Pearson Edexcel

Mark Scheme (Results)

October 2020

Pearson Edexcel International A Level
In Further Pure Mathmatics F3
(WFM03/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = ...

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Notes	Marks
1(a)	$4\sinh^3 x + 3\sinh x = 4\left(\frac{e^x - e^{-x}}{2}\right)^3 + 3\left(\frac{e^x - e^{-x}}{2}\right)$ $= 4\left(\frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{8}\right) + 3\left(\frac{e^x - e^{-x}}{2}\right)$ Uses $\sinh x = \frac{e^x - e^{-x}}{2}$ on both sinh terms and attempts to cube the bracket (min accepted is a linear x a quadratic bracket)		M1
	$= \frac{1}{2}e^{3x} - \frac{3}{2}e^x + \frac{3}{2}e^{-x} - \frac{1}{2}e^{-3x} + \frac{3}{2}e^x - \frac{3}{2}e^{-x}$ $= \frac{e^{3x} - e^{-3x}}{2} = \sinh 3x$*		A1*
			(2)
(b)	$\sinh 3x = 19\sinh x \Rightarrow 4\sinh^3 x + 3\sinh x = 19\sinh x$ $\Rightarrow 4\sinh^3 x - 16\sinh x = 0$ Uses the result from (a) and combines terms		M1
	($\sinh x = 0$ or) $\sinh^2 x = 4$	$\sinh^2 x = 4$ or $\sinh x = (\pm)2$	A1
	(0, 0)	States the origin as one intersection	B1
	$\ln\left(2+\sqrt{5}\right)$ **and** $-\ln\left(2+\sqrt{5}\right)$	Two correct non-zero x values(allow e.g. $\ln\left(-2+\sqrt{5}\right)$ for $-\ln\left(2+\sqrt{5}\right)$)	A1
	$\left(\ln\left(2+\sqrt{5}\right), 38\right)$ **and** $\left(-\ln\left(2+\sqrt{5}\right), -38\right)$	Two correct **points** (allow e.g. $\ln\left(-2+\sqrt{5}\right)$ for $-\ln\left(2+\sqrt{5}\right)$)	A1
			(5)
	Alternative for (b) using exponentials		
	$\sinh 3x = 19\sinh x \Rightarrow \frac{e^{3x} - e^{-3x}}{2} = \frac{19\left(e^x - e^{-x}\right)}{2} \Rightarrow \ldots$ Substitutes the correct exponential forms and collects terms to one side		M1
	$\Rightarrow e^{6x} - 19e^{4x} + 19e^{2x} - 1 = 0$	Correct equation (or equivalent)	A1
	(0, 0)	States the origin as one intersection	B1
	$\frac{1}{2}\ln\left(9+4\sqrt{5}\right)$ **or** $\frac{1}{2}\ln\left(9-4\sqrt{5}\right)$	Two correct non-zero x values (oe)	A1
	$\left(\frac{1}{2}\ln\left(9+4\sqrt{5}\right), 38\right)$ **and** $\left(\frac{1}{2}\ln\left(9-4\sqrt{5}\right), -38\right)$	Two correct **points** (oe)	A1
			Total 7

