Pearson Edexcel

Mark Scheme (Results)

October 2020

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F2
(WFM02/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper or ag- answer given
   - ⎿ or d… The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = ...

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

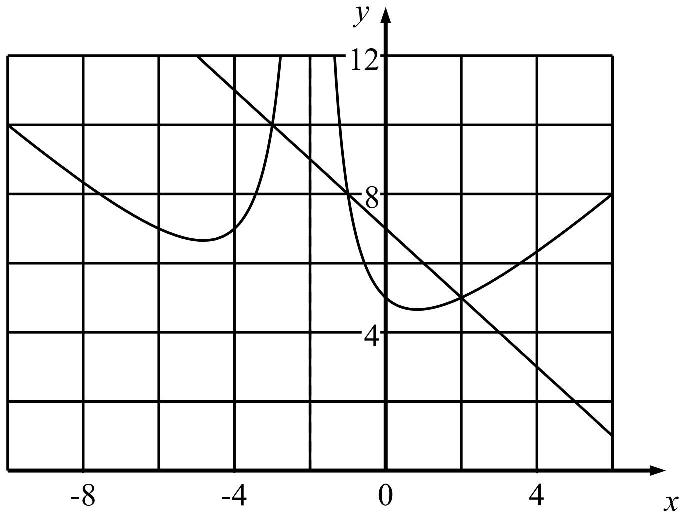
Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1(a)	$\frac{d^3y}{dx^3}+3\frac{dy}{dx}+3x\frac{d^2y}{dx^2}=-2\sin x$	M1M1
	$\frac{d^3y}{dx^3}=-2\sin x-3\frac{dy}{dx}-3x\frac{d^2y}{dx^2}$	A1 (3)
(b)	$\frac{d^3y}{dx^3}=-3\times 5=-15$	B1 (1)
(c)	$\frac{d^2y}{dx^2}=-3\times 0\times 5+2=2$	B1
	$y=2+5x+x^2-\frac{5}{2}x^3$	M1A1 (3)
		[7]
(a)	Accept the dashed notation throughout this question.	
M1	Differentiate $3x\frac{dy}{dx}$ with respect to x. The product rule must be used for $x\frac{dy}{dx}$ with at least one term correct	
M1	Differentiate $\frac{d^2y}{dx^2}$ and $2\cos x$. $\frac{d^2y}{dx^2}\to\frac{d^3y}{dx^3}$ $2\cos x\to\pm 2\sin x$	
A1	$\frac{d^3y}{dx^3}=-3\left(x\frac{d^2y}{dx^2}+\frac{dy}{dx}\right)-2\sin x$. Give A0 if not rearranged to have $\frac{d^3y}{dx^3}=...$	
(b)		
B1	$\frac{d^3y}{dx^3}=-15$ provided 3 terms in result in (a)	
(c)		
B1	$\frac{d^2y}{dx^2}=2$ can be implied by a correct x^2 term in the expansion	
M1	Use of a correct Taylor expansion with their values for $\frac{d^3y}{dx^3}$ and $\frac{d^2y}{dx^2}$ 2! or 2, 3! or 6·	
A1	$y=2+5x+x^2-\frac{5}{2}x^3$ Must include y = ... or f(x) = .. provided f(x) has been defined to be y somewhere in the work.	

Question Number	Scheme	Marks
2 (a)	$\frac{3r+1}{r(r-1)(r+1)}=\frac{A}{r}+\frac{B}{r-1}+\frac{C}{r+1}$	
	$\frac{3r+1}{r(r-1)(r+1)}=-\frac{1}{r}+\frac{2}{r-1}-\frac{1}{r+1}$	M1A1 (2)
(b)	$\frac{2}{1}-\frac{1}{2}-\frac{1}{3}$	
	$\frac{2}{2}-\frac{1}{3}-\frac{1}{4}$ $\quad\frac{2}{n-3}-\frac{1}{n-2}-\frac{1}{n-1}$	
	$\frac{2}{3}-\frac{1}{4}-\frac{1}{5}$ $\quad\frac{2}{n-2}-\frac{1}{n-1}-\frac{1}{n}$	M1
	$\frac{2}{4}-\frac{1}{5}-\frac{1}{6}$ $\quad\frac{2}{n-1}-\frac{1}{n}-\frac{1}{n+1}$	
	$=2-\frac{1}{2}+\frac{2}{2}-\frac{1}{n}-\frac{1}{n}-\frac{1}{n+1}$	dM1A1
	$\frac{5}{2}-\frac{2}{n}-\frac{1}{n+1}=\frac{5n(n+1)-4(n+1)-2n}{2n(n+1)},=\frac{5n^2-n-4}{2n(n+1)}$	M1, A1 cso (5)
(c)	$\sum_{2}^{20}-\sum_{2}^{14}$	
	$=\frac{5\times20^2-20-4}{2\times20\times21}-\frac{5\times14^2-14-4}{2\times14\times15}$	M1
	$=\frac{13}{210}$	A1 (2)
		[9]

