Pearson Edexcel

Mark Scheme (Results)

January 2020

Pearson Edexcel International Advanced Level In Further Pure Mathematics F1 (WFM01) Paper 01

January 2020
Publications Code WFM01_01_2001_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- o.e. – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper or ag- answer given
- ⎣ or d… The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao), unless shown, for example, as A1ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q),\ \text{where}\left|pq\right|=\left|c\right|$, leading to $x=\ldots$

$(ax^2+bx+c)=(mx+p)(nx+q),\ \text{where}\left|pq\right|=\left|c\right|\text{ and }\left|mn\right|=\left|a\right|$, leading to $x=\ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\ \ q\neq 0$, leading to $x=\ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n\rightarrow x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n\rightarrow x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

January 2020
WFM01/01 Further Pure Mathematics F1
Mark Scheme

Question Number	Scheme	Notes	Marks
1.	(a) $\mathbf{A} = \begin{pmatrix} p & -5 \\ -2 & p+3 \end{pmatrix}$ (b) $p=3$; $\mathbf{A} = \begin{pmatrix} a & -5 \\ -2 & d \end{pmatrix}$		
(a)	$\det(\mathbf{A}) = p(p+3)-(-5)(-2)\ \{= p(p+3)-10\}$	Applies $p(p+3) \pm (-5)(-2)$	M1
	$p^2+3p-10=0 \Rightarrow (p+5)(p-2)=0 \Rightarrow p=...$	Obtains a correct expression for $\det(\mathbf{A})$, sets their $\det(\mathbf{A})=0$ and solves their 3TQ = 0 by any valid method to give $p = ...$	M1
	$p=-5,\,2$	$p=-5,\,2$	A1
			(3)
(b)	$\left\{ p=3 \Rightarrow \mathbf{A} = \begin{pmatrix} 3 & -5 \\ -2 & 6 \end{pmatrix} \right\}$		
	For either $\begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix}$ or $\det(\mathbf{A}) = 3(3+3)-10$ or 8	For **either** $\begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix}$ **or** a correct numerical expression or value for $\det(\mathbf{A})$, which can be seen or implied	B1
	$\mathbf{A}^{-1} = \dfrac{1}{3(3+3)-(-5)(-2)}\begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix}$	$\dfrac{1}{ad \pm (-5)(-2)}\text{Adj}(\mathbf{A})$, where a correct method has been employed for finding their Adj($\mathbf{A}$)	M1
	$\mathbf{A}^{-1} = \dfrac{1}{8}\begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix}$ or $= \begin{pmatrix} \frac{3}{4} & \frac{5}{8} \\ \frac{1}{4} & \frac{3}{8} \end{pmatrix}$ or $= \begin{pmatrix} 0.75 & 0.625 \\ 0.25 & 0.375 \end{pmatrix}$ or $= \begin{pmatrix} \frac{6}{8} & \frac{5}{8} \\ \frac{2}{8} & \frac{3}{8} \end{pmatrix}$	Correct $\mathbf{A}^{-1}$	A1
			(3)
			6

		Question 1 Notes
1. (b)	**Note**	$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \text{Adj}(\mathbf{A}) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ is a correct method for finding their Adj($\mathbf{A}$)
	Note	Allow B1 M1 A0 for just writing $\dfrac{1}{3(3+3)-(-5)(-2)}\begin{pmatrix} p+3 & 5 \\ 2 & p \end{pmatrix}$
	Note	Allow B0 M1 A0 for just writing $\dfrac{1}{3(3+3)+(-5)(-2)}\begin{pmatrix} p+3 & 5 \\ 2 & p \end{pmatrix}$
	Note	Allow B0 M1 A0 for just writing $\dfrac{1}{p(p+3) \pm (-5)(-2)}\begin{pmatrix} p+3 & 5 \\ 2 & p \end{pmatrix}$
	Note	Allow M1 for evidence of a correct numerical expression for $\det \mathbf{A} = ad \pm (-5)(-2)$ followed by $\dfrac{1}{\text{their } \det(A)}\text{Adj}(\mathbf{A})$ where a correct method has been employed for finding their Adj($\mathbf{A}$)
	Note	Give final A0 for $\dfrac{1}{18-10}\begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix}$ without reference to $\dfrac{1}{8}\begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix}$ or any other acceptable answer
	Note	Give B1 M1 A1 for writing down a correct final answer for $\mathbf{A}^{-1}$ from no working

Question Number	Scheme	Notes	Marks
2.	Let $\mathrm{f}(x)=3x^3+kx^2+33x+13$; $k\in\mathbb{R}$; $x=-\frac{1}{3}$ is a root of $\mathrm{f}(x)=0$		
	Note: Ignore labelling of parts when marking Q2		
(a) **Way 1**	$3\left(-\frac{1}{3}\right)^3+k\left(-\frac{1}{3}\right)^2+33\left(-\frac{1}{3}\right)+13=0 \Rightarrow k=...$	Some evidence of substituting $x=-\frac{1}{3}$ into the given equation and solves to find $k=...$	M1
	$\left\{-\frac{1}{9}+\frac{1}{9}k-11+13=0\Rightarrow -1+k+18=0\Rightarrow\right\}$ $k=-17$	$k=-17$	A1
			(2)
(a) **Way 2**	$\mathrm{f}(x)=(3x+1)(x^2+Ax+13)$ x: $3(13)+A=33\Rightarrow A=-6$ x^2: $k=1+(\text{"}-6\text{"})(3)$	Expresses $\mathrm{f}(x)=(3x\pm1)(x^2+Ax\pm13)$, equates x terms to find A and equates x^2 terms to find k	M1
	$k=-17$	$k=-17$	A1
			(2)
(b)	$\{\mathrm{f}(x)=\}$ $(3x+1)(x^2-6x+13)$ or $\{\mathrm{f}(x)=\}$ $\left(x+\frac{1}{3}\right)(3x^2-18x+39)$	Attempts to find the quadratic factor e.g. using long division to obtain $(3x\pm1)$ with $(x^2\pm qx+...)$ or $\left(x\pm\frac{1}{3}\right)$ with $(3x^2\pm qx+...)$; $q=$ value $\neq 0$ e.g. factorising/equating coefficients to obtain $\mathrm{f}(x)=(3x\pm1)(x^2\pm qx\pm r)$ or $\mathrm{f}(x)=\left(x\pm\frac{1}{3}\right)(3x^2\pm qx\pm r)$, $q=$ value $\neq 0$, r can be 0	M1
		$x^2-6x+13$ or $3x^2-18x+39$ seen in their working	A1
	$\{x^2-6x+13=0$ or $3x^2-18x+39=0\Rightarrow\}$		
	e.g. • $x=\frac{--6\pm\sqrt{(-6)^2-4(1)(13)}}{2(1)}$ • $(x-3)^2-9+13=0\Rightarrow x=...$	**dependent on the previous M mark** Correct method of applying the quadratic formula or completing the square **for solving their 3TQ** on their quadratic factor	dM1
	$\{x=\}$ $3\pm2\mathrm{i}$ (or $3\pm\mathrm{i}2$)	$3+2\mathrm{i}$ **and** $3-2\mathrm{i}$	A1
			(4)
			6

	Question 2 Notes	
2. (b)	**Note**	You can assume $z\equiv x$ for solutions in this part
	Note	Give final dM1A1 for $x^2-6x+13=0$ or $3x^2-18x+39=0\Rightarrow x=3+2\mathrm{i}, 3-2\mathrm{i}$ with no intermediate working.
	Note	Give M1 A1 dM1 A1 for $3x^3-17x^2+33x+13=0\Rightarrow x=3+2\mathrm{i}, 3-2\mathrm{i}$ with no intermediate working.
	Note	They must be solving a 3TQ $\text{"}A\text{"}x^2+\text{"}B\text{"}x+\text{"}C\text{"}$ where A, B, C ***are all numerical values*** $\neq 0$ for the final dM1 mark.
	Note	**Special Case:** If ***their quadratic*** factor $x^2+\text{"}B\text{"}x+\text{"}C\text{"}$ **can** be factorised then allow dM1 for correct factorisation leading to $x=...$ Otherwise, give dM0 for applying a method of factorisation to solve their 3TQ = 0.