Question Number	Scheme	Notes	Marks
2(i)	$3x^2+12x+24=3\left(x^2+4x+8\right)$ $=3\left(\left(x+2\right)^2+4\right)$	Obtains $3\left(\left(x+2\right)^2+...\right)$ or $3\left(x+2\right)^2+...$ Must include 3 now or later	M1
	$3\left(\left(x+2\right)^2+4\right)$ or $3\left(x+2\right)^2+12$		A1
	$\int \frac{1}{3x^2+12x+24}\,\mathrm{d}x=\frac{1}{3}\int \frac{1}{\left(x+2\right)^2+4}\,\mathrm{d}x=\frac{1}{6}\arctan\frac{x+2}{2}\left(+c\right)$ M1: Use of arctan A1: Fully correct expression (condone omission of $+c$)		M1A1
			(4)
(ii)	$27-6x-x^2=-\left(x^2+6x-27\right)$ $=-\left(\left(x+3\right)^2-36\right)$	Obtains $-\left(\left(x+3\right)^2+...\right)$ or $-\left(x+3\right)^2+...$	M1
	$-\left(\left(x+3\right)^2-36\right)$ or $36-\left(x+3\right)^2$		A1
	$\int \frac{1}{\sqrt{27-6x-x^2}}\,\mathrm{d}x=\int \frac{1}{\sqrt{36-\left(x+3\right)^2}}\,\mathrm{d}x=\arcsin\left(\frac{x+3}{6}\right)\left(+c\right)$ (Or $=-\arccos\left(\frac{x+3}{6}\right)\left(+c\right)$) M1: Use of arcsin (or – arccos) A1: Fully correct expression (condone omission of $+c$)		M1A1
			(4)
			Total 8

Question Number	Scheme	Notes	Marks
3	$\mathbf{M}=\begin{pmatrix}3 & -4 & k\\ 1 & -2 & k\\ 1 & -5 & 5\end{pmatrix}$		
(a)	$\lvert\mathbf{M}-\lambda\mathbf{I}\rvert=\mathbf{M}=\begin{vmatrix}3-\lambda & -4 & k\\ 1 & -2-\lambda & k\\ 1 & -5 & 5-\lambda\end{vmatrix}=\begin{vmatrix}0 & -4 & k\\ 1 & -5 & k\\ 1 & -5 & 2\end{vmatrix}$ $(0)+4[2-k]+k[-5+5]$ Attempts $\lvert\mathbf{M}-\lambda\mathbf{I}\rvert$ using $\lambda=3$		M1
	$(0)+4[2-k]+k[-5+5]=0\Rightarrow k=...$ Uses $\lvert\mathbf{M}-\lambda\mathbf{I}\rvert=0$ and solves for k		M1
	$k=2$	Cao	A1
			(3)
(b)	$(3-\lambda)\left[(\lambda+2)(\lambda-5)+10\right]+4(5-\lambda-2)+2(-5+2+\lambda)=0$ Attempts $\lvert\mathbf{M}-\lambda\mathbf{I}\rvert=0$ using their value of k		M1
	$\Rightarrow(3-\lambda)\left[(\lambda+2)(\lambda-5)+12\right]=0$ $(\lambda+2)(\lambda-5)+12\Rightarrow\lambda^2-3\lambda+2=0\Rightarrow(\lambda-2)(\lambda-1)=0\Rightarrow\lambda=...$ Uses $\lambda=3$as a factor to obtain and solve a 3TQ to find the other eigenvalues (Alternatively may use calculator to solve $\lambda^3-6\lambda^2+11\lambda-6=0$)		M1
	$\lambda=1,\ 2$	Correct values	A1
			(3)
(c)	$\begin{pmatrix}3 & -4 & 2\\ 1 & -2 & 2\\ 1 & -5 & 5\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=3\begin{pmatrix}x\\ y\\ z\end{pmatrix}\Rightarrow\begin{matrix}3x-4y+2z=3x\\ x-2y+2z=3y\\ x-5y+5z=3z\end{matrix}$	Uses the eigenvalue 3 and their k to form at least 2 equations in x, y and z	M1
	$\alpha\begin{pmatrix}1\\ 1\\ 2\end{pmatrix}$ (α a constant)	Any correct eigenvector. Allow any constant multiple of $\mathbf{i}+\mathbf{j}+2\mathbf{k}$	A1
	$\frac{1}{\sqrt{6}}\begin{pmatrix}1\\ 1\\ 2\end{pmatrix}$	Correct normalised vector	A1
			(3)
			Total 9

