OXFORD

INTERNATIONAL
AQA EXAMINATIONS

International AS

Further Mathematics

FM02 – Unit 1 Further Pure, Statistics and Mechanics

Mark scheme

9665

June 2018

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
ft	Follow through from previous incorrect result
CAO	Correct and answer only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

No method shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Answer	Mark	Total	Comments
1(a)	1, 0.5, 0.2	B1	2	
	13.69, 9.61, 6.76	B1		1 or 2 d.p.
1(b)	Three points plotted	M1 A1	3	M1 A0 if two correct
	Straight line drawn through points from $x = 0$ to $x = 1$	B1		
1(c)	$b = 5.2$	B1	3	[5, 5.5] Must equal the intercept on their graph
	Gradient found	M1		
	$a = 8.5$	A1		[8.2, 8.9]
1(d)	$y^2 = \frac{8.5}{x} + 5.2$	B1F	1	ft on 'their' a and b
1(e)	$\left[y^2 = \frac{8.5}{0.5} + 5.2\right]$		1	
	$y = 4.7$	B1F		ft if between 3.7 and 10
Total	**10**			

Q	Answer	Mark	Total	Comments
2	$hy'(2) = 0.2 \times \sqrt{(2^3 - 3 \times 2)}$ $(= 0.2828427...)$	M1 A1	5	
	$y_2 = 5 + 0.2828427 = 5.2828427$	dM1		5 + 'their' value of hy'
	$hy'(2.2) = 0.2 \times \sqrt{(2.2^3 - 3 \times 2.2)}$ $(= 0.4023928)$	dM1		
	5.6852	A1		CAO
Total	**5**			

Q	Answer	Mark	Total	Comments
3(a)	Shear	B1	1	
3(b)	$\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$	B1	1	
3(c)	$A^{-1} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$	B1	4	
	$B^{-1} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$	B1ft		
	$B^{-1}A^{-1} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{3\sqrt{3}}{2} - \frac{1}{2} \\ \frac{1}{2} & -\frac{3}{2} + \frac{\sqrt{3}}{2} \end{bmatrix}$	M1 A1		M1 for one row correct. Must be exact answers. $A^{-1}B^{-1} = \begin{bmatrix} \frac{\sqrt{3}}{2} - \frac{3}{2} & -\frac{3\sqrt{3}}{2} - \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ is SC2
Total	**6**			

Q	Answer	Mark	Total	Comments
4(a)	$f(2) = 0.4$ and $f(3) = -1.1$	B1	2	
	Sign change and f is continuous	B1		
4(b)	Tangent at $x = 3$ that meets x-axis	B1	4	
	Tangent at $x = 2$ that meets x-axis	B1		
	Explanation that tangent at 3 meets axis closer to α (than 3).	E1		Allow alternative explanations
	Explanation that tangent at 2 converges on the other root.	E1		
4(c)	$f'(x) = -0.4x^3 + 2x$	M1	4	
	$f'(2.5) = -0.4(2.5^3) + 2 \times 2.5$ $[= -1.25]$	M1dep		
	$2.5 - \frac{(0.34375)}{(-1.25)}$	M1dep		
	2.775	A1		or 111/40
Total	**10**			

Q	Answer	Mark	Total	Comments
5(a)	Reflection in the line $y = x\tan\theta$	B1	1	
5(b)	$x\cos 2\theta + (mx + c)\sin 2\theta = X$ $x\sin 2\theta - (mx + c)\cos 2\theta = mX + c$	M1 A1	4	Using x instead of X is incorrect – there must be two different variables
	Eliminating X and gathering terms in x to give required result	M1 A1		
5(c)(i)	$\sin 2\theta = \frac{\sqrt{3}}{2}$ **and** $\cos 2\theta = \frac{1}{2}$	B1	3	PI
	$\sqrt{3}m^2 + 2m - \sqrt{3} = 0$ Or $\left(\sqrt{3}m - 1\right)\left(m + \sqrt{3}\right) = 0$	M1		oe
	$m = -\sqrt{3}$ Or $m = \frac{\sqrt{3}}{3}$	A1		for both
5(c)(ii)	$\begin{vmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{vmatrix}$ $= -1$	B1	1	
Total	**9**			