<table>
<tr><td></td><td colspan="4">Question 2 Notes Continued</td></tr>
<tr><td>2. (b)</td><td>Note</td><td colspan="3"><u>Reminder: Method mark for solving a 3TQ = 0</u>
Formula: $Ax^2+Bx+C=0 \Rightarrow$ Attempt to use the correct formula (with values for A, B, C)
Completing the Square: $x^2+Bx+C=0 \Rightarrow \left(x \pm \frac{B}{2}\right)^2 \pm q \pm C = 0,\ q \neq 0,$ leading to $x = ...$</td></tr>
<tr><td></td><td>Note:</td><td colspan="3"><u>Comparing coefficients:</u> $\text{f}(x) = (3x+1)(x^2+\alpha x+\beta) \equiv 3x^3-17x^2+33x+13$
$x^2: 3\alpha+1=-17 \Rightarrow \alpha=-6;\ \ z: 3\beta+\alpha=33 \Rightarrow 3\beta-6=33 \Rightarrow \beta=13;\ \ \text{constant}: \beta=13$
yielding quadratic factor = $x^2-6x+13$</td></tr>
<tr><td></td><td>Note</td><td colspan="3">The solutions $3 \pm 2\text{i}$ need to follow on from a correct $x^2-6x+13=0$ or $3x^2-18x+39=0$ in order to gain the final A mark.</td></tr>
<tr><td></td><td>Note</td><td colspan="3">Give final A0 for writing $\frac{6 \pm 4\text{i}}{2}$ followed by either $3 \pm 4\text{i}$ or $6 \pm 2\text{i}$</td></tr>
<tr><td>2. (a)
ALT 1</td><td>Note</td><td colspan="3"><u>Long division:</u>
$$\begin{array}{r} 3x^2 \quad -18x \quad +39 \\ x+\tfrac{1}{3}\,|\,\overline{3x^3 + kx^2 + 33x + 13} \\ \underline{3x^3 + x^2} \\ (k-1)x^2 + 33x \\ \underline{-18x^2 - 6x} \\ 39x + 13 \\ \underline{39x + 13} \\ 0 \end{array} \qquad \text{or} \qquad \begin{array}{r} x^2 \quad - \quad 6x \quad +13 \\ 3x+1\,|\,\overline{3x^3 + kx^2 + 33x + 13} \\ \underline{3x^3 + x^2} \\ (k-1)x^2 + 33x \\ \underline{-18x^2 - 6x} \\ 39x + 13 \\ \underline{39x + 13} \\ 0 \end{array}$$</td></tr>
<tr><td></td><td></td><td>$(k-1)--18=0 \Rightarrow k = ...$</td><td>Full complete method of dividing by either $x+\frac{1}{3}$ or $(3x+1)$, applying remainder = 0 and solving a relevant equation to find $k = ...$</td><td>M1</td></tr>
<tr><td></td><td></td><td>$k=-17$</td><td>$k=-17$</td><td>A1</td></tr>
<tr><td></td><td></td><td></td><td></td><td>(2)</td></tr>
<tr><td></td><td>Note</td><td colspan="3">Give M0 for dividing by either $x-\frac{1}{3}$ or $3x-1$</td></tr>
</table>

		Question 2 Notes Continued		
2. (a) **ALT 2**	**Note**	**Long division:** $\begin{array}{rl} & x^2 + \left(\frac{k-1}{3}\right)x + \left(\frac{100-k}{9}\right) \\ \hline 3x+1 \mid & 3x^3 + kx^2 + 33x + 13 \\ & \underline{3x^3 + x^2} \\ & (k-1)x^2 + 33x \\ & \underline{(k-1)x^2 + \left(\frac{k-1}{3}\right)x} \\ & \left(\frac{100-k}{3}\right)x + 13 \\ & \underline{\left(\frac{100-k}{3}\right)x + \left(\frac{100-k}{9}\right)} \\ & 13 - \left(\frac{100-k}{9}\right) \end{array}$ or $\begin{array}{rl} & 3x^2 + (k-1)x + \left(\frac{100-k}{3}\right) \\ \hline x+\frac{1}{3} \mid & 3x^3 + kx^2 + 33x + 13 \\ & \underline{3x^3 + x^2} \\ & (k-1)x^2 + 33x \\ & \underline{(k-1)x^2 + \left(\frac{k-1}{3}\right)x} \\ & \left(\frac{100-k}{3}\right)x + 13 \\ & \underline{\left(\frac{100-k}{3}\right)x + \left(\frac{100-k}{9}\right)} \\ & 13 - \left(\frac{100-k}{9}\right) \end{array}$		
		$13-\left(\frac{100-k}{9}\right)=0 \Rightarrow k=...$ or $33-\left(\frac{k-1}{3}\right)=39 \Rightarrow k=...$	Full complete method of dividing by either $x+\frac{1}{3}$ or $(3x+1)$, applying remainder $=0$ and solving a relevant equation to find $k=...$	M1
		$\left\{\frac{117-100+k}{9}=0 \Rightarrow\right\} k=-17$	$k=-17$	A1
				(2)
	Note	Give M0 for dividing by either $x-\frac{1}{3}$ or $3x-1$		

Question Number	Scheme	Notes	Marks
3. (a)	$\sum_{r=1}^{n} r^2(2r+3) = 2\sum_{r=1}^{n} r^3 + 3\sum_{r=1}^{n} r^2$		
	$= 2\left(\frac{1}{4}n^2(n+1)^2\right) + 3\left(\frac{1}{6}n(n+1)(2n+1)\right)$	Attempts to expand $r^2(2r+3)$ and attempts to substitute at least one correct formula for either $\sum_{r=1}^{n} r^3$ or $\sum_{r=1}^{n} r^2$ into their resulting expression	M1
		Obtains an expression of the form $\alpha n^2(n+1)^2 + \beta n(n+1)(2n+1);\ \alpha, \beta \neq 0$	M1
		$2\left(\frac{1}{4}n^2(n+1)^2\right) + 3\left(\frac{1}{6}n(n+1)(2n+1)\right)$ which can be simplified or un-simplified	A1
	$= \frac{1}{2}n(n+1)\big(n(n+1)+(2n+1)\big)$ $= \frac{1}{2}n(n+1)(n^2+3n+1)$ *	Achieves the given result via an appropriate intermediate step with no algebraic errors seen in their working	A1 * **cso**
			(4)
(b)	$\left\{\sum_{r=10}^{25} r^2(2r+3) = \right\}$	{**Note:** Let $\mathrm{f}(n) = \frac{n}{2}(n+1)(n^2+3n+1)$ or their answer to part (a) or their un-simplified expression (for $\mathrm{f}(n)$) of the form $\alpha n^2(n+1)^2 + \beta n(n+1)(2n+1);\ \alpha, \beta \neq 0$}	
	$= \frac{25}{2}(25+1)((25)^2+3(25)+1) - \frac{9}{2}(9+1)((9)^2+3(9)+1)$	Applies $\mathrm{f}(25) - \mathrm{f}(9)$ **Note:** Give M0 for applying $\mathrm{f}(25) - \mathrm{f}(10)$	M1
	$\left\{= \frac{25}{2}(26)(701) - \frac{9}{2}(10)(109) = 227825 - 4905\right\}$		
	$= 222920$	222920 **cao**	A1
			(2)
			6

		Question 3 Notes
3. (a)	**Note**	**<u>Final A mark:</u>** $\text{LHS} = \frac{1}{2}n^2(n+1)^2 + \frac{1}{2}n(n+1)(2n+1) = \frac{1}{2}n^2(n^2+2n+1) + \frac{1}{2}n(2n^2+3n+1)$ $= \frac{1}{2}n^4 + n^3 + \frac{1}{2}n^2 + n^3 + \frac{3}{2}n^2 + \frac{1}{2}n \ = \frac{1}{2}n^4 + 2n^3 + 2n^2 + \frac{1}{2}n$ $\text{RHS} = \frac{n}{2}(n+1)(n^2+3n+1) = \frac{n}{2}(n^3+3n^2+n+n^2+3n+1) = \frac{n}{2}(n^3+4n^2+4n+1)$ $= \frac{1}{2}n^4 + 2n^3 + 2n^2 + \frac{1}{2}n$ Give final A1 cso for using algebra to show that the LHS and RHS are the same with some acknowledgment (e.g. ‘proved’, LHS = RHS, QED or □) that their proof is complete.

	Question 3 Notes Continued	
3. (a)	**Note**	Give final A0 for • jumping from $\frac{1}{2}n^4+2n^3+2n^2+\frac{1}{2}n$ to $\frac{n}{2}(n+1)(n^2+3n+1)$ with no intermediate working
	Note	Condone final A1 for • jumping from $\frac{n}{2}(n^3+4n^2+4n+1)$ to $\frac{n}{2}(n+1)(n^2+3n+1)$ with no intermediate working
	Note	Achieving the given result via an appropriate intermediate step with no algebraic errors seen in their working includes e.g. • $2\left(\frac{1}{4}n^2(n+1)^2\right)+3\left(\frac{1}{6}n(n+1)(2n+1)\right)=\frac{1}{2}n^2(n+1)^2+\frac{1}{2}n(n+1)(2n+1)$ $=\frac{1}{2}n(n+1)(n^2+3n+1)$ • $2\left(\frac{1}{4}n^2(n+1)^2\right)+3\left(\frac{1}{6}n(n+1)(2n+1)\right)=\frac{1}{2}n(n+1)(n^2+n)+\frac{1}{2}n(n+1)(2n+1)$ $=\frac{1}{2}n(n+1)(n^2+3n+1)$ • $2\left(\frac{1}{4}n^2(n+1)^2\right)+3\left(\frac{1}{6}n(n+1)(2n+1)\right)=\frac{1}{2}n(n+1)[n(n+1)]+\frac{1}{2}n(n+1)(2n+1)$ $=\frac{1}{2}n(n+1)(n^2+3n+1)$
3. (b)	**Note**	Allow M1 for $227825-4905$ and A1 for obtaining 222920
	Note	Allow M1 for $\left(\frac{1}{2}(25)^2(26)^2+\frac{1}{2}(25)(26)(51)\right)-\left(\frac{1}{2}(9)^2(10)^2+\frac{1}{2}(9)(10)(19)\right)$ $\{=(211250+16575)-(4050+855)=227825-4905\}$ and A1 for obtaining 222920
	Note	Give M0 A0 for writing 222920 by itself with no supporting working
	Note	Allow M1 A1 for writing $\sum_{r=1}^{25} r^2(2r+3)-\sum_{r=1}^{9} r^2(2r+3)=222920$
	Note	Give M0 A0 for listing individual terms i.e. $\sum_{r=10}^{25} r^2(2r+3)=(10)^2(23)+(11)^2(25)+(12)^2(27)+...+(25)^2(53)$ $=2300+3025+3888+...+33125=222920$ by itself is M0 A0
	Note	Give M0 A0 for applying $\mathrm{f}(25)-\mathrm{f}(10)=\frac{25}{2}(25+1)((25)^2+3(25)+1)-\frac{10}{2}(10+1)((10)^2+3(10)+1)$ $=\frac{25}{2}(26)(701)-5(11)(131)=227825-7205=220620$
	Note	For M1 allow only one slip when substituting in $n=25$ and $n=9$
	Note	Give M0 for • $\frac{25}{2}(25+1)((25)^2+3(25)+1)-\frac{9}{2}(9+1)((10)^2+3(10)+1)\ \{=227825-5895=221930\}$