Question Number	Scheme	Notes	Marks
4.	$I_n = \int x^n \cos x \,dx$		
(a)	$\int x^n \cos x \,dx = x^n \sin x - \int nx^{n-1} \sin x \,dx$ M1: Parts in the correct direction A1: Correct expression		M1A1
	$= x^n \sin x - \left\{-nx^{n-1}\cos x + \int n(n-1)x^{n-2}\cos x \,dx\right\}$ Uses integration by parts again (**dependent on the first M**)		dM1
	$= x^n \sin x + nx^{n-1}\cos x - n(n-1)I_{n-2}$ * Fully correct proof with no errors		A1*
			(4)
ALT			
	$I_n = \int x^n \cos x \,dx = \int x^{n-1}(x\cos x)\,dx$		
	$= x^n \sin x + x^{n-1}\cos x - (n-1)\int x^{n-2}(x\sin x + \cos x)\,dx$ M1: Parts in the correct direction A1: Correct expression		M1A1
	$= x^n \sin x + x^{n-1}\cos x - (n-1)\int x^{n-1}\sin x\,dx - (n-1)I_{n-2}$		
	$= x^n \sin x + x^{n-1}\cos x - (n-1)\left\{-x^{n-1}\cos x + (n-1)I_{n-2}\right\} - (n-1)I_{n-2}$ Uses integration by parts again (**dependent on the first M**)		dM1
	$= x^n \sin x + nx^{n-1}\cos x - n(n-1)I_{n-2}$ * Fully correct proof with no errors		A1*
(b)	$I_0 = \sin x \quad (+k)$		B1
	$I_4 = x^4 \sin x + 4x^3 \cos x - 12I_2$	Applies the reduction formula once for I_4 or I_2	M1
	$= x^4 \sin x + 4x^3 \cos x - 12\left(x^2 \sin x + 2x\cos x - 2I_0\right)$ Applies the reduction formula again and obtains an expression for I_4 which can include I_0 but not I_2		M1
	$= \left(x^4 - 12x^2 + 24\right)\sin x + \left(4x^3 - 24x\right)\cos x + c$ Award A1 for either bracket and A1 for the other If the answer is not factorised but is otherwise correct, award A1A0		A1A1
			(5)
			Total 9

Question Number	Scheme	Notes	Marks
5	$\frac{x^2}{25}-\frac{y^2}{4}=1 \quad y=mx+c$		
(a)	$\frac{x^2}{25}-\frac{(mx+c)^2}{4}=1 \Rightarrow 4x^2-25(m^2x^2+2cmx+c^2)=100$ Substitutes to obtain a quadratic in x and eliminates fractions		M1
	$4x^2-25(m^2x^2+2cmx+c^2)=100$ $(\Rightarrow (25m^2-4)x^2+50cmx+25c^2+100=0)$ Correct 3TQ		A1
	$"b^2=4ac" \Rightarrow (50cm)^2=4(25m^2-4)(25c^2+100)$ Uses '$b^2=4ac$' or equivalent		M1
	$2500c^2m^2=2500c^2m^2+10000m^2-400c^2-1600$ $10000m^2=400c^2+1600$ $25m^2=c^2+4$* Fully correct proof with no errors		A1*
			(4)
ALT 1	Using hyperbolic parameters:		
	$x=5\cosh t, y=2\sinh t \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\cosh t}{5\sinh t}$		
	$\frac{2\cosh t}{5\sinh t}(x-5\cosh t)=y-2\sinh t$ M1: Attempts the equation of the tangent A1: Correct equation (no simplification needed)		M1A1
	$y=\frac{2\cosh t}{5\sinh t}x-\frac{2\cosh^2 t-25\sinh^2 t}{\sinh t}$		
	$25m^2=\frac{4\cosh^2 t}{\sinh^2 t},\ 4+c^2=4+\frac{4}{\sinh^2 t}=\frac{4(\sinh^2 t+1)}{\sinh^2 t}=\frac{4\cosh^2 t}{\sinh^2 t}$ Extracts $25m^2$ and $4+c^2$ from their equation		M1
	$\therefore 25m^2=4+c^2$ * Fully correct proof with no errors		A1*
			(4)
ALT 2	Using trigonometric parameters:		
	$x=5\sec t, y=2\tan t \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\sec t}{5\tan t}$		
	$\frac{2\sec t}{5\tan t}(x-5\sec t)=y-2\tan t$ M1: Attempts the equation of the tangent A1: Correct equation (no simplification needed)		M1A1
	$y=\frac{2\sec t}{5\tan t}x+\frac{2\tan^2 t-2\sec^2 t}{\tan t}$		
	$25m^2=\frac{4\sec^2 t}{\tan^2 t}=\frac{4}{\sin^2 t} \quad 4+c^2=4\left(1+\frac{1}{\tan^2 t}\right)=4\left(\frac{\sin^2 t+\cos^2 t}{\sin^2 t}\right)=\frac{4}{\sin^2 t}$ Extracts $25m^2$ and $4+c^2$ from their equation		M1
	$\therefore 25m^2=4+c^2$ * Fully correct proof with no errors		A1*
			(4)