Q	Answer	Mark	Total	Comments
6(a)	$P(B \cap L') = P(B)P(L'\|B)$ $= 0.875(1 - 0.12)$	M1	2	Use of the Multiplication Rule of probability May be implied by a correct answer
	$= 0.77$	A1		CAO (Accept 77/100 OE)
6(b)	$P(L\|B)P(B) = 0.12 \times 0.875$ $(= 0.105)$	M1	4	Finds numerator of Bayes Theorem unsimplified or simplified May use $P(L \cap B) = P(L\|B)P(B)$ instead
	$P(L\|B)P(B) + P(L\|B')P(B')$ $= 0.12 \times 0.875 + 0.34(1 - 0.875)$	M1		Finds denominator of Bayes Theorem unsimplified or simplified May use $P(L) = P(L \cap B) + P(L \cap B')$ instead
	$P(B\|L) = \frac{P(L\|B)P(B)}{P(L\|B)P(B)+P(L\|B')P(B')}$ $= \frac{0.12 \times 0.875}{0.12 \times 0.875 + 0.34(1 - 0.875)}$	M1		Applies Bayes Theorem unsimplified or simplified May use $P(B\|L) = \frac{P(L \cap B)}{P(L)}$ instead
	$= \frac{42}{59}$ OE	A1		AWRT 0.712
Total	**6**			

<table>
<tr><th>Q</th><th>Answer</th><th>Mark</th><th>Total</th><th>Comments</th></tr>
<tr><td rowspan="2">7(a)</td><td>$\mathrm{E}(T) = 2.5$</td><td>B1</td><td rowspan="2">2</td><td>CAO (Accept 5/2 OE)</td></tr>
<tr><td>$\mathrm{Var}(T) = 1.25$</td><td>B1</td><td>CAO (Accept 5/4 OE)</td></tr>
<tr><td rowspan="4">7(b)</td><td>$\mathrm{E}(T^2) = \mathrm{Var}(T) + \left(\mathrm{E}(T)\right)^2$
Or
$\mathrm{E}(T^2) = \sum t^2 P(T = t)$</td><td>M1</td><td rowspan="5">4</td><td>Correct expression
Can be implied by further working
May be seen in Question 7(a)</td></tr>
<tr><td>$\mathrm{E}(T^2) = 1.25 + 2.5^2$
Or
$\mathrm{E}(T^2) =$
$1 \times \frac{1}{4} + 2^2 \times \frac{1}{4} + 3^2 \times \frac{1}{4} + 4^2 \times \frac{1}{4}$</td><td>A1ft</td><td>Substitutes $\mathrm{E}(T)$ and $\mathrm{Var}(T)$ into the first expression (ft answers from Question 7(a)) or substitutes values into $\sum t^2 P(T = t)$
Can be implied by $\mathrm{E}(T^2) = 7.5$
May be seen in Question 7(a)</td></tr>
<tr><td>$\mathrm{E}(T + S) = \mathrm{E}(T + T^2 + 1)$
$= \mathrm{E}(T) + \mathrm{E}(T^2) + 1$</td><td>M1</td><td>Correct expression
Can be implied by further working</td></tr>
<tr><td>$= 11$</td><td>A1</td><td>CSO</td></tr>
<tr><td rowspan="3">7(b)
Alternative method</td><td><table><tr><td>t</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$t + t^2 + 1$</td><td>3</td><td>7</td><td>13</td><td>21</td></tr></table></td><td>M1
A1</td><td>Calculate $t + t^2 + 1$ for every value of t
Can make one slip for M mark
All values correct for A mark</td></tr>
<tr><td>$\mathrm{E}(T + S) = \sum (t + t^2 + 1) P(T = t)$
$= 3 \times \frac{1}{4} + 7 \times \frac{1}{4} + 13 \times \frac{1}{4} + 21 \times \frac{1}{4}$</td><td>M1</td><td>Applies expectation formula with their values of $t + t^2 + 1$</td></tr>
<tr><td>$= 11$</td><td>A1</td><td>CSO</td></tr>
<tr><td>Total</td><td colspan="4">6</td></tr>
</table>