Question Number	Scheme	Notes	Marks
4.	$z_1 = p+5\text{i},\ z_2 = 9+8\text{i},\ z_3 = \frac{z_1}{z_2}$; $\arg(z_1)=\frac{\pi}{3}$		
(a) **Way 1**	$z_3 = \frac{(p+5\text{i})}{(9+8\text{i})}\times\frac{(9-8\text{i})}{(9-8\text{i})}$	Multiplies numerator **and** denominator of z_3 by $9-8\text{i}$	M1
	$=\frac{9p-8p\text{i}+45\text{i}+40}{81+64}$	Applies $\text{i}^2=-1$ to give either • a correct expression in terms of p for the numerator **or** • a correct numerical expression or value for the denominator	A1
	$=\frac{9p+40}{145}+\left(\frac{-8p+45}{145}\right)\text{i}$	Correct answer written in the form $x+\text{i}y$ o.e. **or** writes a correct $x=\frac{9p+40}{145}$, $y=\frac{-8p+45}{145}$	A1
			(3)
(a) **Way 2**	$z_3 = \frac{(p+5\text{i})}{(9+8\text{i})}\times\frac{(-9+8\text{i})}{(-9+8\text{i})}$	Multiplies numerator **and** denominator of z_3 by $-9+8\text{i}$	M1
	$=\frac{-9p+8p\text{i}-45\text{i}-40}{-81-64}$	Applies $\text{i}^2=-1$ to give either • a correct expression in terms of p for the numerator **or** • a correct numerical expression or value for the denominator	A1
	$=\frac{-9p-40}{-145}+\left(\frac{8p-45}{-145}\right)\text{i}$	Correct answer written in the form $x+\text{i}y$ o.e. **or** writes a correct $x=\frac{-9p-40}{-145}$ and $y=\frac{8p-45}{-145}$	A1
			(3)
(b)	$\left\{\lvert z_2\rvert=\sqrt{9^2+8^2}\ \Rightarrow\right\}\ \lvert z_2\rvert=\sqrt{145}$	$\sqrt{145}$	B1
			(1)
(c)(i) **Way 1**	$\left\{\arg(z_1)=\frac{\pi}{3}\Rightarrow\right\}$		
	e.g. $\arctan\left(\frac{5}{p}\right)=\frac{\pi}{3}$ or $\tan\left(\frac{\pi}{3}\right)=\frac{5}{p}$ or $\sqrt{3}=\frac{5}{p}$	Uses trigonometry to form a correct equation in p	M1
	$p=\frac{5}{\sqrt{3}}$ or $\frac{5}{3}\sqrt{3}$ or $\sqrt{\frac{25}{3}}$	Correct exact value for p **Note:** You can apply isw	A1
(c)(i) **Way 2**	$\left\{z_1 = \sqrt{p^2+25}\left(\cos\frac{\pi}{3}+\text{i}\sin\frac{\pi}{3}\right)= p+5\text{i}\Rightarrow\right\}$		
	e.g. $\sqrt{p^2+25}\left(\cos\frac{\pi}{3}\right)=p$ or $\sqrt{p^2+25}\left(\sin\frac{\pi}{3}\right)=5$	Uses trigonometry to form a correct equation in p	M1
	$p=\frac{5}{\sqrt{3}}$ or $\frac{5}{3}\sqrt{3}$ or $\sqrt{\frac{25}{3}}$	Correct exact value for p **Note:** You can apply isw	A1
(ii)	• $\lvert z_3\rvert = \frac{\lvert z_1\rvert}{\lvert z_2\rvert} = \frac{\sqrt{\left(\frac{5}{\sqrt{3}}\right)^2+(5)^2}}{\sqrt{145}} = \frac{\sqrt{\frac{100}{3}}}{\sqrt{145}}$ • $z_3=\frac{8+3\sqrt{3}}{29}+\frac{27-8\sqrt{3}}{87}\Rightarrow\lvert z_3\rvert=\sqrt{\left(\frac{8+3\sqrt{3}}{29}\right)^2+\left(\frac{27-8\sqrt{3}}{87}\right)^2}$		
	$\lvert z_3\rvert=\frac{10}{\sqrt{435}}$ or $\frac{10}{435}\sqrt{435}$ or $\frac{2}{87}\sqrt{435}$ or $\frac{2\sqrt{435}}{87}$	Correct exact answer written in the form $\frac{a}{\sqrt{b}}$ or $c\sqrt{b}$; $a,b\in\mathbb{Z},\ c\in\mathbb{Q}$	B1
	Note: Give B1 for $\lvert z_3\rvert=\sqrt{\frac{20}{87}}$		**(3)**
			7

	Question 4 Notes	
4. (a)	**Note**	Give 2nd A0 for $z_3 = \frac{9p+40}{81+64} + \left(\frac{-8p+45}{81+64}\right)\mathrm{i}$ without reference to $z_3 = \frac{9p+40}{145} + \left(\frac{-8p+45}{145}\right)\mathrm{i}$
	Note	$\frac{9p+40+(45-8p)\mathrm{i}}{145}$ is not considered to be in the form $x+\mathrm{i}y$
	Note	Allow final A1 for $z_3 = \frac{9p}{145} + \frac{8}{29} + \left(\frac{9}{29} - \frac{8p}{145}\right)\mathrm{i}$
	Note	Allow final A1 for $z_3 = \frac{9p+40}{145} - \left(\frac{8p-45}{145}\right)\mathrm{i}$
	Note	y written as $y = \left(\frac{-8p+45}{145}\right)\mathrm{i}$ is incorrect
	Note	M1 A1 can be implied for writing $z_3 = \frac{(p+5\mathrm{i})}{(9+8\mathrm{i})} = \frac{9p-8p\mathrm{i}}{145} + \frac{8+9\mathrm{i}}{29}$ and final A1 is then given for $z_3 = \frac{9p}{145} + \frac{8}{29} + \left(\frac{9}{29} - \frac{8p}{145}\right)\mathrm{i}$
(b)	**Note**	You can apply isw after seeing $\sqrt{145}$
	Note	Give B0 for writing 12, 12.0 or awrt 12.0 without reference to $\sqrt{145}$
(c)(i)	**Note**	Give M1 for any of $\arctan\left(\frac{5}{p}\right) = 60$, $\tan 60 = \frac{5}{p}$, $\arctan\left(\frac{p}{5}\right) = \frac{\pi}{6}$, $\tan 30 = \frac{p}{5}$
	Note	Give M1 A0 for $p = 2.88$ (truncated) or $p =$ awrt 2.89 without reference to a correct exact value
	Note	Give A0 for $p = \pm\frac{5}{\sqrt{3}}$ with no evidence of rejecting the negative value of p
(c)(ii)	**Note**	Allow B1 for $\lvert z_3 \rvert = \frac{\sqrt{1740}}{87}$

Question Number	Scheme	Notes	Marks
5.	$\mathrm{f}(x)=x^4-12x^{\frac{3}{2}}+7\,;\quad x\geqslant 0$		
(a) **Way 1**	$\mathrm{f}(2)=-10.9411255...$ $\mathrm{f}(3)=\ \ 25.64617093...$	Attempts to evaluate both $\mathrm{f}(2)$ and $\mathrm{f}(3)$ **and** either $\mathrm{f}(2)=-10$ (truncated) or awrt -11 **or** $\mathrm{f}(3)=25$ (truncated) or awrt 26	M1
	Sign change {negative, positive} {and $\mathrm{f}(x)$ is continuous} therefore a root $\{\alpha\}$ exists in the interval $\{[2,3]\}$	Both values correct awrt (or truncated) to 2 sf, reason and a valid conclusion	A1 **cso**
			(2)
(b)	$\mathrm{f}'(x)=4x^3-18x^{\frac{1}{2}}$	At least one of either $x^4\to\pm Ax^3$ or $-12x^{\frac{3}{2}}\to\pm Bx^{\frac{1}{2}}\,;\ A,B\neq 0$	M1
		Correct differentiation, which can be un-simplified or simplified	A1
	$\left\{\alpha\simeq 2.5-\dfrac{\mathrm{f}(2.5)}{\mathrm{f}'(2.5)}\Rightarrow\right\}\alpha\simeq 2.5-\dfrac{(2.5)^4-12(2.5)^{\frac{3}{2}}+7}{4(2.5)^3-18(2.5)^{\frac{1}{2}}}$	**dependent on the previous M mark** Valid attempt at Newton-Raphson using the applied $\mathrm{f}(2.5)$ and their applied $\mathrm{f}'(2.5)$	dM1
	$\left\{\alpha\simeq 2.5-\dfrac{-1.3716649...}{34.0395011...}=2.5+0.0402962...\right\}$		
	$\alpha=2.54$ (2 dp)	**dependent on all 3 previous marks** 2.54 on first iteration (Ignore any subsequent iterations)	A1 **cao** **cso**
	Correct differentiation followed by 2.54 (with no working seen) scores full marks in part (b)		**(4)**
(c) **Way 1**	$\mathrm{f}(2.535)=-0.137392933...$ $\mathrm{f}(2.545)=\ \ 0.231219419...$	Chooses a suitable interval $[x_L\,,x_U]$ for x, which is within ±0.005 and either side of their answer to (b) and attempts to find either $\mathrm{f}(x_L)$ or $\mathrm{f}(x_U)$	M1
	Sign change {negative, positive} {and $\mathrm{f}(x)$ is continuous} therefore (a root) $\alpha=2.54$ {2 dp}	Both values correct awrt 1 sf, reason and a valid conclusion	A1
			(2)
(c) **Way 2**	**Condoned Method: Applying Newton-Raphson again.** E.g. Using $\alpha=2.54,\ 2.5402962...$		
	- $\alpha\simeq 2.54-\dfrac{0.046101609...}{36.8609766...}=2.538751631...$ - $\alpha\simeq 2.5402962...-\dfrac{0.05693746...}{36.88822382...}=2.538752436...$ So $\alpha=2.54$ (2 dp)	Evidence of applying Newton-Raphson for a second time on their answer to part (b)	M1
		Obtains either a truncated 2.538 or awrt 2.539 and a valid conclusion	A1
	Note: Work for Way 2 can be recovered in part (b)		**(2)**
			8
	Question 5 Notes		
5. (a)	**Note**	**Way 1: A1, correct solution only** Required to state **both** values for $\mathrm{f}(2)$ and $\mathrm{f}(3)$ correct awrt (or truncated) to 2 sf along with **a reason and a conclusion.** Reference to change of sign **or** e.g. $\mathrm{f}(2)\times\mathrm{f}(3)<0$ **or** $\mathrm{f}(2)<0<\mathrm{f}(3)$ **or** a diagram **or** <0 and >0 **or** one negative, one positive are sufficient reasons. There must be a conclusion, e.g. $\{x$ or$\}\ \alpha\in[2,3]$ or $\{x$ or$\}\,\alpha\in(2,3)$ or root lies between 2 and 3. Ignore the presence or absence of any reference to continuity.	
	Note	A minimal acceptable reason and conclusion is "change of sign, so $\alpha\in[2,3]$" **or** "change of sign, so root is between 2 and 3" **or** "change of sign, so root" **or** "$\mathrm{f}(2)=-10.9<0,\ \mathrm{f}(3)=25.6>0$, so root" **or** "change of sign, so in the interval"	

