied by correct conclusion in context
	Evidence to suggest/support that the mean mass of jars of honey is the same for both suppliers	E1	Must be consistent with their conclusion on whether to accept H_0 or not or their z and z_{crit} if not explicitly stated
7(b)	t test/values instead of z test/values	E1	oe
	with dof v = 20	E1	oe
	Calculate pooled estimate of variance S_P^2	E1	oe
	Replace $\dfrac{\sigma_A^2}{n_A} + \dfrac{\sigma_B^2}{n_B}$ with $s_p^2\left(\dfrac{1}{n_A} + \dfrac{1}{n_B}\right)$	E1	oe
	Total	**11**	

<table>
<tr><th>Q</th><th>Answer</th><th>Marks</th><th>Comments</th></tr>
<tr><td>8(a)</td><td>H_0: $\mu_B = \mu_A$
H_1: $\mu_B < \mu_A$</td><td>B1</td><td>Both hypotheses</td></tr>
<tr><td></td><td><table><tr><th>Student</th><th>Difference</th></tr><tr><td>1</td><td>4</td></tr><tr><td>2</td><td>1</td></tr><tr><td>3</td><td>0</td></tr><tr><td>4</td><td>-1</td></tr><tr><td>5</td><td>2</td></tr><tr><td>6</td><td>3</td></tr></table></td><td>B1</td><td>All differences found</td></tr>
<tr><td></td><td>$\bar{x} = 1.5$</td><td>B1</td><td></td></tr>
<tr><td></td><td>$s^2 = \frac{1}{6-1}\left(31 - \frac{9^2}{6}\right)$</td><td>M1</td><td>Attempt at variance formula
Allow one slip
Implied by correct answer</td></tr>
<tr><td></td><td>= 3.5</td><td>A1</td><td>Accept AWRT s = 1.87</td></tr>
<tr><td></td><td>dof v = 5</td><td>B1</td><td></td></tr>
<tr><td></td><td>$t = \frac{1.5}{\sqrt{\frac{3.5}{6}}}$</td><td>M1</td><td>Using their s^2</td></tr>
<tr><td></td><td>= 1.96</td><td>A1</td><td>AWRT</td></tr>
<tr><td></td><td>t_5 cv = 2.02</td><td>B1</td><td>AWRT</td></tr>
<tr><td></td><td>Accept H_0</td><td>A1ft</td><td>Follow through their t and t_5
Implied by correct conclusion in context</td></tr>
<tr><td></td><td>No evidence to suggest/support that the tutorial company's claim is true</td><td>E1</td><td>Must be consistent with their conclusion on whether to accept H_0 or not or their t and t_5 if not explicitly stated</td></tr>
<tr><td>8(b)</td><td>The difference between test scores before and after the revision course</td><td>E1</td><td>oe</td></tr>
<tr><td></td><td>are normally distributed</td><td>E1</td><td>oe</td></tr>
<tr><td></td><td>Total</td><td>13</td><td></td></tr>
</table>

Q	Answer	Marks	Comments
9(a)	$M'_X(t) = \ldots e^t(1-0.8e^t)^{-3}$	M1	$M'_X(t) = Ae^t(1-0.8e^t)^{-3}$ where A is a constant
	$M'_X(t) = 0.064e^t(1-0.8e^t)^{-3}$	A1	oe May be unsimplified
	Mean = $M'_X(0) = 0.064e^0(1-0.8e^0)^{-3}$	M1	Attempts $M'_X(0)$
	= 8	A1	
9(b)	$M''_X(t) = \ldots e^t(1-0.8e^t)^{-3} + \ldots e^{2t}(1-0.8e^t)^{-4}$	M1	$M''_X(t) = Ae^t(1-0.8e^t)^{-3} + Be^{2t}(1-0.8e^t)^{-4}$ where A and B are constants oe
	$M''_X(t) = 0.064e^t(1-0.8e^t)^{-3} + 0.1536e^{2t}(1-0.8e^t)^{-4}$	A1	oe May be unsimplified
	Variance = $M''_X(0) - (M'_X(0))^2$ $= 0.064e^0(1-0.8e^0)^{-3} + 0.1536e^{2\times 0}(1-0.8e^0)^{-4} - 8^2$	M1	Applies variance formula with their $M''_X(t)$ and mean
	= 40	A1	
9(c)	$M_{2+3X}(t) = e^{2t}M_X(3t)$	M1	
	$= e^{2t} \times 0.04(1-0.8e^{3t})^{-2}$	M1	
	$= \left(\dfrac{0.2e^t}{1-0.8e^{3t}}\right)^2$	A1	
9(d)	$M_{X+Y}(t) = M_X(t)\,M_Y(t)$ $= 0.04(1-0.8e^t)^{-2}(0.8+0.2e^t)^2$	M1	
	$= \left(\dfrac{0.16+0.04e^t}{1-0.8e^t}\right)^2$	A1	
	Total	**13**	