Q	Answer	Mark	Total	Comments
8(a)(i)	$P(X = 8) = (1 - 0.045)^7 \times 0.045$	M1	2	Correct expression Can be implied by correct answer
	$= 0.0326$	A1		AWRT (From calculator 0.03260138983)
8(a)(ii)	$P(X > 5) = (1 - 0.045)^5$	M1	2	Correct expression Can be implied by correct answer
	$= 0.794$	A1		AWRT (From calculator 0.7943590686)
8(b)	$G_X(t) = E(t^x) = \sum_{x=1}^{\infty} t^x \times p(1-p)^{x-1}$	M1	4	Applies probability generating function formula unsimplified or simplified
	$= pt(1 + (p-1)t + (p-1)^2t^2 + \cdots)$	M1		Simplifies expression Constants may be unsimplified or simplified
	$= pt\left(\frac{1}{1-(1-p)t}\right)$	M1		Applies sum to infinity formula to evaluate the summation Constants may be unsimplified or simplified
	$= \frac{0.045t}{1 - 0.955t}$	A1		CAO
Total	**8**			

Q	Answer	Mark	Total	Comments
9	$v^2 = 150^2 + 40^2 - 2 \times 150 \times 40 \times \cos(45°)$ $v = 125 ms^{-1}$	M1 A1	2	M1 Uses cosine rule to find resultant speed. A1 Correct speed.
Total	**2**			

Q	Answer	Mark	Total	Comments
10	$[G] = \frac{[F][d^2]}{[M][m]} = \frac{MLT^{-2}L^2}{M^2} = L^3T^{-2}M^{-1}$ Units are $m^3s^{-2}kg^{-1}$	M1 M1 A1	3	M1 Makes G the subject of the equation. M1 Uses dimensional analysis or equivalent. A1 Correct units in any order.
Total	**3**			

Q	Answer	Mark	Total	Comments
11	$r_A = (30 + t)i + (90 - 2t)j$ $r_B = (5 + 3t)i + (70 + 2t)j$	B1	6	Forms correct expressions for position vectors of A and B.
	$r_A - r_B = (25 - 2t)i + (20 - 4t)j$ $s^2 = (25 - 2t)^2 + (20 - 4t)^2$	M1		Finds distance between A and B.
	$\frac{d}{dt}(s^2) = -4(25 - 2t) - 8(20 - 4t)$	M1		Uses differentiation to find the time for the minimum distance.
	$0 = -260 + 40t$ $t = \frac{260}{40} = 6.5$	A1		Correct time.
	$s = \sqrt{12^2 + (-6)^2} = 13.4m$	M1 A1		M1 Finds minimum distance. A1 Correct minimum distance.
Total	**6**			

Q	Answer	Mark	Total	Comments
12(a)	$4 \times 1 + 1 \times 5 = v_P + 5v_Q$ $9 = v_P + 5v_Q$	M1 A1	**6**	M1 Conservation of momentum equation. A1 Correct equation.
	$v_P - v_Q = -0.8(4 - 1)$ $v_P - v_Q = 2.4$	M1 A1		M1 Uses coefficient of restitution. A1 Correct equation.
	$v_P = -0.5, \lvert v_P \rvert = 0.5$	A1		Correct speed for P.
	$v_Q = 1.9$	A1		Correct speed for Q.
12(b)	$1.9e > 0.5$ $e > \frac{5}{19}$	M1 A1 A1	**3**	M1 Uses e for collision with wall. A1 Correct inequality. A1 Correct inequality for e.
Total	**9**			