<table>
<tr><th></th><th colspan="2">Question 5 Notes Continued</th></tr>
<tr><td>5. (a)</td><td>Note</td><td>Give final A0 for writing as their conclusion "root lies between $\mathrm{f}(2)$ and $\mathrm{f}(3)$ "</td></tr>
<tr><td>5. (a)</td><td>Note</td><td><u>ALT</u>
The root of $\mathrm{f}(x) = 0$ is $2.5388...$, so they can choose x_1 which is less than $2.5388...$,
and choose x_2 which is greater than $2.5388...$ with both x_1 and x_2 lying in the interval [2, 3].
M1: Finds $\mathrm{f}(x_1)$ and $\mathrm{f}(x_2)$ with one of these values correct awrt (or truncated) to 2 sf
A1: Both values correct awrt (or truncated) to 2 sf, reason (e.g. sign change) and conclusion</td></tr>
<tr><td></td><td>Note</td><td><u>Helpful Table</u>
<table>
<tr><th>x</th><th>$\mathrm{f}(x)$</th></tr>
<tr><td>2</td><td>$-10.9411255...$</td></tr>
<tr><td>2.1</td><td>$-10.0701694...$</td></tr>
<tr><td>2.2</td><td>$-8.731928012...$</td></tr>
<tr><td>2.3</td><td>$-6.873372451...$</td></tr>
<tr><td>2.4</td><td>$-4.439168148...$</td></tr>
<tr><td>2.5</td><td>$-1.371664903...$</td></tr>
<tr><td>2.6</td><td>$2.389111651...$</td></tr>
<tr><td>2.7</td><td>$6.90546741...$</td></tr>
<tr><td>2.8</td><td>$12.24204622...$</td></tr>
<tr><td>2.9</td><td>$18.46583545...$</td></tr>
<tr><td>3</td><td>$25.64617093...$</td></tr>
</table></td></tr>
<tr><td>(b)</td><td>dM1</td><td>This mark can be implied by applying at least one correct <i>value</i> of either $\mathrm{f}(2.5)$ or their $\mathrm{f}'(2.5)$ (where $\mathrm{f}'(2.5)$ is found using their $\mathrm{f}'(x)$) to awrt 2 significant figures in $2.5 - \frac{\mathrm{f}(2.5)}{\mathrm{f}'(2.5)}$.
So <i>just writing</i> $2.5 - \frac{\mathrm{f}(2.5)}{\mathrm{f}'(2.5)}$ with an incorrect ft answer on their $\mathrm{f}'(2.5)$ scores dM0 A0.</td></tr>
<tr><td></td><td>Note</td><td>Allow M1 A1 dM1 A1 for $2.5 - \frac{\mathrm{f}(2.5)}{\mathrm{f}'(2.5)} = 2.54$ with no algebraic differentiation</td></tr>
<tr><td></td><td>Note</td><td>Allow M1 A1 dM1 A1 for correct answer 2.54 given with no other working</td></tr>
<tr><td></td><td>Note</td><td>You can imply the M1 A1 marks for the absence of algebraic differentiation by either
• $\mathrm{f}'(2.5) = 4(2.5)^3 - 18(2.5)^{\frac{1}{2}}$
• $\mathrm{f}'(2.5)$ applied correctly in $\alpha \simeq 2.5 - \frac{(2.5)^4 - 12(2.5)^{\frac{3}{2}} + 7}{4(2.5)^3 - 18(2.5)^{\frac{1}{2}}}$
• $\mathrm{f}'(2.5) = \text{awrt } 34$</td></tr>
<tr><td></td><td>Note</td><td>Differentiating INCORRECTLY to give $\mathrm{f}'(x) = 4x^3 + 18x^{\frac{1}{2}}$ leads to
$\alpha \simeq 2.5 - \frac{-1.3716649...}{90.9604989...} = 2.51507978... = 2.52 \text{ (2 dp)}$
This response should be given M1 A0 dM1 A0</td></tr>
<tr><td></td><td>Note</td><td>Differentiating INCORRECTLY to give $\mathrm{f}'(x) = 4x^3 + 18x^{\frac{1}{2}}$ and
$\alpha \simeq 2.5 - \frac{\mathrm{f}(2.5)}{\mathrm{f}'(2.5)} = 2.52$ is M1 A0 dM1 A0</td></tr>
</table>

<table>
<tr><td></td><td colspan="2">Question 5 Notes Continued</td></tr>
<tr><td rowspan="5">5. (c)</td><td>Note</td><td>If they obtain a correct answer 2.54 by an incorrect method in part (b) then M1 A1 is allowed in part (c).</td></tr>
<tr><td>Note</td><td>Way 1: A1, correct solution only
Required to state both values for $\mathrm{f}(x_L)$ and $\mathrm{f}(x_U)$ correct awrt (or truncated) to 1 sf along with a reason and a conclusion. Reference to change of sign or e.g. $\mathrm{f}(2.535)\times\mathrm{f}(2.545)<0$ or $\mathrm{f}(2.535)<0<\mathrm{f}(2.545)$ or a diagram or <0 and >0 or one negative, one positive are sufficient reasons. There must be a (minimal, not incorrect) conclusion e.g. $\alpha=2.54$, root (or α to part (b)) is correct, QED or □ are all acceptable. Ignore the presence or absence of any reference to continuity.</td></tr>
<tr><td>Note</td><td>A minimal acceptable reason and conclusion is any of
• “change of sign, hence root”
• “change of sign, so $\alpha=2.54$”
• “change of sign, so $x=2.54$”
• “change of sign, so α is correct {to 2 decimal places}”
• “$\mathrm{f}(2.535)=-0.1<0,\ \mathrm{f}(2.545)=0.2>0,$ so root”
• “$\mathrm{f}(2.535)=-0.1<0,\ \mathrm{f}(2.545)=0.2>0,$ so $\alpha=2.54$”</td></tr>
<tr><td>Note</td><td>No explicit reference to 2 decimal places is necessary for the conclusion</td></tr>
<tr><td>Note</td><td>Give A0 for stating “root is in between 2.535 and 2.545” or “root lies in the given interval” without reference to either $\alpha=2.54$, root (or α to part (b)) is correct, QED or □</td></tr>
<tr><td rowspan="2">(c)</td><td>Note</td><td><u>Way 1: ALT</u>
The root of $\mathrm{f}(x)=0$ is $2.5388...$, so they can choose x_L which is less than $2.5388...$, and choose x_U which is greater than $2.5388...$ with both x_L and x_U lying in the interval $[2.535,\ 2.545]$ and evaluate $\mathrm{f}(x_L)$ and $\mathrm{f}(x_U)$
M1: Chooses a suitable interval $[x_L,\ x_U]$ and attempts to find either $\mathrm{f}(x_L)$ or $\mathrm{f}(x_U)$
A1: Both values correct awrt (or truncated) to 1 sf, reason (e.g. sign change) and conclusion</td></tr>
<tr><td>Note</td><td><u>Helpful Table</u>
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<tr><th>x</th><th>$\mathrm{f}(x)$</th></tr>
<tr><td>2.535</td><td>$-0.137392933...$</td></tr>
<tr><td>2.536</td><td>$-0.100854301...$</td></tr>
<tr><td>2.537</td><td>$-0.064244144...$</td></tr>
<tr><td>2.538</td><td>$-0.027562401...$</td></tr>
<tr><td>2.539</td><td>0.00919099...</td></tr>
<tr><td>2.54</td><td>0.046016091...</td></tr>
<tr><td>2.541</td><td>0.082912964...</td></tr>
<tr><td>2.542</td><td>0.119881671...</td></tr>
<tr><td>2.543</td><td>0.156922274...</td></tr>
<tr><td>2.544</td><td>0.194034836...</td></tr>
<tr><td>2.545</td><td>0.231219419...</td></tr>
</table></td></tr>
<tr><td rowspan="2">(c)
Way 2</td><td>Note</td><td>If $\alpha=2.54$ in part (b), then give M1 A1 in part (c) for any of
• “$\alpha_2=2.538\Rightarrow\alpha_2=2.54$”
• “$\alpha_2=2.539\Rightarrow\alpha_2=2.54$”
• “$\alpha_2=2.539$, so answer to part (b) is correct”</td></tr>
<tr><td>Note</td><td>If $\alpha=2.54$ in part (b), then give M1 A0 in part (c) for writing “$\alpha\simeq2.54-\dfrac{\mathrm{f}(2.54)}{\mathrm{f}'(2.54)}=2.54$”</td></tr>
</table>