(a)	
M1	Correct method for obtaining the PFs
A1	Correct PFs
(b)	
M1	Show sufficient terms at both ends (eg 3 at start and 2 at end) to demonstrate the cancelling. (This can be implied by correct work at the next line) Must be using PFs of the correct form and start at $r = 2$ unless extra terms are ignored at next stage. Can be split into $\sum\left(\frac{1}{r-1}-\frac{1}{r}\right)+\sum\left(\frac{1}{r-1}-\frac{1}{r+1}\right)$
dM1	Extract the non-cancelled terms (min 4 correct terms but 5/2 counts as 3 correct) Depends on first M of (b)
A1	Correct terms extracted
M1	Write terms using the common denominator, numerator need not be simplified. Must start with a min of 3 terms inc terms with denominators n and $(n+1)$
A1cso	Correct answer from correct working
(c)	
M1	Form and use the difference of the 2 summations shown using their result from (b) or an earlier form seen in (b)
A1	Correct **exact** answer, as shown or equivalent

Question Number	Scheme	Marks
3		This sketch on its own scores no marks, but it may be seen in the work
	$\dfrac{x^2+3x+10}{x+2}=7-x$	
	$x^2+3x+10=14+5x-x^2$	M1
	$x^2-x-2=0 \quad (x-2)(x+1)=0$	dM1
	CVs $2,-1$	A1A1
	$\dfrac{-(x^2+3x+10)}{x+2}=7-x$	
	$-x^2-3x-10=14+5x-x^2$	M1
	$8x=-24 \quad$ CV -3	A1
	$x<-3 \quad -1<x<2$	dddM1A1A1
		[9]
NB	**No algebra implies no marks**	
M1	Form a quadratic equation or inequality, no simplification needed	
dM1	Solve the 3TQ any valid method Depends on the first M mark.	
A1	Either CV	
A1	Both CVs	
M1	Change the sign of LHS or RHS and obtain an equation (quadratic or linear, no simplification needed)	
A1	Correct CV from solving the linear equation	
dddM1	x < their smallest CV and x between their other 2 CVs All M marks above needed	
A1	Either inequality correct	
A1	Both inequalities correct "and" between the inequalities is acceptable. If $\cap$ used, deduct an A mark.	