(b)	$25m^2 = c^2 + 4$ and $2 = m + c$ $25m^2 = (2-m)^2 + 4$ or $25(2-c)^2 = c^2 + 4$ Uses the given hyperbola and the straight line with the result from (a) to obtain an equation in *m* or *c*		M1
	$24m^2 + 4m - 8 = 0$ or $24c^2 - 100c + 96 = 0$	Correct 3TQ in *m* or *c*	A1
	$24m^2 + 4m - 8 = 0 \Rightarrow m = \frac{1}{2}, -\frac{2}{3}$ Or $24c^2 - 100c + 96 = 0 \Rightarrow c = \frac{3}{2}, \frac{8}{3}$	Solves their 3TQ in *m* or *c*	M1
	$y = \frac{1}{2}x + \frac{3}{2}$ **or** $y = -\frac{2}{3}x + \frac{8}{3}$	One correct tangent	A1
	$y = \frac{1}{2}x + \frac{3}{2}$ **and** $y = -\frac{2}{3}x + \frac{8}{3}$	Both correct tangents	A1
			(5)
(c)	$m = \frac{1}{2}, c = \frac{3}{2} \Rightarrow \frac{9}{4}x^2 + \frac{75}{2}x + \frac{625}{4} = 0 \Rightarrow x = \ldots$ **or** $m = -\frac{2}{3}, c = \frac{8}{3} \Rightarrow \frac{64}{9}x^2 - \frac{800}{9}x + \frac{2500}{9} = 0 \Rightarrow x = \ldots$ Uses one of their *m* and *c* pairs and solves for *x*		M1
	$x = -\frac{25}{3}, y = -\frac{8}{3}$ **or** $x = \frac{25}{4}, y = -\frac{3}{2}$	One correct point	A1
	$x = -\frac{25}{3}, y = -\frac{8}{3}$ **and** $x = \frac{25}{4}, y = -\frac{3}{2}$	Both correct points	A1
			(3)
			Total 12