Question Number	Scheme	Notes	Marks
6.	$\mathbf{A}=\begin{pmatrix}2&3\\1&-4\end{pmatrix}$; $A: R(3p-13,\ p-4)\mapsto R'(7,-2)$		
(a) **Way 1**	$\left\{\begin{pmatrix}x_{R'}\\y_{R'}\end{pmatrix}=\begin{pmatrix}2&3\\1&-4\end{pmatrix}\begin{pmatrix}3p-13\\p-4\end{pmatrix}=\right\}$ $=\begin{pmatrix}2(3p-13)+3(p-4)\\1(3p-13)-4(p-4)\end{pmatrix}$	Correct method of multiplying out either $\begin{pmatrix}2&3\end{pmatrix}\begin{pmatrix}3p-13\\p-4\end{pmatrix}$ or $\begin{pmatrix}1&-4\end{pmatrix}\begin{pmatrix}3p-13\\p-4\end{pmatrix}$ to give a linear expression in terms of p for either $x_{R'}$ or $y_{R'}$ **Note:** Allow one slip in their multiplication	M1
	- $2(3p-13)+3(p-4)=7\Rightarrow p=\dots$ - $1(3p-13)-4(p-4)=-2\Rightarrow p=\dots$	**dependent on the previous M mark** Solves either their $x_{R'}=7$ or their $y_{R'}=-2$ to give $p=\dots$	dM1
	$\{9p-38=7$ or $-p+3=-2\Rightarrow\}$ $p=5$	$p=5$	A1
			(3)
(a) **Way 2**	$\{\mathbf{AR}=\mathbf{R}'\Rightarrow\mathbf{R}=\mathbf{A}^{-1}\mathbf{R}'\Rightarrow\}$		
	$\mathbf{R}=\frac{1}{-8-3}\begin{pmatrix}-4&-3\\-1&2\end{pmatrix}\begin{pmatrix}7\\-2\end{pmatrix}=\begin{pmatrix}2\\1\end{pmatrix}$	Applies $\mathbf{A}^{-1}\begin{pmatrix}7\\-2\end{pmatrix}$ to find the value for either x_R or y_R **Note:** Allow one slip in finding $\mathbf{A}^{-1}$	M1
	- $3p-13=2\Rightarrow p=\dots$ - $p-4=1\Rightarrow p=\dots$	**dependent on the previous M mark** Solves either $3p-13=$ their x_R or $p-4=$ their y_R to give $p=\dots$	dM1
	$p=5$	$p=5$	A1
			(3)
(a) **Way 3**	$\{\mathbf{AR}=\mathbf{R}'\Rightarrow\}$ $\begin{pmatrix}2&3\\1&-4\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}7\\-2\end{pmatrix}$ $\begin{aligned}2a+3b&=7\\a-4b&=-2\end{aligned}\Rightarrow a=2$ or $b=1$	Correct method of applying $\begin{pmatrix}2&3\\1&-4\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}7\\-2\end{pmatrix}$ to form a pair of simultaneous equations and attempts to find either $a=\dots$ or $b=\dots$ **Note:** Allow one slip in their multiplication	M1
	- $3p-13=2\Rightarrow p=\dots$ - $p-4=1\Rightarrow p=\dots$	**dependent on the previous M mark** Solves either $3p-13=$ their a or $p-4=$their b to give $p=\dots$	dM1
	$p=5$	$p=5$	A1
			(3)
(b) **Way 1**	$\{R(3(5)-13,\ 5-4)=R(2,1)\}$ $\{\text{Area}(ORS)=\}$ $\frac{1}{2}(7)(\text{"}2\text{"})$	A correct method for finding their x_R **and** applies $\frac{1}{2}(7)(\text{their } x_R)$	M1
	$=7\ (\text{units})^2$	7	A1 **cao**
			(2)
(c)	$\{\text{Area}(OR'S')=\}$ $\lvert 2(-4)-3(1)\rvert\times(7)$	$\pm(2(-4)-3(1))\times(\text{their area}(ORS))$	M1
	$=\ 77$	Correct answer of 77, which must be positive Only allow follow through of the value for $11\times$ their positive answer to (b)	A1 ft
			(2)
			7

Question Number	Scheme	Notes	Marks
6. (b) **Way 2**	{Area (ORS)} $=\frac{1}{2}\begin{vmatrix}0&2&0&0\\0&1&7&0\end{vmatrix}=\frac{1}{2}\lvert(0+14+0)-(0+0+0)\rvert$	A correct method for finding their $R(2,\,1)$ with a complete applied method for finding area(ORS) using $S(0,\,7)$ and their $R(2,\,1)$	M1
	$= 7$ (units)^2	7	A1 **cao**
			(2)

Question 6 Notes

6.	**Note**	$ORS \mapsto OR'S' \Rightarrow \begin{pmatrix}0&2&0\\0&1&7\end{pmatrix} \mapsto \begin{pmatrix}0&7&21\\0&-2&-28\end{pmatrix}$
(b) **Way 1**	**Note**	A correct method for finding their x_R includes any of • $x_R = 3(\text{"5"}) - 13 = 2$, where $p = \text{"5"}$ is found using part (a), Way 1 • their x_R found by applying $\mathbf{A}^{-1}\mathbf{R}'$ using part (a), Way 2 • $x_R =$ their a found using part (a), Way 3
(b) **Way 2**	**Note**	Give M1 A1 for $\frac{1}{2}\begin{vmatrix}2&0\\1&7\end{vmatrix}=\frac{1}{2}\lvert 14-0\rvert = 7$
	Note	Give M0 A0 for $\begin{vmatrix}0&2&0&0\\0&1&7&0\end{vmatrix}=\lvert\lvert(0+14+0)-(0+0+0)\rvert\rvert = 14$
	Note	There are other ways to find Area(ORS). All ways require a complete correct method for the M mark and a correct area of 7 for the A mark.
	Note	Give M1 for $\frac{1}{2}(1)(\text{"2"}) + \frac{1}{2}(6)(\text{"2"})$ as this method is equivalent to writing $\frac{1}{2}(7)(\text{"2"})$
	Note	Give M0 for the calculation $\frac{1}{2}(7)(7)\ \left\{=\frac{49}{2}\right\}$
(c)	**Note**	Give M1 A0 for applying $(2(-4)-3(1))\times(7)$ to give -77 with no reference to 77
	Note	Part (c) requires the use of the answer to part (b). So give M0 A0 for • Area $(OR'S') = \frac{1}{2}\begin{vmatrix}0&7&21&0\\0&-2&-28&0\end{vmatrix}=\frac{1}{2}\lvert(0-196+0)-(0-42+0)\rvert=\frac{1}{2}(154)=77$ • Area $(OR'S') = \frac{1}{2}\begin{vmatrix}7&21\\-2&-28\end{vmatrix}=\frac{1}{2}\lvert(-196)-(-42)\rvert=\frac{1}{2}(154)=77$ • Area $(OR'S') = (28)(21)-\frac{1}{2}(21)(28)-\frac{1}{2}(7)(2)-\frac{1}{2}(2+28)(14)$ $= 588-294-7-210 = 77$
	Note	Allow M1 A1 for • $\dfrac{\begin{vmatrix}7&21\\-2&-28\end{vmatrix}}{\begin{vmatrix}2&0\\1&7\end{vmatrix}}\times 7 = \dfrac{\lvert(-196)-(-42)\rvert}{\lvert 14-0\rvert}\times 7 = \dfrac{154}{14}\times 7 = 11\times 7 = 77$