Question Number	Scheme	Marks
4		
(a)	$\lvert 18\sqrt{3}-18\mathrm{i}\rvert = 18\sqrt{(3+1)} = 36$	B1
	$\tan\theta = \dfrac{-18}{18\sqrt{3}} \quad \theta = -\dfrac{\pi}{6}, \quad 18\sqrt{3}-18\mathrm{i} = 36\left(\cos\left(-\dfrac{\pi}{6}\right)+\mathrm{i}\sin\left(-\dfrac{\pi}{6}\right)\right)$	M1,A1cao (3)
(b)	$z^4 = 36\left(\cos-\dfrac{\pi}{6}+\mathrm{i}\sin-\dfrac{\pi}{6}\right) = 36\left(\cos\left(2k\pi-\dfrac{\pi}{6}\right)+\mathrm{i}\sin\left(2k\pi-\dfrac{\pi}{6}\right)\right)$	M1
	$z = \sqrt{6}\left(\cos\left(\dfrac{12k\pi-\pi}{24}\right)+\mathrm{i}\sin\left(\dfrac{12k\pi-\pi}{24}\right)\right)$	M1
	$k=0 \quad z_0 = \sqrt{6}\left(\cos\left(\dfrac{-\pi}{24}\right)+\mathrm{i}\sin\left(\dfrac{-\pi}{24}\right)\right) = \sqrt{6}\,\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{24}\right)}$	B1
	$k=1 \quad z_1 = \sqrt{6}\left(\cos\left(\dfrac{11\pi}{24}\right)+\mathrm{i}\sin\left(\dfrac{11\pi}{24}\right)\right) = \sqrt{6}\mathrm{e}^{\mathrm{i}\frac{11\pi}{24}}$	A1ft
	$k=2 \quad z_2 = \sqrt{6}\left(\cos\left(\dfrac{23\pi}{24}\right)+\mathrm{i}\sin\left(\dfrac{23\pi}{24}\right)\right) = \sqrt{6}\mathrm{e}^{\mathrm{i}\frac{23\pi}{24}}$	
	$k=-1 \quad z_3 = \sqrt{6}\left(\cos\left(-\dfrac{13\pi}{24}\right)+\mathrm{i}\sin\left(-\dfrac{13\pi}{24}\right)\right) = \sqrt{6}\mathrm{e}^{\mathrm{i}\left(-\frac{13\pi}{24}\right)}$	A1ft (5)
		[8]

(a)	
B1	Correct modulus
M1	Attempt argument using $\tan\theta = \dfrac{\pm 18}{18\sqrt{3}}$ or other valid method. Can be implied by $\theta = \pm\dfrac{\pi}{6}$
A1cao	Correct answer in the required form.
(b)	
M1	Valid method for generating at least 2 roots, rotation through $\dfrac{\pi}{2}$ accepted
M1	Apply de Moivre or use the rotation method
B1	Any one correct root
A1ft	Second root in required form
A1ft	All 4 roots in the required form Follow through their $\sqrt[4]{36}$ but 36 not acceptable.
NB	Argument in degrees – M1M1B1A0A0 (ie treat as mis-read) Incorrect argument: B0A1ftA1ft available Answers in $r(\cos\theta+\mathrm{i}\sin\theta)$ form – deduct final A marks

Question Number	Scheme	Marks
5	$w=\frac{z-3\mathrm{i}}{z+2\mathrm{i}}$	
	$w(z+2\mathrm{i})=z-3\mathrm{i} \quad z=\frac{\mathrm{i}(2w+3)}{1-w}$	M1
	$\lvert z\rvert=1 \quad \left\lvert\frac{\mathrm{i}(2w+3)}{1-w}\right\rvert=1$	dM1
	$\lvert \mathrm{i}(2w+3)\rvert=\lvert 1-w\rvert$	
	$w=u+\mathrm{i}v \quad (2u+3)^2+4v^2=(1-u)^2+v^2$	ddM1
	$4u^2+12u+9+4v^2=1-2u+u^2+v^2$	
	$3u^2+3v^2+14u+8=0$	dddM1
	$u^2+v^2+\frac{14}{3}u+\frac{8}{3}=0$	A1
	$\left(u+\frac{7}{3}\right)^2+v^2=-\frac{8}{3}+\frac{49}{9}=\frac{25}{9}$	
(i)	Centre $\left(-\frac{7}{3},0\right)$	A1
(ii)	Radius $\frac{5}{3}$	A1 (7)
		[7]

(a) M1	re-arrange to $z=$ ....
dM1	dep (on first M1) using $\lvert z\rvert=1$ with their previous result
ddM1	dep (on both previous M marks) use $w=u+\mathrm{i}v$ (or any other pair of letters inc (x, y)) and find the moduli (or square of it)
dddM1	dep (on all previous M marks) re-arrange to the form of the equation of a circle (same coeffs for the squared terms)
A1	for a correct equation in u and v with coeffs of u^2 and v^2 both 1
A1	Correct centre, must be in coordinate brackets. Completion of square need not be shown.
A1	Correct radius **Centre and radius must come from a correct circle equation for the A marks**