Question Number	Scheme	Notes	Marks
6(a)	$\mathbf{A}=\begin{pmatrix}1 & -1 & 1\\ 1 & 1 & 1\\ 1 & 2 & a\end{pmatrix}$		
	$\lvert\mathbf{A}\rvert = a-2+a-1+2-1(=2a-2)$	Correct determinant in any form	B1
	$\begin{pmatrix}1 & -1 & 1\\ 1 & 1 & 1\\ 1 & 2 & a\end{pmatrix}\to\begin{pmatrix}a-2 & a-1 & 1\\ -a-2 & a-1 & 3\\ -2 & 0 & 2\end{pmatrix}\to\begin{pmatrix}a-2 & 1-a & 1\\ a+2 & a-1 & -3\\ -2 & 0 & 2\end{pmatrix}$ Applies the correct method to reach at least a matrix of cofactors 2 correct rows or 2 correct columns needed		M1
	$\begin{pmatrix}a-2 & 1-a & 1\\ a+2 & a-1 & -3\\ -2 & 0 & 2\end{pmatrix}\to\begin{pmatrix}a-2 & a+2 & -2\\ 1-a & a-1 & 0\\ 1 & -3 & 2\end{pmatrix}$ Correct transpose of cofactors		A1
	$\mathbf{A}^{-1}=\frac{1}{2a-2}\begin{pmatrix}a-2 & a+2 & -2\\ 1-a & a-1 & 0\\ 1 & -3 & 2\end{pmatrix}$	Correct inverse	A1
			(4)
(b)	$a=4\Rightarrow \mathbf{A}^{-1}=\frac{1}{6}\begin{pmatrix}2 & 6 & -2\\ -3 & 3 & 0\\ 1 & -3 & 2\end{pmatrix}$	Correct inverse (follow through their matrix from (a))	B1ft
	$=\frac{1}{6}\begin{pmatrix}2 & 6 & -2\\ -3 & 3 & 0\\ 1 & -3 & 2\end{pmatrix}\begin{pmatrix}12-6\lambda\\ 4+2\lambda\\ 6+3\lambda\end{pmatrix}=...$	Attempt to multiply the parametric form of l_2 by their inverse	M1
	$=\begin{pmatrix}6-\lambda\\ -4+4\lambda\\ 2-\lambda\end{pmatrix}$	Correct parametric form	A1
	$\mathbf{r}=\begin{pmatrix}6\\ -4\\ 2\end{pmatrix}+\lambda\begin{pmatrix}-1\\ 4\\ -1\end{pmatrix}$	Correct equation (allow equivalent forms) but if given as l = ... award A0	A1
			(4)
			Total 8

	Alternatives for (b)		
(i)	$a = 4 \Rightarrow \mathbf{A}^{-1} = \frac{1}{6}\begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}$	Correct inverse (follow through their matrix from (a))	B1ft
	$\frac{1}{6}\begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}\begin{pmatrix} 12 \\ 4 \\ 6 \end{pmatrix} = \frac{1}{6}\begin{pmatrix} 36 \\ -24 \\ 12 \end{pmatrix}$ $\frac{1}{6}\begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}\begin{pmatrix} -6 \\ 2 \\ 3 \end{pmatrix} = \frac{1}{6}\begin{pmatrix} -6 \\ 24 \\ -6 \end{pmatrix}$	Attempt $\mathbf{A}^{-1}$(point on l_2) **and** $\mathbf{A}^{-1}$(direction of l_2) **Both** correct (NB No ft)	M1 A1
	$\mathbf{r} = \begin{pmatrix} 6 \\ -4 \\ 2 \end{pmatrix} + \lambda\begin{pmatrix} -1 \\ 4 \\ -1 \end{pmatrix}$	Correct equation (allow equivalent forms) but if given as $l = \ldots$ award A0	A1
			(4)
(ii)	$a = 4 \Rightarrow \mathbf{A}^{-1} = \frac{1}{6}\begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}$	Correct inverse (follow through their matrix from (a))	B1ft
	$\frac{1}{6}\begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}\begin{pmatrix} 12 \\ 4 \\ 6 \end{pmatrix} = \frac{1}{6}\begin{pmatrix} 36 \\ -24 \\ 12 \end{pmatrix}$ $\frac{1}{6}\begin{pmatrix} 2 & 6 & -2 \\ -3 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}\begin{pmatrix} 6 \\ 6 \\ 9 \end{pmatrix} = \frac{1}{6}\begin{pmatrix} 30 \\ 0 \\ 6 \end{pmatrix}$	Attempt $\mathbf{A}^{-1}$(point on l_2) for 2 points **Both** correct (NB No ft)	M1 A1
	$\mathbf{r} = \begin{pmatrix} 6 \\ -4 \\ 2 \end{pmatrix} + \lambda\begin{pmatrix} -1 \\ 4 \\ -1 \end{pmatrix}$	Obtain the direction vector and deduce correct equation (allow equivalent forms) but if given as $l = \ldots$ award A0	A1
			(4)