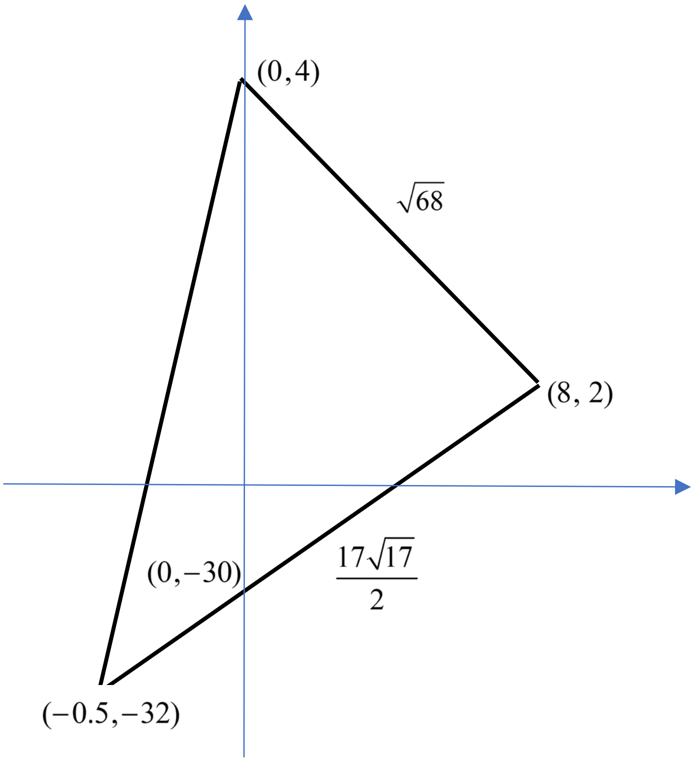
Question Number	Scheme	Notes	Marks
7.	$3x^2 + px - 5 = 0$ has roots α, β ; p is a constant (c) $\left(\alpha + \frac{1}{\beta}\right) + \left(\beta + \frac{1}{\alpha}\right) = 2\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$		
(a) (i)	$\alpha\beta = -\frac{5}{3}$	$\alpha\beta = -\frac{5}{3}$	B1
(ii)	$\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ $= \alpha\beta + 2 + \frac{1}{\alpha\beta} \quad = -\frac{5}{3} + 2 + \frac{1}{\left(-\frac{5}{3}\right)}$	Expands to give $\frac{1}{\alpha\beta} + 1 + 1 + \alpha\beta$; and uses their value of $\alpha\beta$ at least once in a resulting expression	M1
	$= -\frac{4}{15}$	$-\frac{4}{15}$	A1
			(3)
(b)(i)	$\alpha + \beta = -\frac{p}{3}$	$\alpha + \beta = -\frac{p}{3}$ (may be recovered from (a))	B1
(ii)	$\left(\alpha + \frac{1}{\beta}\right) + \left(\beta + \frac{1}{\alpha}\right) = \alpha + \beta + \frac{\alpha + \beta}{\alpha\beta}$	Evidence of $\frac{1}{\beta} + \frac{1}{\alpha}$ rewritten as $\frac{\alpha + \beta}{\alpha\beta}$ Can be implied	M1
	$= -\frac{p}{3} + \frac{-\frac{p}{3}}{-\frac{5}{3}}$ or $-\frac{p}{3} + \frac{p}{5}$ or $-\frac{2p}{15}$	$-\frac{p}{3} + \frac{-\frac{p}{3}}{-\frac{5}{3}}$ or $-\frac{p}{3} + \frac{p}{5}$ or $-\frac{2p}{15}$ or an equivalent fraction in terms of p **Note:** You can apply isw	A1
			(3)
(c)	$-\frac{2p}{15} = 2\left(-\frac{4}{15}\right) \Rightarrow p = 4$	Correctly obtains $p = 4$	B1
			(1)
(d)	$\sum = 2\left(-\frac{4}{15}\right) \quad = -\frac{8}{15}$; $\prod = -\frac{4}{15}$		
	$x^2 - -\frac{8}{15}x - \frac{4}{15} = 0$	Valid method for finding (their sum) **and applies** $x^2 - (\text{their sum})x + \text{their product}$ (can be implied), for their numerical values of the sum and product. **Note:** "$=0$" is not required for this mark **Note:** E.g. Using $(\text{their sum}) = \alpha + \beta = -\frac{p}{3} = -\frac{4}{3}$ is not considered a valid method for finding (their sum)	M1
	$15x^2 + 8x - 4 = 0$	***Any integer multiple*** of $15x^2 + 8x - 4 = 0$, including the "$=0$"	A1 **cso**
			(2)
			9

Question Number	Scheme	Notes	Marks
(a)(ii) **Way 2**	$\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$ $=\frac{(\alpha\beta+1)(\alpha\beta+1)}{\alpha\beta}=\frac{\left(-\frac{5}{3}+1\right)\left(-\frac{5}{3}+1\right)}{\left(-\frac{5}{3}\right)}=\frac{\frac{4}{9}}{-\frac{5}{3}}$	Expands to give $\frac{(\alpha\beta+1)(\alpha\beta+1)}{\alpha\beta}$ and uses their value of $\alpha\beta$ at least once in a resulting expression	M1
	$=-\frac{4}{15}$	$-\frac{4}{15}$	A1
(b)(ii) **Way 2**	$\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)$ $=\frac{(\alpha\beta+1)}{\beta}+\frac{(\alpha\beta+1)}{\alpha}=\frac{\alpha^2\beta+\alpha+\alpha\beta^2+\beta}{\alpha\beta}$	Embedded evidence of $\frac{1}{\beta}+\frac{1}{\alpha}$ rewritten as $\frac{\alpha+\beta}{\alpha\beta}$ Can be implied	M1
	$=\frac{\alpha\beta(\alpha+\beta)+\alpha+\beta}{\alpha\beta}$		
	$=\frac{(-\frac{5}{3})(-\frac{p}{3})+(-\frac{p}{3})}{(-\frac{5}{3})}$ or $\frac{\frac{5p}{9}-\frac{p}{3}}{-\frac{5}{3}}$ or $\frac{\frac{2p}{9}}{-\frac{5}{3}}$ or $-\frac{2p}{15}$	Correct expression in terms of p **Note:** You can apply isw	A1

	Question 7 Notes	
7. (d)	**Note**	Valid method for finding (their sum) includes • applying their $p=\dots$ in (c) to $\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)=$ their $-\frac{2p}{15}$ found in (b)(ii) • applying $\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)=2\left(\text{their } -\frac{4}{15} \text{ from (a)(ii)}\right)$
	Note	Defining a quadratic equation $px^2+qx+r=0$ and a correct method leading to $p=15,\ q=8,\ r=-4$ without writing a final answer of $15x^2+8x-4=0$ is final M1 A0
	Note	Give M0 for $\sum=-\frac{8}{15}$, $\mathbf{\Pi}=-\frac{4}{15}$ leading to $x^2+\frac{8}{15}-\frac{4}{15}=0$ (without recovery)
	Note	Allow M1 for $\sum=-\frac{8}{15}$, $\mathbf{\Pi}=-\frac{4}{15}$ with $x^2-(\text{sum})x+(\text{product})$ leading to $x^2+\frac{8}{15}-\frac{4}{15}=0$
	Note	Give A1 for $15y^2+8y-4=0$ (i.e. writing their answer completely in another variable)
	Note	$\alpha,\beta=\frac{-2\pm\sqrt{19}}{3}$ and $\alpha+\frac{1}{\beta},\beta+\frac{1}{\alpha}=\frac{-4\pm2\sqrt{19}}{15}$ may be used in (d) to find the sum and product of $\alpha+\frac{1}{\beta}$ and $\beta+\frac{1}{\alpha}$

	Question 7 Notes Continued	
7.	**ALT**	For finding $\alpha, \beta = \dfrac{-p+\sqrt{p^2+60}}{6}, \dfrac{-p-\sqrt{p^2+60}}{6}$
(a) (i)	**Note**	Give B1 for $\alpha, \beta = \dfrac{-p+\sqrt{p^2+60}}{6}, \dfrac{-p-\sqrt{p^2+60}}{6}$ and then finding $\alpha\beta = -\dfrac{5}{3}$ or $-\dfrac{60}{36}$
(b) (i)	**Note**	Give B1 for $\alpha, \beta = \dfrac{-p+\sqrt{p^2+60}}{6}, \dfrac{-p-\sqrt{p^2+60}}{6}$ and then finding $\alpha+\beta = -\dfrac{p}{3}$
	Note	Allow B1 for writing $\alpha+\beta = \dfrac{-p+\sqrt{p^2+60}}{6} + \dfrac{-p-\sqrt{p^2+60}}{6}$
(b)(ii)	**Note**	Allow M1 A1 for writing $\left(\alpha+\dfrac{1}{\beta}\right)+\left(\beta+\dfrac{1}{\alpha}\right)$ as $\dfrac{-p+\sqrt{p^2+60}}{6} + \dfrac{-p-\sqrt{p^2+60}}{6} + \dfrac{6}{-p+\sqrt{p^2+60}} + \dfrac{6}{-p-\sqrt{p^2+60}}$