Question Number	Scheme	Marks
6.	$\frac{dy}{dx}+\frac{(x\cot x+2)}{x}y=\frac{4\sin x}{x^2}$	B1
	IF $=e^{\int\frac{(x\cot x+2)}{x}dx}$	M1
	$=e^{(\ln\sin x+2\ln x)}$	A1
	$=x^2\sin x$	A1
	$\frac{d}{dx}(\text{their IF}\times y)=\text{their IF}\times\text{"}\frac{4\sin x}{x^2}\text{"}$	M1
	$yx^2\sin x=\int 4\sin^2 x\,dx=4\int\frac{1-\cos 2x}{2}dx=4\left(\frac{x}{2}-\frac{1}{4}\sin 2x\right)\quad(+C)$	dM1A1
	$y=\frac{2x-\sin 2x+C}{x^2\sin x}$ oe	A1cao **[8]**

B1	Divide through by x^2
M1	Attempt an IF of the form $e^{\int\frac{k(x\cot x+2)}{x}dx}$
A1	$(\ln\sin x+2\ln x)$
A1	Correct IF
M1	Multiply through by their IF and write LHS in form shown – can be implied by next line. Allow if IF is seen instead of their function provided an IF has been attempted. Allow use of their RHS
dM1	Attempt to integrate $\sin^2 x$, including using $\sin^2 x=\frac{1}{2}(1\pm\cos 2x)$ $\cos 2x\to k\sin 2x$ depends on previous M mark
A1	Correct integration, constant not needed
A1	Include the constant and treat it correctly. Must have $y=...$

Question Number	Scheme	Marks
7 **(a)**	$r\sin\theta = 2a\sin\theta + 2a\sin\theta\cos\theta$ OR $r\sin\theta = 2a\sin\theta + a\sin 2\theta$	B1
	$\frac{\mathrm{d}(r\sin\theta)}{\mathrm{d}\theta} = 2a\cos\theta + 2a\cos^2\theta - 2a\sin^2\theta$ OR $\frac{\mathrm{d}(r\sin\theta)}{\mathrm{d}\theta} = 2a\cos\theta + 2a\cos 2\theta$	M1 A1
	$2\cos^2\theta + \cos\theta - 1 = 0$ terms in any order	
	$(2\cos\theta - 1)(\cos\theta + 1) = 0$	
	$\cos\theta = \frac{1}{2}$ $\theta = \frac{\pi}{3}$ $(\theta = \pi$ need not be seen$)$	dM1A1
	$r = 2a \times \frac{3}{2} = 3a$	A1 (6)
(b)	Area $= \frac{1}{2}\int r^2 \mathrm{d}\theta = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 4a^2(1+\cos\theta)^2 \mathrm{d}\theta$	
	$= 2a^2\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}(1 + 2\cos\theta + \cos^2\theta)\mathrm{d}\theta$	M1
	$= 2a^2\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left(1 + 2\cos\theta + \frac{1}{2}(\cos 2\theta + 1)\right)\mathrm{d}\theta$	M1
	$= 2a^2\left[\theta + 2\sin\theta + \frac{1}{2}\left(\frac{1}{2}\sin 2\theta + \theta\right)\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	dM1A1
	$= 2a^2\left[\frac{\pi}{3} + \sqrt{3} + \frac{1}{4}\times\frac{\sqrt{3}}{2} + \frac{\pi}{6} - \left(\frac{\pi}{6} + 1 + \frac{1}{4}\times\frac{\sqrt{3}}{2} + \frac{\pi}{12}\right)\right]$	M1 NB: A1 on e-PEN
	$= 2a^2\left(\frac{\pi}{4} + \sqrt{3} - 1\right)$	
	Area of $\Delta OAB = \frac{1}{2}\times 3a \times (2+\sqrt{3})a \times \sin\frac{\pi}{6}$ $\left(= \frac{3}{4}a^2(2+\sqrt{3})\right)$	
	Shaded area $= 2a^2\left(\frac{\pi}{4} + \sqrt{3} - 1\right) - \frac{3}{4}a^2(2+\sqrt{3}) = \frac{a^2}{4}(2\pi - 14 + 5\sqrt{3})$	M1A1cao (7)
		[13]