Question Number	Scheme	Notes	Marks
7	$x = \cosh t + t, \quad y = \cosh t - t$		
(a)	$\frac{dx}{dt} = \sinh t + 1, \quad \frac{dy}{dt} = \sinh t - 1$	Correct derivatives	B1
	$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \sinh^2 t + 2\sinh t + 1 + \sinh^2 t - 2\sinh t + 1$ $= 2\sinh^2 t + 2$ M1: Squares correctly, cancels and collects terms		M1
	$= 2\left(1 + \sinh^2 t\right) = 2\cosh^2 t$ *	Uses $\cosh^2 t = 1 + \sinh^2 t$ to complete the proof with no errors	A1*
			(3)
(b)	$S = 2\pi\int y\, ds = 2\pi\int(\cosh t - t)\sqrt{2}\cosh t\, dt$	Uses $S = 2\pi\int y\, ds$ with the given y and the result from part (a)	M1
	$= 2\sqrt{2}\pi\int_0^{\ln 3}\left(\cosh^2 t - t\cosh t\right)dt$ *	Correct proof with no errors	A1*
			(2)
(c)	$\int \cosh^2 t\, dt = \int \pm\frac{1}{2} \pm \frac{1}{2}\cosh 2t\, dt$	Uses $\cosh^2 t = \pm\frac{1}{2} \pm \frac{1}{2}\cosh 2t$	M1
	$\int t\cosh t\, dt = t\sinh t - \int \sinh t\, dt$	Attempts integration by parts the right way round on $t\cosh t$	M1
		Correct expression	A1
	$S = \left(2\sqrt{2}\pi\right)\int \left(\cosh^2 t - t\cosh t\right)dt = \left(2\sqrt{2}\pi\right)\left[\frac{1}{2}t + \frac{1}{4}\sinh 2t - t\sinh t + \cosh t\right]$ A1: 2 correct terms A1: All correct		A1A1
	$(S =)\, 2\sqrt{2}\pi\left\{\left(\frac{1}{2}\ln 3 + \frac{10}{9} - \frac{4}{3}\ln 3 + \frac{5}{3}\right) - (1)\right\}$ dM1: Correct use of limits 0 and ln3 depends on both preceding M marks		dM1
	$S = \frac{1}{9}\sqrt{2}\pi(32 - 15\ln 3)$	cao	A1 **(7)**
			Total 12
	Alternative for (c)		
	$\int \cosh^2 t\, dt = \int\left(\frac{e^t + e^{-t}}{2}\right)^2 dt$ $= \frac{1}{4}\int\left(e^{2t} + 2 + e^{-2t}\right)dt$	Substitutes the exponential form and attempts to square	M1
	$\int t\cosh t\, dt = \frac{1}{2}\int t\left(e^t + e^{-t}\right)dt$ $= \frac{1}{2}te^t - \frac{1}{2}\int te^t dt - \left\{\frac{1}{2}te^{-t} - \frac{1}{2}\int e^{-t}dt\right\}$	Substitutes the exponential form and attempts integration by parts the right way round Correct expression	M1 A1
	$(S =)\left(2\sqrt{2}\pi\right)\left\{\frac{1}{4}\left(\frac{1}{2}e^{2t} + 2t - \frac{1}{2}e^{-2t}\right) - \frac{1}{2}te^t + \frac{1}{2}e^t + \frac{1}{2}te^{-t} - \frac{1}{2}e^{-t}\right\}$ A1: either integral correct A1: other integral correct but both must be in a complete expression for S		A1A1
	Depends on both M marks above	Correct use of limits 0 and ln3	dM1
	$S = \frac{1}{9}\sqrt{2}\pi(32 - 15\ln 3)$	cao	A1