Question Number	Scheme	Notes	Marks
8.	$H: xy = 16;\ P\left(4t, \frac{4}{t}\right), t \neq 0,$ and $A: t = 2$ lies on H. $A(8, 2)$		
(a)	$y = \frac{16}{x} = 16x^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -16x^{-2}$ or $-\frac{16}{x^2}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = \pm k\,x^{-2};\ k \neq 0$	M1
	$xy = 16 \Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 0$	Uses implicit differentiation to give $\pm x\frac{\mathrm{d}y}{\mathrm{d}x} \pm y$	
	$x = 4t,\ y = \frac{4}{t} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t}.\frac{\mathrm{d}t}{\mathrm{d}x} = -\left(\frac{4}{t^2}\right)\left(\frac{1}{4}\right)$	their $\frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{1}{\text{their } \frac{\mathrm{d}y}{\mathrm{d}t}}$; **Condone** $p \equiv t$	
	So at P, $m_T = -\frac{1}{t^2}$	Correct calculus work leading to $m_T = -\frac{1}{t^2}$	A1
	So, $m_N = t^2$	Applies $m_N = \frac{-1}{m_T}$, where m_T is found using calculus	M1
	• $y - \frac{4}{t} = "t^2"(x - 4t)$ • $\frac{4}{t} = "t^2"(4t) + c \Rightarrow y = "t^2"x +$ their c	Correct straight line method for an equation of a normal where $m_N\ (\neq m_T)$ is found by using calculus	M1
	Correct algebra leading to $ty - t^3x = 4 - 4t^4$ *	Correct solution only	A1 cso
			(5)
(b)	$\{t = 2 \Rightarrow\}$ **N**: $2y - 8x = 4 - 64\ \{\Rightarrow y = 4x - 30\}$	Uses $t = 2$ to find the equation of the normal to H at A	M1
	• $x(4x - 30) = 16\ \{\Rightarrow 2x^2 - 15x - 8 = 0\}$ • $\left(\frac{y + 30}{4}\right)y = 16\ \{\Rightarrow y^2 + 30y - 64 = 0\}$ • $\frac{4}{t} = 4(4t) - 30\ \{\Rightarrow 8t^2 - 15t - 2 = 0\}$	Substitutes the equation of the normal into the equation of the curve H to obtain an equation in x only or y only or t only	M1
	• $(x - 8)(2x + 1) = 0 \Rightarrow x_B = -\frac{1}{2}$ • $(y - 2)(y + 32) = 0 \Rightarrow y_B = -32$ • $(t - 2)(8t + 1) = 0 \Rightarrow t_B = -\frac{1}{8}$	**dependent on the first two M marks** Solves their 3 TQ = 0 to obtain a value for the x (or y) coordinate of B or a value of t at B	ddM1
	$B(-0.5, -32)$	Correct coordinates for B	A1
	$AB = \sqrt{(8 - -0.5)^2 + (2 - -32)^2}$	**dependent on the second M mark** Correct Pythagoras method to find the length of AB	dM1
	$= \frac{17\sqrt{17}}{2}$ or $\frac{\sqrt{4913}}{2}$ or $\sqrt{\frac{4913}{4}}$ or $\sqrt{1228.25}$	Correct exact length	A1
			(6)
(c)	$y - 2 = -\frac{1}{4}(x - 8)$ and $x = 0 \Rightarrow y_C = 2 + 2 = 4$	Finds the equation of the tangent at $(8, 2)$ to H, and sets $x = 0$ to find $y_C = ...$	M1
	$AC = \sqrt{(8 - 0)^2 + (2 - 4)^2}\ \{= \sqrt{68}\}$ Area $ABC = \frac{1}{2}\left(\frac{17\sqrt{17}}{2}\right)\left(\sqrt{68}\right)$	Uses the points $(8, 2)$, $(-0.5, -32)$ and $(0, 4)$ in a complete method to find the area of triangle ABC	M1
	$= 144.5$ or $\frac{289}{2}$	Correct answer	A1
			(3)
			14

	Question 8 Notes	
8. (b)	**Note**	The correct coordinates of B can be implied. e.g. embedded in the distance expression for AB
	Note	An incorrect **N**: $y = 4x + 30$ leads to the correct length AB for $A(-8, -2)$ and $B(0.5, 32)$
	Note	Condone final dM1 for $x_B = -\frac{1}{2}$ leading to $B(-2, -8)$ and $AB = \sqrt{(8 - -2)^2 + (2 - -8)^2}$
(c)	**Note**	Give 1st M0 for setting $x = 0$ in the equation of the normal to find $y_C = ...$
	Note	The 2nd M mark can only be gained by using all 3 correct points $(8, 2)$, $(-0.5, -32)$ and $(0, 4)$. Complete area methods include • Area $ABC = \frac{1}{2}\left(\frac{17\sqrt{17}}{2}\right)\left(\sqrt{68}\right)\{= 144.5\}$ • AB crosses y-axis at $(0, -30)$ and so Area $ABC = \frac{1}{2}(34)\left(\frac{1}{2}\right) + \frac{1}{2}(34)(8)\{= 8.5 + 136 = 144.5\}$ • Area $ABC = \frac{1}{2}\begin{vmatrix} 8 & -0.5 & 0 & 8 \\ 2 & -32 & 4 & 2 \end{vmatrix} = \frac{1}{2}\lvert(-256 - 2 + 0) - (-1 + 0 + 32)\rvert \left\{= \frac{1}{2}\lvert(-289)\rvert = 144.5\right\}$ • Area $ABC = (32 + 4)\left(\frac{1}{2} + 8\right) - \frac{1}{2}(32 + 2)\left(\frac{1}{2} + 8\right) - \frac{1}{2}(32 + 4)\left(\frac{1}{2}\right) - \frac{1}{2}(2)(8)$ $\{= 306 - 144.5 - 9 - 8 = 144.5\}$ • Area $ABC = \frac{1}{2}(8 + 8.5)(36) - \frac{1}{2}(32 + 2)\left(\frac{1}{2} + 8\right) - \frac{1}{2}(2)(8)$ $\{= 297 - 144.5 - 8 = 144.5\}$
	Note	**<u>Helpful Sketch</u>** 

Question Number	Scheme	Notes	Marks
9.	$\mathrm{f}(n)=7^n(3n+1)-1$ is a multiple of 9	$u_1=2,\ u_2=6,\ u_{n+2}=3u_{n+1}-2u_n \Rightarrow u_n=2(2^n-1)$	
(i) **Way 1**	$\mathrm{f}(1)=7(4)-1=27$ {is a multiple of 9}	$\mathrm{f}(1)=27$ is the minimum	B1
	$\mathrm{f}(k+1)-\mathrm{f}(k)=\underline{7^{k+1}(3(k+1)+1)-1}-(7^k(3k+1)-1)$	Attempts $\mathrm{f}(k+1)-\mathrm{f}(k)$	M1
		A correct expression for $\underline{\mathrm{f}(k+1)}$	A1
	$=7^{k+1}(3k+4)-1-7^k(3k+1)+1=7^k(21k+28)-7^k(3k+1)$		
	$=18k(7^k)+27(7^k)$ or $7^k(18k+27)$	**dependent on the previous M mark** Uses correct algebra to achieve an expression where each term is an obvious multiple of 9	dM1
	$\mathrm{f}(k+1)=9(7^k)(2k+3)+7^k(3k+1)-1$ or $\mathrm{f}(k+1)=18k(7^k)+27(7^k)+\mathrm{f}(k)$	Correct algebra leading to either e.g. $\mathrm{f}(k+1)=9(7^k)(2k+3)+7^k(3k+1)-1$ or $\mathrm{f}(k+1)=18k(7^k)+27(7^k)+\mathrm{f}(k)$	A1
	If the result is $\underline{\text{true for } n=k}$, then it is $\underline{\text{true for } n=k+1}$. As the result has been shown to be $\underline{\text{true for } n=1}$, then the result $\underline{\text{is true for all } n}$ $(\in\mathbb{Z}^+)$		A1 cso
			(6)
(i) **Way 2**	$\mathrm{f}(1)=7(4)-1=27$ {is a multiple of 9}	$\mathrm{f}(1)=27$ is the minimum	B1
	$\mathrm{f}(k+1)=7^{k+1}(3(k+1)+1)-1$	Attempts $\mathrm{f}(k+1)$	M1
		A correct expression for $\mathrm{f}(k+1)$	A1
	$=7^{k+1}(3k+4)-1=7^k(21k+28)-1$		
	$=18k(7^k)+27(7^k)+7^k(3k+1)-1$ **or** $=(7^k)(18k+27)+7^k(3k+1)-1$ **or** $=9(7^k)(2k+3)+7^k(3k+1)-1$	**dependent on the previous M mark** Uses correct algebra to express $\mathrm{f}(k+1)=\mathrm{g}(k)+7^k(3k+1)-1$ or $\mathrm{f}(k+1)=\mathrm{g}(k)+\mathrm{f}(k)$ where each term in $\mathrm{g}(k)$ is an obvious multiple of 9	dM1
		Correct algebra leading to either e.g. $\mathrm{f}(k+1)=9(7^k)(2k+3)+7^k(3k+1)-1$ or $\mathrm{f}(k+1)=18k(7^k)+27(7^k)+\mathrm{f}(k)$	A1
	If the result is $\underline{\text{true for } n=k}$, then it is $\underline{\text{true for } n=k+1}$. As the result has been shown to be $\underline{\text{true for } n=1}$, then the result $\underline{\text{is true for all } n}$ $(\in\mathbb{Z}^+)$		A1 cso
			(6)
(ii)	$\{n=1,\}\ u_1=2(2^1-1)=2$;	Checks that the general formula works for either u_1 or u_2	M1
	$\{n=2,\}\ u_2=2(2^2-1)=6$	Checks that the general formula works for both u_1 and u_2	A1
	$\{u_{k+2}=3u_{k+1}-2u_k \Rightarrow\}$ $u_{k+2}=3(2(2^{k+1}-1))-2(2(2^k-1))$	Finds u_{k+2} by attempting to substitute $u_{k+1}=2(2^{k+1}-1)$ and $u_k=2(2^k-1)$ into $u_{k+2}=3u_{k+1}-2u_k$ Condone one slip	M1
	$\{u_{k+2}\}=6(2^{k+1})-6-4(2^k)+4$		
	$\{u_{k+2}\}=3(2^{k+2})-2^{k+2}-2$	Valid evidence of working in the same power of 2	M1
	$=2(2^{k+2})-2=2(2^{k+2}-1)$	***Uses algebra*** in a complete method to achieve this result with no errors	A1
	If the result is $\underline{\text{true for } n=k \text{ and for } n=k+1}$, then it is $\underline{\text{true for } n=k+2}$. As the result has been shown to be $\underline{\text{true for } n=1 \text{ and } n=2}$, then the result $\underline{\text{is true for all } n}$ $(\in\mathbb{Z}^+)$		A1 **cso**
			(6)
			12