Question Number	Scheme	Marks
(a)		
B1	Multiply r by $\sin\theta$ Award if not seen explicitly but a correct result following use of double angle formula is seen	
M1	Differentiate $r\sin\theta$ or $r\cos\theta$ (using product rule or using double angle formula first)	
A1	Correct derivative for $r\sin\theta$	
dM1	Use $\sin^2\theta+\cos^2\theta=1$ to form a 3TQ in $\cos\theta$ and attempt its solution by a valid method	
A1	Correct value for θ	
A1	Correct r	
(b)		
M1	Use area $=\frac{1}{2}\int r^2\mathrm{d}\theta$ with $r=2a+2a\cos\theta$, no limits needed,	
M1	Use a double angle formula to obtain a function ready for integrating (Alt method uses integration by parts – may be seen)	
dM1	Attempt the integration $\cos 2\theta \rightarrow \frac{1}{k}\sin 2\theta$ $k=\pm 2$ or ± 1	
A1	Correct integration,	
M1	Substitute the limits (need not be simplified). Limits $\frac{\pi}{6}$ and their θ from (a) provided this is $>\frac{\pi}{6}$ NB: A1 on e-PEN	
M1	Obtain the area of ΔOAB and subtract from their previous area	
A1	Correct answer	

Question Number	Scheme	Marks
8 (a)	$x=\mathrm{e}^u \quad \frac{\mathrm{d}x}{\mathrm{d}u}=\mathrm{e}^u$ or $\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^{-u}$ or $\frac{\mathrm{d}x}{\mathrm{d}u}=x$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}u}\times\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^{-u}\frac{\mathrm{d}y}{\mathrm{d}u}$	M1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\mathrm{e}^{-u}\frac{\mathrm{d}u}{\mathrm{d}x}\frac{\mathrm{d}y}{\mathrm{d}u}+\mathrm{e}^{-u}\frac{\mathrm{d}^2y}{\mathrm{d}u^2}\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^{-2u}\left(-\frac{\mathrm{d}y}{\mathrm{d}u}+\frac{\mathrm{d}^2y}{\mathrm{d}u^2}\right)$	M1A1
	$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+3x\frac{\mathrm{d}y}{\mathrm{d}x}-8y=4\ln x$	
	$\mathrm{e}^{2u}\times\mathrm{e}^{-2u}\left(-\frac{\mathrm{d}y}{\mathrm{d}u}+\frac{\mathrm{d}^2y}{\mathrm{d}u^2}\right)+3\mathrm{e}^u\times\mathrm{e}^{-u}\frac{\mathrm{d}y}{\mathrm{d}u}-8y=4\ln\left(\mathrm{e}^u\right)$	dM1
	$\frac{\mathrm{d}^2y}{\mathrm{d}u^2}+2\frac{\mathrm{d}y}{\mathrm{d}u}-8y=4u \quad *$	A1*cso (6)
B1	$\frac{\mathrm{d}x}{\mathrm{d}u}=\mathrm{e}^u$ oe as shown seen explicitly or used	
M1	Obtaining $\frac{\mathrm{d}y}{\mathrm{d}x}$ using chain rule here or seen later	
M1	Obtaining $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ using product rule (penalise lack of chain rule by the A mark)	
A1	Correct expression for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ any equivalent form	
dM1	Substituting in the equation to eliminate x (u and y **only**). Depends on the 2nd M mark	
A1*cso	Obtaining the **given** result from completely correct work	
	ALTERNATIVE 1	
	$x=\mathrm{e}^u \quad \frac{\mathrm{d}x}{\mathrm{d}u}=\mathrm{e}^u=x$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}u}=\frac{\mathrm{d}y}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}u}=x\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$\frac{\mathrm{d}^2y}{\mathrm{d}u^2}=1\frac{\mathrm{d}x}{\mathrm{d}u}\times\frac{\mathrm{d}y}{\mathrm{d}x}+x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\times\frac{\mathrm{d}x}{\mathrm{d}u}=x\frac{\mathrm{d}y}{\mathrm{d}x}+x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	M1A1
	$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\mathrm{d}^2y}{\mathrm{d}u^2}-\frac{\mathrm{d}y}{\mathrm{d}u}$	
	$\left(\frac{\mathrm{d}^2y}{\mathrm{d}u^2}-\frac{\mathrm{d}y}{\mathrm{d}u}\right)+3x\times\frac{1}{x}\frac{\mathrm{d}y}{\mathrm{d}u}-8y=4\ln\left(\mathrm{e}^u\right)$	
	$\frac{\mathrm{d}^2y}{\mathrm{d}u^2}+2\frac{\mathrm{d}y}{\mathrm{d}u}-8y=4u \quad *$	dM1A1*cso (6)