	Alternative for the first 3 marks of (c)		
	$=2\sqrt{2}\pi\int\left(\cosh^2 t - t\cosh t\right)\mathrm{d}t$ $=2\sqrt{2}\pi\int\cosh t\left(\cosh t - t\right)\mathrm{d}t$		
	$2\sqrt{2}\pi\left(\left[\sinh t\left(\cosh t - t\right)\right] - \int\sinh t\left(\sinh t - 1\right)\mathrm{d}t\right)$		
	$2\sqrt{2}\pi\left(\left[\sinh t\left(\cosh t - t\right)\right] - \left[\cosh t\left(\sinh t - 1\right)\right] + \int\cosh^2 t\mathrm{d}t\right)$ M1 (2nd on e-PEN): Use parts twice A1 Correct expression		M1A1
	$\int\cosh^2 t\,\mathrm{d}t = \int\pm\frac{1}{2}\pm\frac{1}{2}\cosh 2t\,\mathrm{d}t$	Uses $\cosh^2 t = \pm\frac{1}{2}\pm\frac{1}{2}\cosh 2t$	M1 (1st on e-PEN)
	Rest as main scheme		

Question Number	Scheme	Notes	Marks
8(a)	$\mathbf{n}=\begin{pmatrix}1\\-5\\3\end{pmatrix}\times\begin{pmatrix}3\\-2\\2\end{pmatrix}=\begin{pmatrix}-10+6\\-(2-9)\\-2+15\end{pmatrix}$	Attempt vector product between normal vectors	M1
	$=\begin{pmatrix}-4\\7\\13\end{pmatrix}$	Correct vector	A1
	$x=0\Rightarrow -5y+3z=11,\quad -2y+2z=7$ $\Rightarrow y=-\frac{1}{4}, z=\frac{13}{4}$ or $y=0\Rightarrow x+3z=11,\quad 3x+2z=7$	Correct strategy to find a point on l	M1
	$\Rightarrow x=-\frac{1}{7}, z=\frac{26}{7}$ or $z=0\Rightarrow x-5y=11, 3x-2y=7$ $\Rightarrow x=1, y=-2$	Correct position vector of point on l	A1
	$\mathbf{r}=\mathbf{i}-2\mathbf{j}+\lambda(-4\mathbf{i}+7\mathbf{j}+13\mathbf{k})$	Correct equation. (follow through their position and direction vectors but must be "$\mathbf{r}$ =")	A1ft
			(5)
ALT	$x=11+5y-3z$		
	$3x-2y+2z=7\Rightarrow 3(11+5y-3z)-2y+2z=7$ $\Rightarrow y-\frac{7z}{13}=-\frac{26}{13}\quad\left(z=\frac{13y+26}{7}\right)$ Eliminate one variable		M1
	$x=11+5\left(-\frac{26}{13}+\frac{7z}{13}\right)\Rightarrow z=\frac{13-13x}{4}$	Obtain 2 correct expressions for one of the variables	A1
	$\frac{x-1}{-\frac{4}{13}}=\frac{y+2}{\frac{7}{13}}=z$	M1 Obtain a Cartesian equation for l A1 Correct equation	M1A1
	$\mathbf{r}=(\mathbf{i}-2\mathbf{j})+\lambda\left(-\frac{4}{13}\mathbf{i}+\frac{7}{13}\mathbf{j}+\mathbf{k}\right)$ oe	Deduce a vector equation for l Follow through their Cartesian equation	A1ft
			(5)