		Question 9 Notes
9. (i)	**Note**	**Final A1** is dependent on all previous marks being scored. It is gained by candidates conveying the ideas of **all** four underlined points **in part (i)** **either** at the end of their solution **or** as a narrative in their solution.
	Note	Shows $\mathrm{f}(k+1)-\mathrm{f}(k)=7^k(18k+27)$ **or** $\mathrm{f}(k+1)-\mathrm{f}(k)=9(7^k)(2k+3)$ and writing if $\mathrm{f}(k+1)-\mathrm{f}(k)=9(7^k)(2k+3)$ o.e. is a multiple of 9 then $\mathrm{f}(k+1)$ is a multiple of 9 is acceptable for the penultimate A mark in part (i). This means that the final A mark can potentially be available.
	Note	Only showing $\mathrm{f}(k+1)=\ 7\mathrm{f}(k)\ +6+21(7^k)$ (see Way 4) does not get the final dM mark because $6+21(7^k)$ is not an obvious multiple of 9
	Note	Allow dM1 for obtaining e.g. $\mathrm{f}(k+1)-\mathrm{f}(k)=18k(7^k)-27(7^k)$ or $\mathrm{f}(k+1)-\mathrm{f}(k)=7^k(18k-27)$
	Note	Allow dM1 for obtaining $\mathrm{f}(k+1)=18k(7^k)-27(7^k)+7^k(3k+1)-1$ or $\mathrm{f}(k+1)=9(7^k)(2k-3)+\mathrm{f}(k)$
(ii)	**Note**	**1st M1:** At least one check is correct. **1st A1:** Both checks are correct - Check 1: Shows $u_1=2$ by writing an intermediate step of e.g. $2(2^1-1)$ or 2×1 - Check 2: Shows $u_2=6$ by writing an intermediate step of e.g. $2(2^2-1)$ or 2×3
	Note	Ignore $u_3=3u_2-2u_1=3(6)-2(2)=14$ as part of their solution to (ii)
	Note	Ignore $\{n=3,\}$ $u_2=2(2^3-1)=14$ as part of their solution to (ii)
	Note	Valid evidence of working in the same power of 2 includes: - $6(2^{k+1})-4(2^k)\rightarrow6(2^{k+1})-2(2^{k+1})$ or $2(3(2^{k+1})-2^{k+1})$ - $3(2(2^{k+1}))-2(2(2^k))\ \rightarrow3(2^{k+2})-(2^{k+2})$ - $3(2(2^{k+1}))-2(2(2^k))\ \rightarrow12(2^k)-4(2^k)$ - $6(2^{k+1})-4(2^k)\rightarrow8(2^k)$ (by implication) - $6(2^{k+1})-4(2^k)\rightarrow4(2^{k+1})$ (by implication)
	Note	Writing $u_{k+2}=3(2(2^{k+1}-1))-2(2(2^k-1))=2(2^{k+2}-1)$ is 2nd M1, 3rd M0, 2nd A0
	Note	Showing $\{\text{RHS}=\}$ $u_{k+2}=2(2^{k+2}-1)=8(2^k)-2$ and writing $\{\text{LHS}=\}$ $u_{k+2}=3(2(2^{k+1}-1))-2(2(2^k-1))$ and using valid algebra to show that $u_{k+2}=8(2^k)-2$ $\{=\text{RHS}\}$ is fine for the 2nd M, 3rd M and 2nd A marks
	Note	**Final A1** is dependent on all previous marks being scored. It is gained by candidates conveying the ideas of **all** four underlined points **in part (ii)** **either** at the end of their solution **or** as a narrative in their solution.
	Note	"Assume for $n=k$, $u_k=2(2^k-1)$ and for $n=k+1$, $u_{k+1}=2(2^{k+1}-1)$" satisfies the requirement "true for $\boldsymbol{n=k}$ and $n=k+1$"
	Note	"For $n\in\mathbb{Z}^+$, $u_n=2(2^n-1)$" satisfies the requirement "true for all *n*"
	Note	Full marks in (ii) can be obtained for an equivalent proof where e.g. - $n=k,\ n=k+1,\ \rightarrow n=k-2,\ n=k-1$; i.e. $k\equiv k-2$ - $n=k,\ n=k+1,\ \rightarrow n=k-1,\ n=k$; i.e. $k\equiv k-1$
(i), (ii)	**Note**	Allow as part of their conclusion "true for all positive values of *n*"
	Note	Allow as part of their conclusion "true for all values of *n*"
	Note	Allow as part of their conclusion "true for all $n\in\mathbb{N}$"
	Note	Condone referring to *n* as any integer in their conclusion for the final A1
	Note	Condone $n\in\mathbb{Z}^*$ as part of their conclusion for the final A1
	Note	Referring to *n* as a real number their conclusion is final A0

Question Number	Scheme	Notes	Marks
9.	$\mathrm{f}(n) = 7^n(3n+1) - 1$ is a multiple of 9; $P \in \mathbb{Z}^+$		
(i) **Way 3**	$\mathrm{f}(1) = 7(4) - 1 = 27$ {is a multiple of 9}	$\mathrm{f}(1) = 27$ is the minimum	B1
	$\mathrm{f}(k+1) - (9P+1)\mathrm{f}(k)$	Attempts $\mathrm{f}(k+1) - (9P+1)\mathrm{f}(k)$	M1
	$= \underline{7^{k+1}(3(k+1)+1) - 1} - (9P+1)(7^k(3k+1) - 1)$	A correct expression for $\underline{\mathrm{f}(k+1)}$	A1
	$= 7^k(21k + 28 - (9P+1)(3k+1)) - 1 + 9P + 1$		
	$= 7^k(21k + 28 - (27Pk + 9P + 3k + 1)) - 1 + 9P + 1$		
	$= 7^k(21k + 28 - 27Pk - 9P - 3k - 1) + 9P$		
	$= 7^k(18k - 27Pk - 9P + 27) + 9P$	**dependent on the previous M mark** Uses correct algebra to achieve an expression where each term is an obvious multiple of 9	dM1
	$\mathrm{f}(k+1) = 7^k(18k - 27PK - 9P + 27) + 9P + (9P+1)\mathrm{f}(k)$	Achieves a correct result for $\mathrm{f}(k+1) = \ldots$	A1
	If the result is <u>true for $n = k$,</u> then it is <u>true for $n = k+1$.</u> As the result has been shown to be <u>true for $n = 1$,</u> then the result <u>is true for all n</u> $(\in \mathbb{Z}^+)$		A1 cso
			(6)
	Note: $P = 0 \Rightarrow \mathrm{f}(k+1) - \mathrm{f}(k) = 7^k(18k + 27)$ $P = 1 \Rightarrow \mathrm{f}(k+1) - 10\mathrm{f}(k) = 7^k(18 - 9k) + 9$ $P = 2 \Rightarrow \mathrm{f}(k+1) - 19\mathrm{f}(k) = 7^k(9 - 36k) + 18$ $P = 3 \Rightarrow \mathrm{f}(k+1) - 28\mathrm{f}(k) = 7^k(-63k) + 27 = 27 - 9k(7^{k+1})$		

Question Number	Scheme	Notes	Marks
9.	$\mathrm{f}(n) = 7^n(3n+1) - 1$ is a multiple of 9		
(i) **Way 4**	$\mathrm{f}(1) = 7(4) - 1 = 27$ {is a multiple of 9}	$\mathrm{f}(1) = 27$ is the minimum	B1
	$\mathrm{f}(k+1) = 7^{k+1}(3(k+1)+1) - 1$	Attempts $\mathrm{f}(k+1)$	M1
		A correct expression for $\mathrm{f}(k+1)$	A1
	$= 7(7^k)(3k+3+1) - 1$		
	$= 7(7^k)(3k+1) + 3(7)(7^k) - 1$		
	$= 7[(7^k)(3k+1) - 1] + 7 + 21(7^k) - 1$ $= 7\mathrm{f}(k) + 6 + 21(7^k)$ Let $\mathrm{g}(n) = 6 + 21(7^n)$ $\mathrm{g}(1) = 6 + 21(7^1) = 153$ {is a multiple of 9} {Assume the result is true for $\boldsymbol{n = k}$} $\mathrm{g}(k+1) = 6 + 21(7^{k+1})$ $= 6 + 147(7^k)$ $= 6 + 21(7^k) + 126(7^k)$ or $= \mathrm{g}(k) + 9(14)(7^k)$	**dependent on the previous M mark** Uses correct algebra to express $\mathrm{f}(k+1) = \alpha(7^k(3k+1) - 1) + \mathrm{g}(k)$ or $\mathrm{f}(k+1) = \alpha\mathrm{f}(k) + \mathrm{g}(k);\ \alpha \neq 0$ and uses correct algebra to achieve an expression for $\mathrm{g}(k+1)$ where each term is an obvious multiple of 9	M1
		Correct algebra leading to $\mathrm{f}(k+1) = 7\mathrm{f}(k) + 6 + 21(7^k)$ o.e. and $\mathrm{g}(k+1) = 6 + 21(7^k) + 126(7^k)$ where $\mathrm{g}(n) = 6 + 21(7^n)$	A1
	Proves that $\mathrm{g}(n) = 6 + 21(7^n)$ is a multiple of 9 and proves that for $\mathrm{f}(n)$ if the result is <u>true for $n = k$,</u> then it is <u>true for $n = k+1$.</u> As the result has been shown to be <u>true for $n = 1$,</u> then the result <u>is true for all n</u> $(\in \mathbb{Z}^+)$		A1 cso
			(6)
	Note: An alternative Way 4 method shows • $\mathrm{f}(k+1) = 7\mathrm{f}(k) + 6 + 21(7^k) = 7\mathrm{f}(k) + 9(7^k + 1) + 3(7^k) - 3$ • Defines $\mathrm{g}(n) = 3(7^n) - 3$ and proceeds to show that $\mathrm{g}(n)$ is also a multiple of 9		

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