Question Number	Scheme	Marks
B1	$\frac{dx}{du} = e^u$ oe as shown seen explicitly or used	
M1	Obtaining $\frac{dy}{du}$ using chain rule here or seen later	
M1	Obtaining $\frac{d^2y}{du^2}$ using product rule (penalise lack of chain rule by the A mark)	
A1	Correct expression for $\frac{d^2y}{du^2}$ any equivalent form	
dM1	Substituting in the equation to eliminate x (u and y **only**). Depends on the 2nd M mark	
A1*cso	Obtaining the **given** result from completely correct work	
	ALTERNATIVE 2:	
	$u = \ln x \quad \frac{du}{dx} = \frac{1}{x}$	B1
	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{x}\frac{dy}{du}$	M1
	$\frac{d^2y}{dx^2} = -\frac{1}{x^2}\frac{dy}{du} + \frac{1}{x}\frac{d^2y}{du^2} \times \frac{du}{dx} = -\frac{1}{x^2}\frac{dy}{du} + \frac{1}{x^2}\frac{d^2y}{du^2}$	M1A1
	$x^2\left(-\frac{1}{x^2}\frac{dy}{du} + \frac{1}{x^2}\frac{d^2y}{du^2}\right) + 3x \times \frac{1}{x}\frac{dy}{du} - 8y = 4u$	
	$\frac{d^2y}{du^2} + 2\frac{dy}{du} - 8y = 4u$ *	M1A1*cso
	Notes as for main scheme	

There are also **other solutions** which will appear, either starting from equation II and obtaining equation I, or mixing letters x, y and u until the final stage.
Mark as follows:

B1 as shown in schemes above
M1 obtaining a first derivative with chain rule
M1 obtaining a second derivative with product rule
A1 correct second derivative with 2 or 3 variables present
dM1 Either substitute in equation I or substitute in equation II according to method chosen **and** obtain an equation with only y and u (following sub in eqn I) or with only x and y (following sub in eqn II)
A1cso Obtaining the required result from completely correct work

Question Number	Scheme	Marks
(b)	$m^2+2m-8=0$	
	$(m+4)(m-2)=0, \quad m=-4,2$	M1A1
	CF $=Ae^{-4u}+Be^{2u}$	A1
	PI: try $y=au+b$ (or $y=cu^2+au+b$ different derivatives, c = 0)	
	$\frac{dy}{du}=a \quad \frac{d^2y}{du^2}=0$	M1
	$0+2a-8(au+b)=4u$	
	$a=-\frac{1}{2} \quad b=-\frac{1}{8}$	dM1A1
	$\therefore y=Ae^{-4u}+Be^{2u}-\frac{1}{2}u-\frac{1}{8}$	B1ft (7)
(c)	$y=Ax^{-4}+Bx^2-\frac{1}{2}\ln x-\frac{1}{8}$	B1 (1)
		[14]

(b) M1	Writing down the correct aux equation and solving to m = … (usual rules)
A1	Correct solution ($m=-4,2$)
A1	Correct CF – can use any (single) variable
M1	Using an appropriate PI and finding $\frac{dy}{du}$ **and** $\frac{d^2y}{du^2}$ Use of $y=\lambda u$ scores M0
dM1	Substitute in the equation to obtain values for the unknowns. Depends on the second M1
A1	Correct unknowns two or three (with c = 0)
B1ft	A complete solution, follow through their CF and a non-zero PI. Must have y = a function of u Allow recovery of incorrect variables.
(c) B1	Reverse the substitution to obtain a correct expression for y in terms of x No ft here x^{-4} or $e^{-4\ln x}$ and x^2 or $e^{2\ln x}$ allowed. Must start y = …

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