(b)	$\begin{pmatrix}3\\2\\1\end{pmatrix}-\begin{pmatrix}2\\0\\3\end{pmatrix}=\begin{pmatrix}1\\2\\-2\end{pmatrix}$	Correct vector joining P to Q	B1
	$\begin{pmatrix}-4\\7\\13\end{pmatrix}\times\begin{pmatrix}1\\2\\-2\end{pmatrix}=\begin{pmatrix}-40\\5\\-15\end{pmatrix}$	Attempt vector product between the direction of l and their $\mathbf{i}+2\mathbf{j}-2\mathbf{k}$	M1
		Correct vector	A1
	$\sin\theta=\frac{\lvert-40\mathbf{i}+5\mathbf{j}-15\mathbf{k}\rvert}{\lvert-4\mathbf{i}+7\mathbf{j}+13\mathbf{k}\rvert\lvert\mathbf{i}+2\mathbf{j}-2\mathbf{k}\rvert}$	Angle between PQ and line n	
	$d=\lvert\overrightarrow{PQ}\rvert\sin\theta$		
	$d=\frac{\lvert-40\mathbf{i}+5\mathbf{j}-15\mathbf{k}\rvert}{\lvert-4\mathbf{i}+7\mathbf{j}+13\mathbf{k}\rvert}=\frac{1}{\sqrt{234}}\sqrt{40^2+5^2+15^2}$	Fully correct method for the distance	M1
	$d=\frac{5\sqrt{481}}{39}$	Cao Allow equivalent **exact** forms e.g. $d=\frac{5\sqrt{74}}{\sqrt{234}}$	A1
			(5)
ALT 1	$\mathbf{r}_m=\begin{pmatrix}2\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}-\frac{4}{7}\\1\\\frac{13}{7}\end{pmatrix}$ or $\mathbf{r}_n=\begin{pmatrix}3\\2\\1\end{pmatrix}+\mu\begin{pmatrix}-\frac{4}{7}\\1\\\frac{13}{7}\end{pmatrix}$	Vector equation for either line with their direction vector from (a)	B1ft
	$\overrightarrow{OP}=\begin{pmatrix}2\\0\\3\end{pmatrix}$ $\overrightarrow{ON}=\begin{pmatrix}3-\frac{4}{7}\mu\\2+\mu\\1+\frac{13}{7}\mu\end{pmatrix}$ $\overrightarrow{NP}=\begin{pmatrix}-1+\frac{4}{7}\mu\\-2-\mu\\2-\frac{13}{7}\mu\end{pmatrix}$	Uses either P and the parametric form of a point on n OR Q and the parametric form of a point on m	
	$\begin{pmatrix}-1+\frac{4}{7}\mu\\-2-\mu\\2-\frac{13}{7}\mu\end{pmatrix}.\begin{pmatrix}-\frac{4}{7}\\1\\\frac{13}{7}\end{pmatrix}=0$	M1: Forms scalar product of vector NP and direction vector of l and equates to zero A1: Correct vectors	M1A1
	$\Rightarrow\mu=\frac{56}{117}$	Solves	M1
	$\Rightarrow d=\sqrt{\left(-\frac{85}{117}\right)^2+\left(-\frac{290}{117}\right)^2+\left(\frac{10}{9}\right)^2}=\frac{5\sqrt{481}}{39}$	Obtains the correct distance	A1
			(5)
	Alternative for M1A1M1		
	$\overrightarrow{NP}=\begin{pmatrix}-1+\frac{4}{7}\mu\\-2-\mu\\2-\frac{13}{7}\mu\end{pmatrix}\Rightarrow d=\sqrt{\left(-1+\frac{4}{7}\mu\right)^2+(-2-\mu)^2+\left(2-\frac{13}{7}\mu\right)^2}\Rightarrow d$ is min when $\Rightarrow\mu=\frac{56}{117}$ M1: Find d in terms of a parameter A1: correct expression M1: use calculus (or simplify and complete the square) to find the parameter corresponding to the min d		

ALT 2	Correct vector *PQ*		B1
	$\begin{pmatrix}1\\2\\-2\end{pmatrix}.\begin{pmatrix}-4\\7\\13\end{pmatrix}=\begin{vmatrix}1\\2\\-2\end{vmatrix}\begin{vmatrix}-4\\7\\13\end{vmatrix}\cos\theta$	Forms the scalar product and attempts to evaluate the LHS	M1
	$\cos\theta=\frac{-16}{3\sqrt{234}}$	Correct value for $\cos\theta$ exact or decimal	A1
	$d=\lvert PQ\rvert\sin\theta=3\sqrt{1-\left(\frac{-16}{3\sqrt{234}}\right)^2}=\frac{5\sqrt{74}}{\sqrt{234}}$	M1: Correct method for the distance. A1: Correct **EXACT** distance	M1A1
			(5)
			Total 10

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